

$$(5) \int_1^2 \int_0^{2-y} xy \, dx \, dy$$

Evaluate.

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$$\text{let } I = \int_1^2 \int_0^{2-y} xy \, dx \, dy$$

$$= \int_1^2 \left[y \frac{x^2}{2} \right]_0^{2-y} dy$$

$$= \frac{1}{2} \int_1^2 y [(2-y)^2 - 0] dy$$

$$= \frac{1}{2} \int_1^2 y [4 + y^2 - 4y] dy$$

$$= \frac{1}{2} \int_1^2 (4y + y^3 - 4y^2) dy$$

$$= \frac{1}{2} \left[\frac{4y^2}{2} + \frac{y^4}{4} - \frac{4y^3}{3} \right]_1^2$$

$$= \frac{1}{2} \left[2(2^2 - 1^2) + \frac{1}{4}(2^4 - 1^4) - \frac{4}{3}(2^3 - 1^3) \right]$$

$$= \frac{1}{2} \left[(8 - 2) + \frac{1}{4}(16 - 1) - \frac{4}{3}(8 - 1) \right]$$

$$= \frac{5}{24}$$