

Advanced Mathematics

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions.

- 1 a. If $Z = \frac{(2+3i)(1-i)}{(1+i)}$, find $z + \bar{z}$ and $z\bar{z}$. (06 Marks)
- b. Express the complex number $1 + \sqrt{3}i$ in the polar form and exponential form. (07 Marks)
- c. Find the modulus and amplitude of $\frac{(1+2i)(2-i)}{4+3i}$. (07 Marks)
- 2 a. Find the n^{th} derivative of $y = \frac{x+2}{x+1} + \log\left(\frac{x+2}{x+1}\right) + \cos^2 2x$. (06 Marks)
- b. If $y = a \cos(\log x) + b \sin(\log x)$, show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. (07 Marks)
- c. Find the angle between the curves $\gamma = a(1 + \cos \theta)$ and $\gamma = a(1 - \cos \theta)$. (07 Marks)
- 3 a. Obtain the Maclaurin's series for $\sqrt{1 + \sin 2x}$ upto the term involving x^4 . (06 Marks)
- b. If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x+y}\right)$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$. (07 Marks)
- c. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$, evaluate $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at $(1, -1, 0)$. (07 Marks)
- 4 a. If ϕ is the angle between the radius vector and tangent to the polar curve $r = t(\theta)$ at (r, θ) , show that $\tan \phi = r \frac{d\theta}{dr}$. (06 Marks)
- b. If $v = f(x-y, y-z, z-x)$, show that $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} = 0$. (07 Marks)
- c. State and prove Euler's theorem for a homogeneous function u in x and y of degree n . (07 Marks)
- 5 a. Obtain the reduction formula for $\int \sin^n x dx$ and hence evaluate $\int \sin^3 x dx$. (06 Marks)
- b. Evaluate $\int_0^{2a} x^2 \sqrt{2ax - x^2} dx$. (07 Marks)
- c. Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dy dx$. (07 Marks)

- 6 a. Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z) dy dx dz$. (06 Marks)
- b. Show that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (07 Marks)
- c. Evaluate $\int_0^{\pi/2} \sqrt{\tan \theta} d\theta$. (07 Marks)
- 7 a. Solve $e^x \tan y dx + (1+e^x) \sec^2 y dy = 0$. (06 Marks)
- b. Solve $x \frac{dy}{dx} + y = x^3 y^6$. (07 Marks)
- c. Solve $(3x^2 + 6xy^2) dx + (6x^2y + 4y^3) dy = 0$. (07 Marks)
- 8 a. Solve $\frac{d^3y}{dx^3} + 4 \frac{dy}{dx} = \sin 2x$. (06 Marks)
- b. Solve $(D-2)^2 y = 8(e^{2x} + x^2)$. (07 Marks)
- c. Solve $\frac{d^2y}{dx^2} + 5 \frac{dy}{dx} + 6y = e^{-2x} \cos 2x$. (07 Marks)
