

② Find pedal equation to curve $r = a(1 + \sin \theta)$

Q.4 $r = a(1 + \sin \theta)$

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$$\boxed{\frac{dr}{d\theta} = a \cos \theta}$$

We have, $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} \left(\frac{dr}{d\theta} \right)^2$

$$= \frac{1}{r^2} + \frac{1}{r^4} (a \cos \theta)^2$$

$$= \frac{1}{r^2} + \frac{1}{r^4} (a^2 \cos^2 \theta)$$

$$\boxed{\frac{1}{p^2} = \frac{1}{r^2} + \frac{a^2}{r^4} (1 - \sin^2 \theta)} \quad \text{--- (1)}$$

Given, $r = a(1 + \sin \theta)$

$$\frac{r}{a} = 1 + \sin \theta$$

$$\sin \theta = \frac{r}{a} - 1 = \frac{r - a}{a} \quad \text{--- (2)}$$

Substituting (2) in (1) we get

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{a^2}{r^4} \left[1 - \left(\frac{r - a}{a} \right)^2 \right]$$

$$= \frac{1}{r^2} + \frac{a^2}{r^4} \left[1 - \frac{(r - a)^2}{a^2} \right]$$

$$= \frac{1}{r^2} + \frac{a^2}{r^4} \left[\frac{a^2 - (r^2 + a^2 - 2ar)}{a^2} \right]$$

$$= \frac{1}{r^2} + \frac{a^2 + 2ar - r^2 - r^2}{r^4}$$

$$= \frac{1}{r^2} + \frac{2ar}{r^4} - \frac{r^2}{r^4} \cdot \frac{1}{r^2} = \frac{1}{r^2} + \frac{2ar}{r^4} - \frac{1}{r^2}$$

$$\frac{1}{p^2} = \frac{2a}{r^3}$$

$$\boxed{2ap^2 = r^3}$$