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① modulus & amplitude of each of the complex numbers.

①  $z = 1 + \cos \theta + i \sin \theta$

$$= 2 \cos^2 \left[ \frac{\theta}{2} \right] + i 2 \sin \left[ \frac{\theta}{2} \right] \cdot \cos \left[ \frac{\theta}{2} \right]$$

$$z = 2 \cos \left[ \frac{\theta}{2} \right] \left[ \cos \left[ \frac{\theta}{2} \right] + i \sin \left[ \frac{\theta}{2} \right] \right]$$

therefore  $|z| = 2 \cos \left[ \frac{\theta}{2} \right]$  and  $\arg z = \left[ \frac{\theta}{2} \right]$

⑤ Evaluate  $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} xy \, dy \, dx$

Let  $I = \int_0^{2a} \int_0^{\sqrt{2ax-x^2}} xy \, dy \, dx$

$$= \int_0^{2a} x \left[ \frac{y^2}{2} \right]_0^{\sqrt{2ax-x^2}} dx = \frac{1}{2} \int_0^{2a} x [(2ax-x^2) - 0] dx$$

$$= \frac{1}{2} \int_0^{2a} (2ax^2 - x^3) dx = \frac{1}{2} \left[ 2a \frac{x^3}{3} - \frac{x^4}{4} \right]_0^{2a}$$

$$= \frac{1}{2} \left[ \frac{2a}{3} [(2a)^3 - 0] - \frac{1}{4} [(2a)^4 - 0] \right]$$

$$= \frac{1}{2} \left[ \frac{16a^4}{3} - \frac{16a^4}{4} \right] = \frac{16a^4}{2} \left[ \frac{1}{3} - \frac{1}{4} \right]$$

$$= 8a^4 \left[ \frac{4-3}{12} \right] = \frac{8a^4}{3}$$