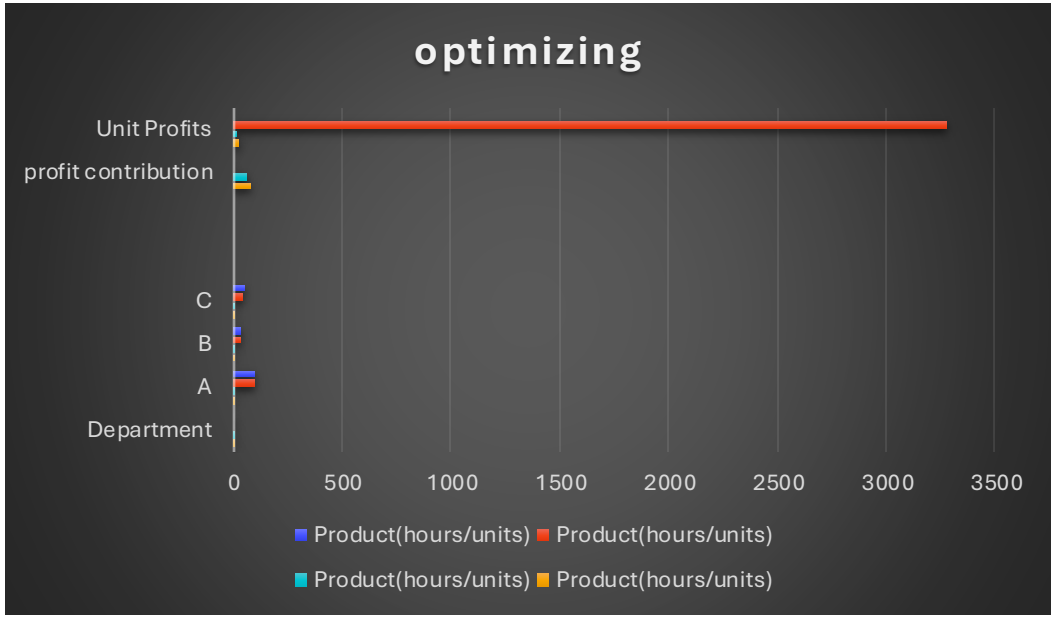
Optimizing Production With Linear Programming: A Case Study From Hartman Company

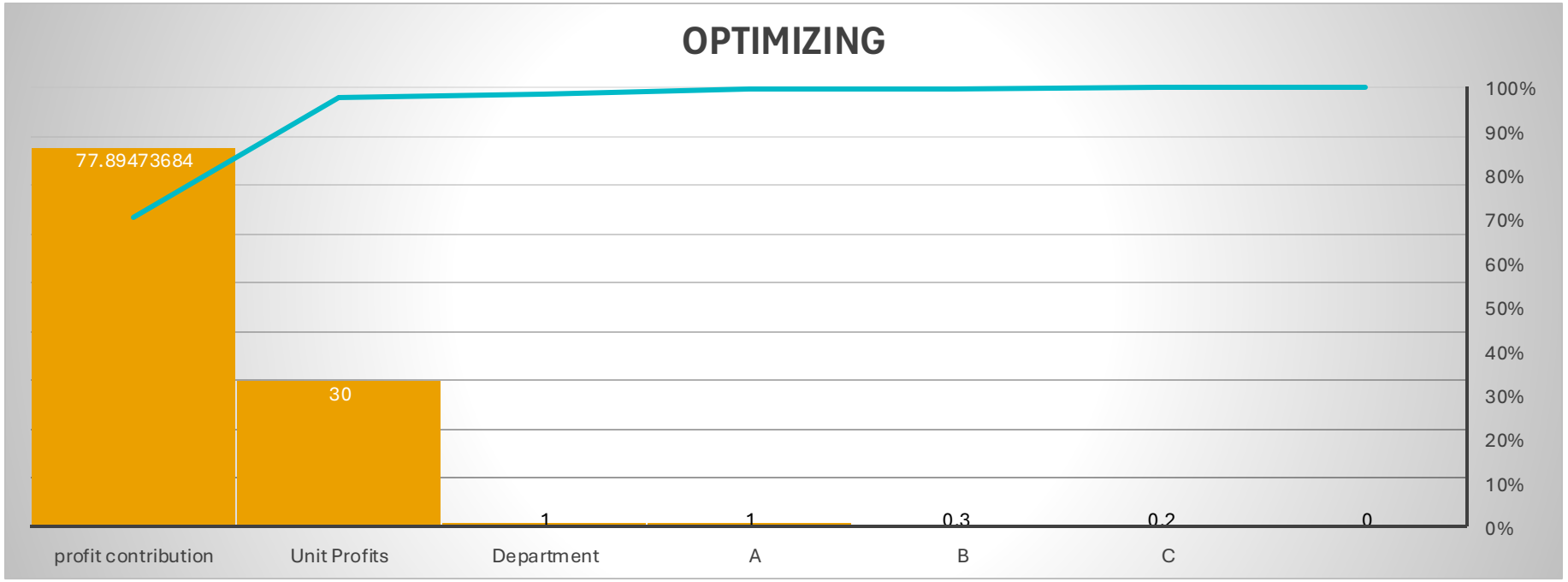
Optimizing Production with Linear Programming:

- At Hartman Company, we recently tackled a classic operations research challenge: **how to maximize profit while managing labor constraints across three departments**.

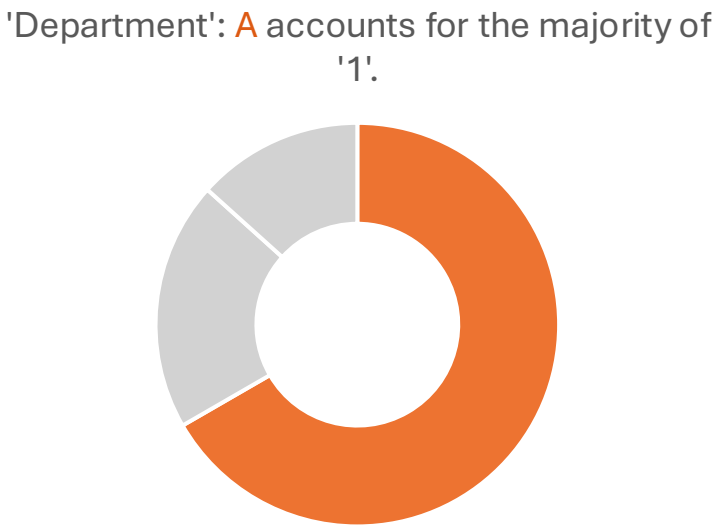
	A	B	C	D	E
1					
2					
3		Product(hours/units)			
4	**Department**	**1**	**2**	**Used**	**Available**
5	**A**	**1**	**0.35**	**100**	**100**
6	**B**	**0.3**	**0.2**	**36**	**36**
7	**C**	**0.2**	**0.5**	**47.1578947**	**50**
8					
9					
10	**profit contribution**	**78**	**63**	**Total Profit**	
11	**Unit Profits**	**$30**	**$15**	**$3,284**	
12					

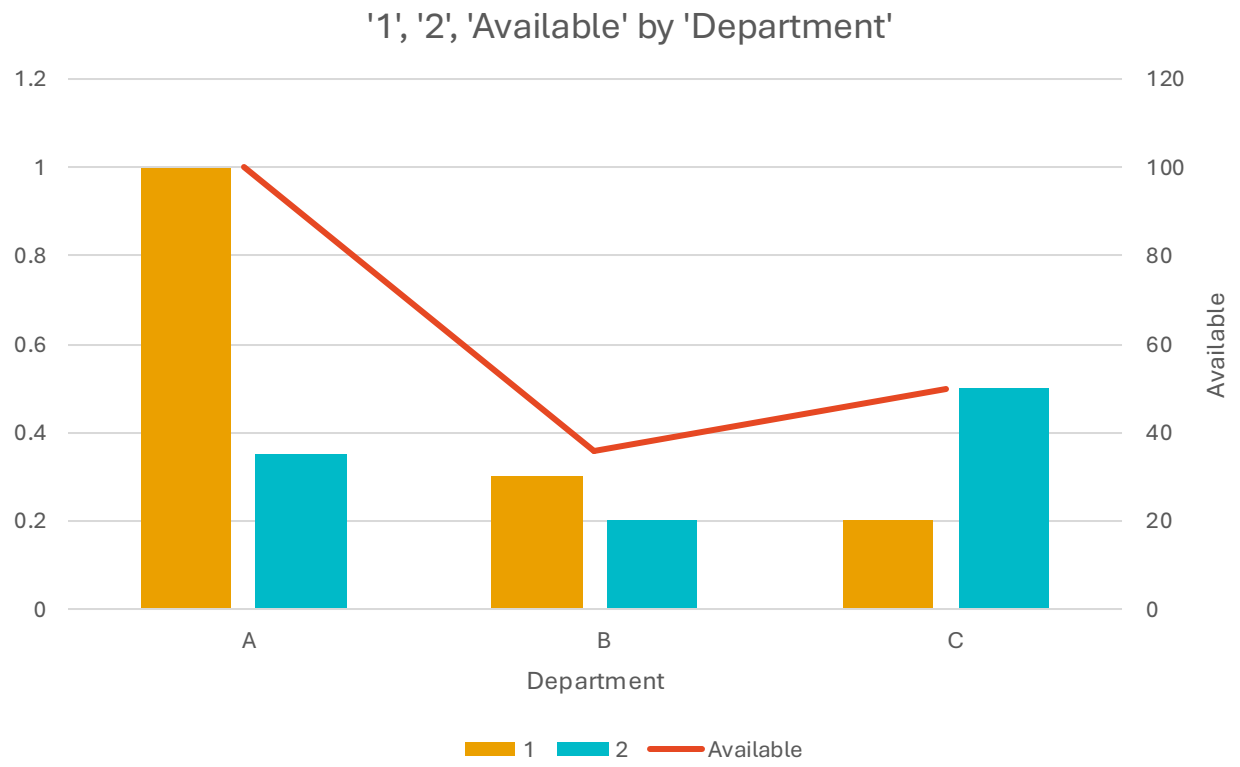
Linear Programming Model (No Overtime)
Objective Function:
Maximize
Z=30X1+15X2Z=30$X$1+15$X$2
Subject to labor constraints:
Department A: 1X1+0.35X2≤1001$X$1+0.35$X$2≤100
Department B: 0.3X1+0.2X2≤360.3$X$1+0.2$X$2≤36
Department C: 0.2X1+0.5X2≤500.2$X$1+0.5$X$2≤50
X1,X2≥0$X$1,$X$2≥0
Solution:
Product 1: 78 units
Product 2: 63 units
Total Profit: $3,284



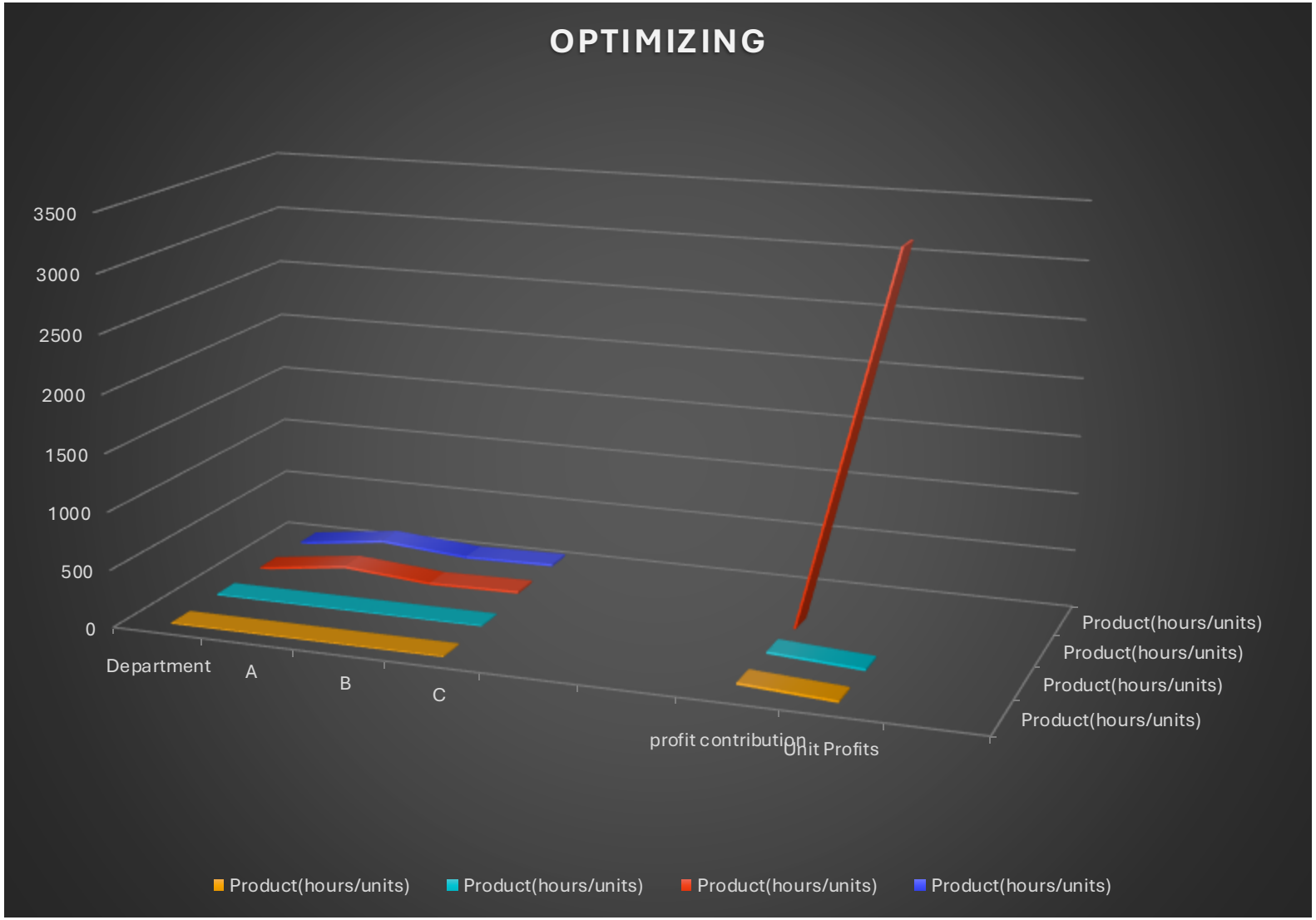


Department	Sum of 1
A	1
B	0.3
C	0.2
Grand Total	1.5





🔍 **The Problem:** We needed to determine the optimal production quantities for two products, considering:
Labor availability in Departments A, B, and C
Profit contributions of $30 and $15 per unit
Fixed labor hours and potential for paid overtime



Sensitivity Analysis (No Overtime)

💡 **The Solution: Using linear programming and Solver, we developed and analyzed multiple scenarios: overtime recommendations based on the Sensitivity Report:**
from the constraint table, we analyze the shadow prices to determine which departments would benefit from overtime and how much we should pay per extra labor hour:
department A: shadow price: $15.79, adding one more hour of labor increases profit by 15.79.
department B: shadow price: $47.37, adding one more hour of labor increases profit by 47.37.
department C: shadow price: $0, no increase.
I recommend department B: highest shadow price: $47.37, can pay up to 47.37 per hour for overtime.

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$D$11	Unit Profits Total Profit	$0	$3,284

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$10	profit contribution Product(hours/units)	0	78	Contin
$C$10	profit contribution	0	63	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$5	A Used	100	$D$5<=$E$5	Binding	0
$D$6	B Used	36	$D$6<=$E$6	Binding	0
$D$7	C Used	47.15789474	$D$7<=$E$7	Not Binding	2.842105263

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$10	profit contribution Product(hours/units)	77.8947368	0	30	12.8571429	7.5
$C$10	profit contribution	63.1578947	0	15	5	4.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$5	A Used	100	15.7894737	100	20	2.45454545
$D$6	B Used	36	47.3684211	36	0.62790698	6
$D$7	C Used	47.1578947	0	50	1E+30	2.84210526

Extended Model with Overtime

Overtime is allowed with the following limits and costs:

- **New Objective Function**: Maximize Z=30X1+15X2−18oA−22.5oB−12oC*Z*=30*X*1+15*X*2−18*oA*−22.5*oB*−12*oC*
- **Solution**:
- Product 1: 81 units
- Product 2: 84 units
- Gross Profit: $3,677
- Overtime Cost: $276
- **Net Profit**: $3,400.5

	A	B	C	D	E
3					
4		Product(hours/units)			
5	**Department**	**1**	**2**	**Used**	**Available**
6	**A**	**1**	**0.35**	**110**	**110**
7	**B**	**0.3**	**0.2**	**40.95348837**	**42**
8	**C**	**0.2**	**0.5**	**58**	**58**
9					
10					
11	**profit contrib**	**81**	**84**	**Total Profit**	
12	**Unit Profits**	**$30**	**$15**	**$3,677**	

Updated Sensitivity Analysis (With Overtime)

🔍 **Insight: Overtime in Department A is highly profitable. Department C is marginally profitable. Department B should avoid overtime.**

Department	Shadow Price ($/hr)	Overtime Cost	Profitable?
A	27.91	18	✅ Yes
B	0	22.5	❌ No
C	10.47	12	✅ Yes (marginal)

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$D$12	Unit Profits Total Profit	$0	$3,677

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$11	profit contribution Product(hours/units)	0	81	Contin
$C$11	profit contribution	0	84	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$6	A Used	110	$D$6<=$E$6	Binding	0
$D$7	B Used	40.95348837	$D$7<=$E$7	Not Binding	1.046511628
$D$8	C Used	58	$D$8<=$E$8	Binding	0

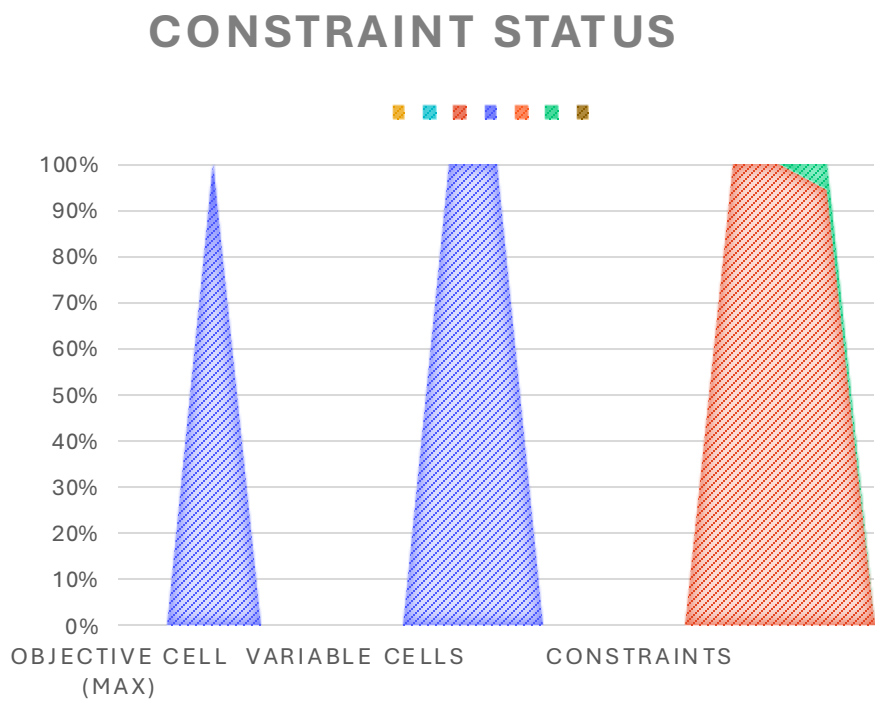
Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$11	profit contribution Product(hours/units)	80.6976744	0	30	12.8571429	24
$C$11	profit contribution	83.7209302	0	15	60	4.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$6	A Used	110	27.9069767	110	4.09090909	69.4
$D$7	B Used	40.9534884	0	42	1E+30	1.04651163
$D$8	C Used	58	10.4651163	58	4.73684211	36

- overtime recommendations based on the Sensitivity Report:
- **Shadow Prices & Recommendations for Overtime:**
- department A: shadow price: $27.91, adding one more hour of increases profit by 27.91.
- department B: shadow price: 0,
- department C: shadow price: $10.47, adding one more hour of increases profit by 10.47.
- Pay for overtime in Dept A up to $27.91 per hour (cost is $18, so it's profitable).
- **Without Overtime:** $3,677
- **With Overtime (A = 10 hours, C = 8 hours):** $3,400.5



- 📈 **Visual Insights**
- **Profit comparison charts**
- **Labor usage vs availability**
- **Profit vs overtime hours**
- **Shadow price and profitability tables**
- **Slack and constraint binding tables**