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MATDIP301

Third Semester B.E. Degree Examination, June/July 2016
Advanced Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions.

- 1
 - a. Express the complex number $\frac{(1+i)(1+3i)}{(1+5i)}$ in the form $a + ib$. (06 Marks)
 - b. Find the modulus and amplitude of $1 + \cos\theta + i \sin\theta$. (07 Marks)
 - c. Find the cube root of $1 - i$. (07 Marks)
- 2
 - a. Find the n^{th} derivative of $e^{ax} \cos (bx + c)$. (06 Marks)
 - b. Find the n^{th} derivative of $\frac{6x}{(x-2)(x+2)(x-1)}$. (07 Marks)
 - c. If $y = \sin^{-1}x$, prove that $(1-x^2)y_{n+2} - (2n+1)x y_{n+1} - n^2 y_n = 0$. (07 Marks)
- 3
 - a. Find the angle of intersection of the curves $r^2 \sin 2\theta = a^2$, $r^2 \cos 2\theta = b^2$. (06 Marks)
 - b. Find the nodal equation of the curve $r(1 - \cos\theta) = 2a$. (07 Marks)
 - c. Expand $\log (\sec x)$ upto the term containing x^4 using Maclaurin's series. (07 Marks)
- 4
 - a. If $u = x^3 - 3xy^2 + x + e^x \cos y + 1$, show that $u_{xx} + u_{yy} = 0$. (06 Marks)
 - b. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $xu_x + yu_y + zu_z = 0$. (07 Marks)
 - c. Find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$, where $u = x + y + z$, $v = y + z$, $w = z$. (07 Marks)
- 5
 - a. Obtain reduction formula for $\int \cos^n x \, dx$, where n is positive integer. (06 Marks)
 - b. Evaluate $\int_0^2 \frac{x^4}{\sqrt{4-x^2}} dx$. (07 Marks)
 - c. Evaluate $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dz dy dx$. (07 Marks)

- 6 a. Prove that: i) $\Gamma(n+1) = n \Gamma(n)$ and ii) $\Gamma(n+1) = n!$ for a positive integer n . (06 Marks)
- b. Prove that $\beta(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}$. (07 Marks)
- c. Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} \cdot \int_0^{\pi/2} \sqrt{\sin \theta} d\theta = \pi$. (07 Marks)
- 7 a. Solve $\frac{dy}{dx} = (9x + y + 1)^2$. (06 Marks)
- b. Solve $ye^{xy} dx + (xe^{xy} + 2y) dy = 0$. (07 Marks)
- c. Solve $\frac{dy}{dx} + y \cot x = \cos x$. (07 Marks)
- 8 a. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 5e^{-2x}$. (06 Marks)
- b. Solve $(D^2 - 4D + 13)y = \cos 2x$. (07 Marks)
- c. Solve $(D^2 + 2D + 1)y = x^2 + 2x$. (07 Marks)
