

1. Write regular expressions for language that accept all strings of letters in which the letters are in ascending lexicographic order.

**Solution:**

$(A|a)^*(B|b)^*(C|c)^*(D|d)^* \dots (Y|y)^*(Z|z)^*$

2. Consider the grammar:

$S \rightarrow (L) \mid a$

$L \rightarrow L, S \mid S$

Find parse trees for the following sentences:

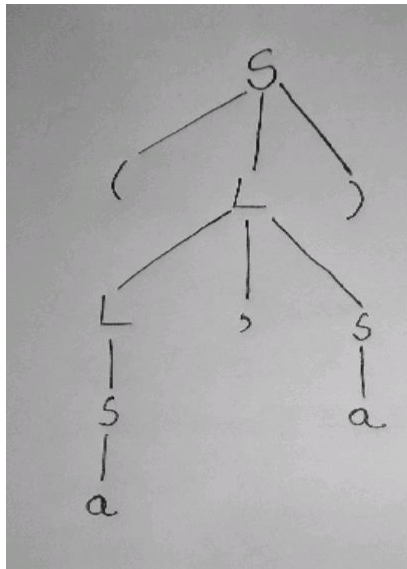
i)  $(a, a),$

ii)  $(a, (a, a)),$

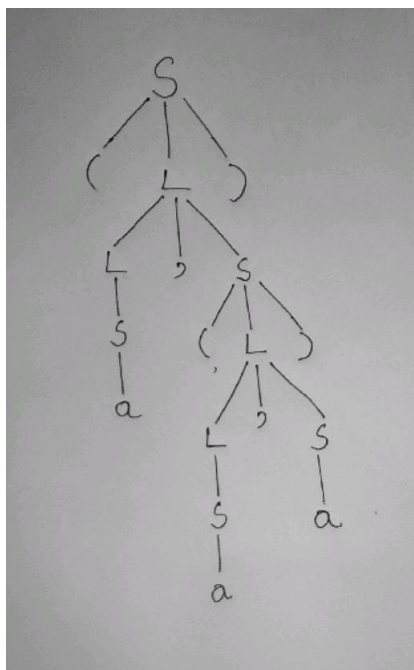
iii)  $(a, ((a, a), (a, a)))$

**Solution:**

i)



ii)



iii)



ii) Construct the corresponding rightmost derivation for *abab*.

iii) Construct the parse tree for *abab*.

**Solution:**

i) The two different leftmost deviation is constructed below with deviation tree for each of them:

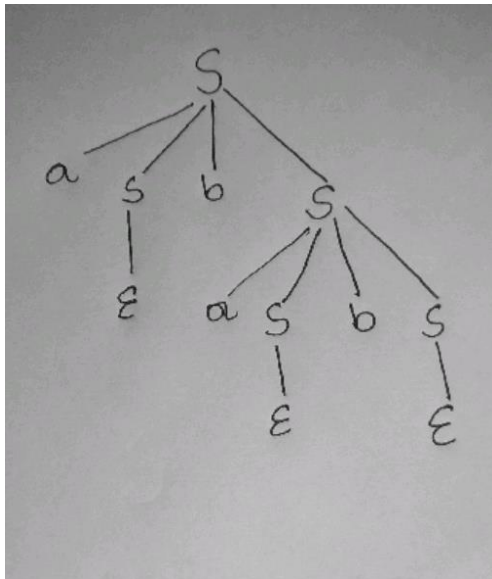
[1]  $S \rightarrow aSbS$

$\rightarrow abS$  (by  $S \rightarrow \epsilon$ )

$\rightarrow abaSbS$  (by  $S \rightarrow aSbS$ )

$\rightarrow ababS$  (by  $S \rightarrow \epsilon$ )

$\rightarrow abab$  (by  $S \rightarrow \epsilon$ )



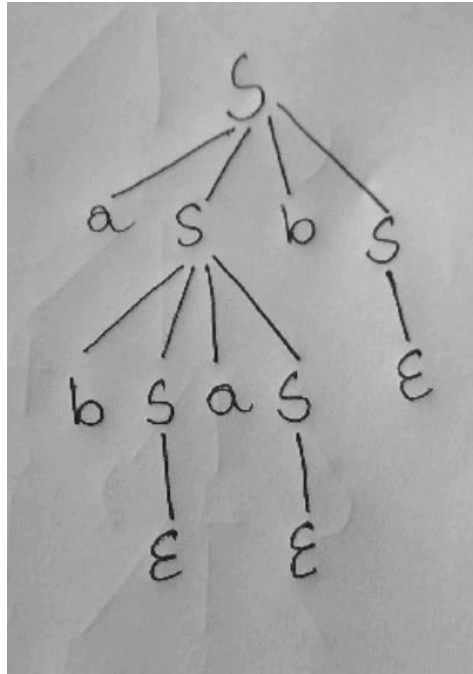
[2]  $S \rightarrow aSbS$

$\rightarrow abSaSbS$  (by  $S \rightarrow bSaS$ )

$\rightarrow abaSbS$  (by  $S \rightarrow \epsilon$ )

-> ababS (by  $S \rightarrow \epsilon$ )

-> abab (by  $S \rightarrow \epsilon$ )



As, the string **abab** constructs two different trees, the grammar  $S \rightarrow aSbS \mid bSaS \mid \epsilon$  is ambiguous.

ii) For rightmost derivation, derivation should be started from right side. For the string **abab**, this can be done in two ways –

a)  $S \rightarrow aSbS$

-> aSbaSbS (by  $S \rightarrow aSbS$ )

-> aSbaSb (by  $S \rightarrow \epsilon$ )

-> aSbab (by  $S \rightarrow \epsilon$ )

-> abab (by  $S \rightarrow \epsilon$ )

b)  $S \rightarrow aSbS$

$\rightarrow aSb$  (by  $S \rightarrow \epsilon$ )

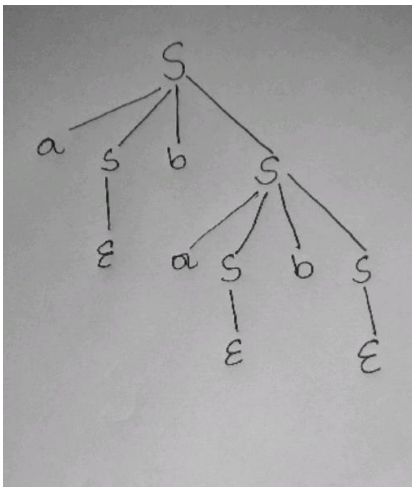
$\rightarrow abSaSb$  (by  $S \rightarrow bSaS$ )

$\rightarrow aSab$  (by  $S \rightarrow \epsilon$ )

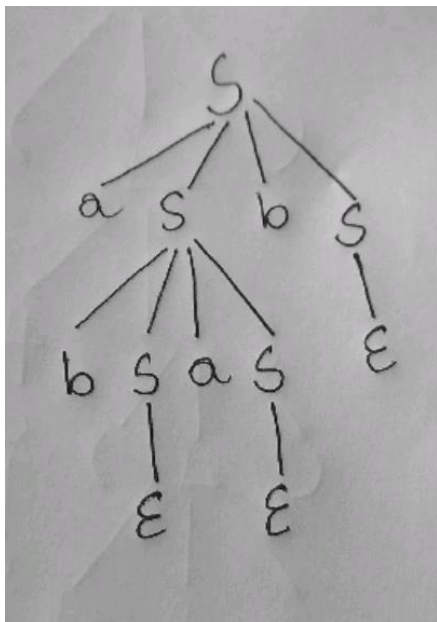
$\rightarrow abab$  (by  $S \rightarrow \epsilon$ )

iii) Parse tree for the string **abab** –

a)



b)



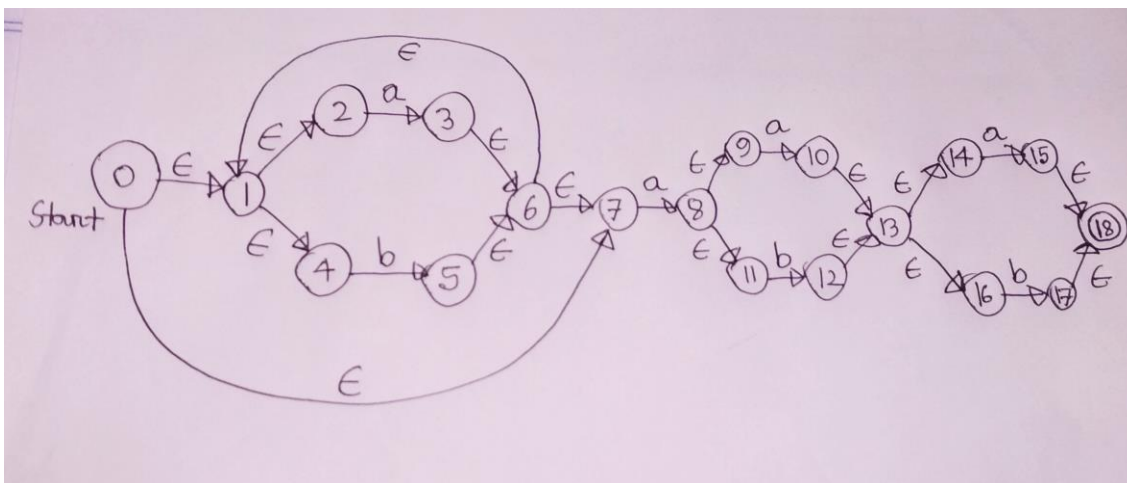
5. Construct an NFA using Thompson's construction from the following regular expression:

$(a|b)^*a(a|b)(a|b)$

Convert the resulting NFA to DFA using Subset construction rule.

**Solution:**

**NFA:**



**DFA:**

$\epsilon$ -closure  $\{0\} = \{0,1,2,4,7\} = A$

Move  $(A, a) = \{3,8\}$

$\epsilon$ -closure  $\{3,8\} = \{1,2,3,4,6,7,8,9,11\} = B$

Move  $(A, b) = \{5\}$

$\epsilon$ -closure  $\{5\} = \{1,2,4,5,6,7\} = C$

Move  $(B, a) = \{3,8,10\}$

$\epsilon$ -closure  $\{3,8,10\} = \{1,2,3,4,6,7,8,9,10,11,13,14,16\} = D$

Move  $(B, b) = \{5,12\}$

$\epsilon$ -closure  $\{5,12\} = \{1,2,4,5,6,7,12,13,14,16\} = E$

Move  $(C, a) = \{3,8\}$

$\epsilon$ -closure  $\{3,8\} = B$

Move  $(C, b) = \{5\}$

$\epsilon$ -closure  $\{5\} = C$

Move  $(D, a) = \{3,8,10,15\}$

$\epsilon$ -closure  $\{3,8,10,15\} = \{1,2,3,4,6,7,8,9,10,11,13,14,15,16,18\} = F$

Move  $(D, b) = \{5,12,17\}$

$\epsilon$ -closure  $\{5,12,17\} = \{1,2,4, 5,6,7,12,13,14,16,17,18\} = G$

Move  $(E, a) = \{3,8,15\}$

$\epsilon$ -closure  $\{3,8,15\} = \{1,2,3,4,6,7,8,9,11,15,18\} = H$

Move  $(E, b) = \{5,17\}$

$\epsilon$ -closure  $\{5,17\} = \{1,2,4,5,6,7,17,18\} = I$

Move  $(F, a) = \{3,8,10,15\}$



$\epsilon$ -closure  $\{3,8,10,15\} = F$

Move  $(F, b) = \{5,12,17\}$

$\epsilon$ -closure  $\{5,12,17\} = G$

Move  $(G, a) = \{3,8,15\}$

$\epsilon$ -closure  $\{3,8,15\} = H$

Move  $(G, b) = \{5,17\}$

$\epsilon$ -closure  $\{5,17\} = I$

Move  $(H, a) = \{3,8,10\}$

$\epsilon$ -closure  $\{3,8,10\} = D$

Move  $(H, b) = \{5,12\}$

$\epsilon$ -closure  $\{5,12\} = E$

Move  $(I, a) = \{3,8\}$

$\epsilon$ -closure  $\{3,8\} = B$

Move  $(I, b) = \{5\}$

$\epsilon$ -closure  $\{5\} = C$

### Transition Table

State	a	b
A	B	C
B	D	E
C	B	C
D	F	G
E	H	I
F	F	G
G	H	I
H	D	E

I	B	C
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