USN | | | | | | | | | | MATDIP301

Third Semester B.E. Degree Examination, June/July 2015
Advanced Mathematics – I

Time: 3 hrs.   Max. Marks:100

Note: Answer any FIVE full questions.

1 a. Express the complex number
$\dfrac{(5-3i)(2+i)}{4+2i}$ in the form x + iy.   (06 Marks)

   b. Find the modulus and the amplitude of $1 + \cos\theta + i\sin\theta$.   (07 Marks)

   c. Find the cube roots of $1 + i$.   (07 Marks)

2 a. Find the nth derivative of $e^{ax}\cos(bx+c)$.   (06 Marks)

   b. Find the nth derivative of $\dfrac{x}{(x+1)(2x+3)}$.   (07 Marks)

   c. If $x = \tan(\log y)$ prove that $(1+x^2)y_{n+1} + (2nx-1)y_n + n(n-1)y_{n-1} = 0$. (07 Marks)

3 a. Find the angle of intersection of the curves $r^n = a^n\cos n\theta$, $r^n = b^n\sin n\theta$. (06 Marks)

   b. Find the Pedal equation of the curve $r = a(1-\cos\theta)$.   (07 Marks)

   c. Using Maclcaurin's series expand $\log(1+x)$ upto the term containing x^4.   (07 Marks)

4 a. If $u = f(x+ct) + g(x-ct)$ show that $\dfrac{\partial^2 u}{\partial t^2} = c^2\dfrac{\partial^2 u}{\partial x^2}$. (06 Marks)

   b. If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$ prove that $xu_x + yu_y + zu_z = 0$. (07 Marks)

   c. If $u = x+y$, $v = y+z$, $w = z+x$ find the value of $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$. (07 Marks)

5 a. Obtain the reduction formula for $\int\cos^n x\,dx$ where n is a positive integer. (06 Marks)

   b. Evaluate $\displaystyle\int_0^a \dfrac{x^4}{\sqrt{a^2-x^2}}dx$.   (07 Marks)

   c. Evaluate $\displaystyle\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$.   (07 Marks)

6 a. Define beta and gamma functions and prove that $\Gamma(n+1) = n\Gamma(n)$.   (06 Marks)

   b. Show that $\displaystyle\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta \times \int_0^{\pi/2}\dfrac{1}{\sqrt{\sin\theta}}\,d\theta = \pi$. (07 Marks)

   c. Prove that $\beta(m,n) = \dfrac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$.   (07 Marks)

7 a. Solve : $\dfrac{dy}{dx} = \cos(x + y + 1)$. **(06 Marks)**

   b. Solve : $(x^2 - y^2)\,dx - xy\,dy = 0$. **(07 Marks)**

   c. Solve : $\dfrac{dy}{dx} + y\cot x = 4x\,\mathrm{cosec}\,x$. **(07 Marks)**

8 a. Solve : $(D^3 - 6D^2 + 11D - 6)\,y = 0$. **(06 Marks)**

   b. Solve : $(D^2 + 2D + 1) = x^2 + e^{+x}$. **(07 Marks)**

   c. Solve : $(D^2 + D + 1)y = \sin 2x$. **(07 Marks)**

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