(5) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\,dy\,dx$.

$$= \int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\,dy\,dx = \int_0^a \int_0^x \int_0^{x+y} e^x e^y e^z\, dz\,dy\,dx$$

$$= \int_0^a \int_0^x e^x e^y e^z \Big]_0^{x+y} dy\,dx$$

$$= \int_0^a \int_0^x e^x e^y \left[e^{x+y} - e^0\right] dy\,dx$$

$$= \int_0^a \int_0^x \left(e^{2x} e^{2y} - e^x e^y\right) dy\,dx.$$

$$= \int_0^a e^{2x} \frac{e^{2y}}{2} - e^x e^y \Big]_0^x dx$$

$$= \int_0^a \left\{ \frac{e^{2x}}{2}\left[e^{2x} - e^0\right] - e^x\left[e^x - e^0\right]\right\} dx$$

$$= \int_0^a \left[\frac{1}{2} e^{4x} - \frac{1}{2} e^{2x} - e^{2x} + e^x\right] dx$$

$$= \int_0^a \left[\frac{1}{2} e^{4x} - \frac{3}{2} e^{2x} + e^x\right] dx$$

$$= \frac{1}{2}\frac{e^{4x}}{4} - \frac{3}{2}\frac{e^{2x}}{2} + e^x \Big]_0^a$$

$$= \frac{1}{8}\left(e^{4a} - 1\right) - \frac{3}{4}\left(e^{2a} - 1\right) + \left(e^a - 1\right)$$

$$= \frac{1}{8} e^{4a} - \frac{3}{4} e^{2a} + e^a - \frac{3}{8}$$