USN | | | | | | | | | | | **MATDIP301**

Third Semester B.E. Degree Examination, December 2011

Advanced Mathematics - I

Time: 3 hrs.   Max. Marks:100

Note: *Answer any FIVE full questions.*

1   a.   Express $\dfrac{1}{(2+i)^2} - \dfrac{1}{(2-i)^2}$ in the form $a + ib$. **(06 Marks)**

   b.   Find the modulus and amplitude of $\dfrac{(3-\sqrt{2}i)^2}{1+2i}$. **(07 Marks)**

   c.   Find the real part of $\dfrac{1}{1+\cos\theta + i\sin\theta}$. **(07 Marks)**

2   a.   Find the nth derivative of $\cos x \cos 2x \cos 3x$. **(06 Marks)**

   b.   If $y = (\sin^{-1} x)^2$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$. **(07 Marks)**

   c.   Find the nth derivative of $\dfrac{x+2}{x+1} + \log\left(\dfrac{x+2}{x+1}\right)$. **(07 Marks)**

3   a.   State and prove Euler's theorem. **(06 Marks)**

   b.   Given $u = \sin\left(\dfrac{x}{y}\right)$, $x = e^t$, $y = t^2$, find $\dfrac{du}{dt}$ as a function of t. **(07 Marks)**

   c.   If $x = r\cos\theta$, $y = r\sin\theta$, find $\dfrac{\partial(x,y)}{\partial(r,\theta)}$ and $\dfrac{\partial(r,\theta)}{\partial(x,y)}$. **(07 Marks)**

4   a.   Find the angle of intersection of the curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$. **(06 Marks)**

   b.   Find the pedal equation of the curve $\dfrac{2a}{r} = 1 - \cos\theta$. **(07 Marks)**

   c.   Expand $e^{\sin x}$ by Maclaurin's series upto the term containing x^4. **(07 Marks)**

5   a.   Obtain the reduction formula for $I_n = \int_0^{\pi/2} \sin^n x\, dx$ where n is a positive integer. **(06 Marks)**

   b.   Evaluate : $\int_1^5 \int_1^{x^2} x(x^2 + y^2)\, dx\, dy$. **(07 Marks)**

   c.   Evaluate : $\int_0^1 \int_0^2 \int_1^2 x^2 yz\, dx\, dy\, dz$. **(07 Marks)**

6 a. Prove that $\beta(m, n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (06 Marks)

   b. Show that $\Gamma(n) = \int\limits_{0}^{1}\left(\log\dfrac{1}{x}\right)^{n-1} dx$. (07 Marks)

   c. Express $\int\limits_{0}^{\pi/2}\sqrt{\tan\theta}\, d\theta$ in terms of Gamma function. (07 Marks)

7 a. Solve : $\dfrac{dy}{dx} = \dfrac{x(2\log x + 1)}{\sin y + y\cos y}$. (06 Marks)

   b. Solve : $(1 + e^{x/y})dx + e^{x/y}\left(1 - \dfrac{x}{y}\right)dy = 0$. (07 Marks)

   c. Solve : $(x^2 - ay)dx = (ax - y^2)dy$. (07 Marks)

8 a. Solve : $\dfrac{d^4 y}{dx^4} + 8\dfrac{d^2 y}{dx^2} + 16y = 0$. (06 Marks)

   b. Solve : $(D - 2)^2 y = 8(e^{2x} + \sin 2x)$. (07 Marks)

   c. Solve : $(D^3 + 4D)y = \sin 2x$. (07 Marks)

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