$$\frac{x}{(x-1)(2x+3)} = \frac{A}{(x-1)} + \frac{B}{(2x+3)}$$

$$\Rightarrow x = A(2x+3) + B(x-1)$$

Put $\boxed{x=1} \Rightarrow 1 = 5A \Rightarrow A = 1/5$

Put $x = -3/2 \Rightarrow -3/2 = -5/2\, B \Rightarrow B = 3/5$

$$\therefore \frac{x}{(x-1)(2x+3)} = \frac{1}{5(x-1)} + \frac{3}{5(2x+3)}$$

$$\therefore y = \frac{1}{5(x-1)} + \frac{3}{5(2x+3)}$$

Differentiating n times, we get.

$$y_n = \frac{1}{5}\frac{(-1)^n n!}{(x-1)^{n+1}} + \frac{3}{5}\frac{(-1)^n n!\, 2^n}{(2x+3)^{n+1}}$$