

MATDIP301

Third Semester B.E. Degree Examination, Dec.2016/Jan.2017
Advanced Mathematics - I

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions.

- 1 a. Express the $\frac{3}{1+i} - \frac{1}{2-i} + \frac{1}{1-i}$ in the form of $a + ib$. (06 Marks)
 b. Find the cube roots of $1 - i$. (07 Marks)
 c. Prove that $\left(\frac{1 + \cos \theta + i \sin \theta}{1 + \cos \theta - i \sin \theta}\right)^n = \cos n \theta + i \sin n \theta$. (07 Marks)
- 2 a. Find the n th derivative of $e^{ax} \cos(bx + c)$. (07 Marks)
 b. Find the n th derivative of $\frac{x}{(x-1)(2x+3)}$. (06 Marks)
 c. If $y = a \cos(\log x) + b \sin(\log x)$ prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. (07 Marks)
- 3 a. With usual notations P.T $\tan \phi = \frac{rd\theta}{dr}$. (06 Marks)
 b. Find the angle between the pairs of curves
 $r = a \log \theta$ $r = \frac{a}{\log \theta}$. (07 Marks)
 c. Find the Pedal equation to the curve $r = a(1 + \sin \theta)$. (07 Marks)
- 4 a. State and prove Euler's theorem of Homogeneous functions. (06 Marks)
 b. If $u = f(x-y, y-z, z-x)$
 P.T $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (07 Marks)
 c. If $u = \tan^{-1} x + \tan^{-1} y$, $V = \frac{x+y}{1-xy}$
 S.T $\frac{\partial(u,v)}{\partial(x,y)} = 0$. (07 Marks)
- 5 a. Obtain the Reduction formula for $\int \sin^m x \cos^n x \, dx$. Where m, n are positive integers. (07 Marks)
 b. Evaluate $\int_1^2 \int_0^{2-y} xy \, dx \, dy$. (06 Marks)
 c. Evaluate $\int_0^3 \int_0^2 \int_0^1 (x+y+z) \, dz \, dx \, dy$. (07 Marks)
- 6 a. Prove that $\left(\frac{1}{2}\right) = \sqrt{\pi}$. (06 Marks)
 b. Prove that $\int_0^x x^2 e^{-x^4} \, dx \times \int_0^x e^{-x^4} \, dx = \frac{\pi}{8\sqrt{2}}$. (07 Marks)

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- c. Evaluate the Integral $\int_0^1 x^5(1-x)^6 dx$. (07 Marks)
- 7 a. Solve $(D^3 - 3D - 2)y = 0$. (06 Marks)
b. Solve $(y'' + y) = e^{-x} + \cos x + x^3$. (07 Marks)
c. Solve $y'' - 2y' + y = xe^x \sin x$. (07 Marks)
- 8 a. Solve $\frac{dy}{dx} = \frac{x(2 \log x + 1)}{\sin y + y \cos y}$. (06 Marks)
b. Solve $x \log x \frac{dy}{dx} + y = 2 \log x$. (07 Marks)
c. Solve $(2xy + y - \tan y) dx + (x^2 - x \tan^2 y + \sec^2 y) dy = 0$. (07 Marks)
