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MATDIP301

Third Semester B.E. Degree Examination, Dec.2014/Jan.2015

Advanced Mathematics – I

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions.

1. a. Express : $\frac{(3+i)(1-3i)}{2+i}$ in the form $x + iy$. (05 Marks)
 b. Find the modulus and amplitude of the complex number $1 + \cos \alpha + i \sin \alpha$. (05 Marks)
 c. If $(3x - 2iy)(2 + i)^2 = 10(1 + i)$, then find the values of x and y . (05 Marks)
 d. Prove that $(\cos \theta_1 + i \sin \theta_1)(\cos \theta_2 + i \sin \theta_2) = \cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)$. (05 Marks)
2. a. Find the n^{th} derivative of $e^{ax} \cos(bx + c)$. (06 Marks)
 b. If $y = a \cos(\log x) + b \sin(\log x)$ prove that $x^2 y_{n+2} + (2n + 1)xy_{n+1} + (n^2 + 1)y_n = 0$. (07 Marks)
 c. Compute the n^{th} derivatives of $\sin x \sin 2x \sin 3x$. (07 Marks)
3. a. With usual notations prove that $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} \left(\frac{dr}{d\theta} \right)^2$. (06 Marks)
 b. Prove that the curves cuts $r^n = a^n \cos n\theta$, and $r^n = b^n \sin n\theta$ orthogonally. (07 Marks)
 c. Expand $\log(1 + \sin x)$ in powers of x by Maclaurin's theorem up to the terms containing x^3 . (07 Marks)
4. a. If $u = x^2y + y^2z + z^2x$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = (x + y + z)^2$. (06 Marks)
 b. If $u = f(x - y, y - z, z - x)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (07 Marks)
 c. If $u = e^x \cos y$, $v = e^x \sin y$, find $J = \frac{\partial(u,v)}{\partial(x,y)}$, $J' = \frac{\partial(x,y)}{\partial(u,v)}$ and verify $JJ' = 1$. (07 Marks)
5. a. Obtain a reduction formula for $\int \sin^n x \, dx$. (06 Marks)
 b. Evaluate : $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dx dy$. (07 Marks)
 c. Evaluate : $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz dy dx$. (07 Marks)
6. a. Define Gamma function. Prove that $\Gamma(n + 1) = n\Gamma(n)$. (06 Marks)
 b. With usual notation prove that : $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (07 Marks)
 c. Prove that $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1} \beta(m, m)$. (07 Marks)

- 7 a. Solve : $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$. (05 Marks)
- b. Solve : $\frac{dy}{dx} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$. (05 Marks)
- c. Solve : $\frac{dy}{dx} + y \cot x = \sin x$. (05 Marks)
- d. Solve : $(x^2 + y)dx + (y^3 + x)dy = 0$. (05 Marks)
- 8 a. Solve : $\frac{d^3 y}{dx^3} - 6\frac{d^2 y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$. (06 Marks)
- b. Solve : $y'' - 6y' + 9y = e^x + 3^x$. (07 Marks)
- c. Solve : $\frac{d^2 y}{dx^2} + 4y = x^2 + \sin 3x$. (07 Marks)

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