

③ $r = a(1 + \cos \theta)$

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$$\frac{dr}{d\theta} = a(-\sin \theta)$$

We have $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} \left[\frac{dr}{d\theta} \right]^2$

$$= \frac{1}{r^2} + \frac{1}{r^4} (a \sin \theta)^2$$

$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} (a \sin \theta)^2$

$$= \frac{1}{r^2} + \frac{a^2}{r^4} (\sin^2 \theta)$$

$$\boxed{\frac{1}{p^2} = \frac{1}{r^2} + \frac{a^2}{r^4} (1 - \cos^2 \theta)} \quad \text{--- (1)}$$

Given $r = a(1 + \cos \theta)$

$$\frac{r}{a} = (1 + \cos \theta)$$

$$\cos \theta = \frac{1 + r}{a} = \left[\frac{a + r}{a} \right] \quad \text{--- (2)}$$

Substituting eq (2) in (1) we get.

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{a^2}{r^4} \left[1 - \left(\frac{a + r}{a} \right)^2 \right]$$

$$= \frac{1}{r^2} + \frac{a^2}{r^4} \left[1 - \frac{(a + r)^2}{a^2} \right]$$

$$= \frac{1}{r^2} + \frac{a^2}{r^4} \left[\frac{a^2 - (a^2 + r^2 + 2ar)}{a^2} \right]$$

$$= \frac{1}{r^2} + \frac{a^2}{r^4} \left[\frac{-a^2 - r^2 + 2ar}{a^2} \right]$$

$$= \frac{1}{r^2} + \frac{2ar}{r^4} - \frac{r^2}{r^4} = \frac{1}{r^2} + \frac{2a}{r^3} - \frac{1}{r^2}$$

$$\frac{1}{p^2} = \frac{2a}{r^3}$$

$$\boxed{2ap^2 = r^3}$$