Concordia University
Department of Computer Science
& Software Engineering

COMP 478/6771 Image Processing

Assignment#1

Due Date: February 11, 2025

1. a) Suppose that a flat area with center at (x0, y0) is illuminated by a light source with intensity distribution:

$$i(x,y) = Ke^{-[(x-x_0)^2+(y-y_0)^2]}$$

Assume for simplicity that the reflectance of the area is constant and equal to 1.0 and let K=255. If the resulting image is digitized with k bits of intensity resolution, and the eye can detect an abrupt change of eight shades of intensities between adjacent pixels, what value of k will cause visible false contouring?

b) Sketch the image in part a) for k=2.

2. The purpose of this question is to perform histogram equalization to a given histogram and plot the resulting histogram. Given the following histogram where GL is Gray level, and NP is Number of pixels.

GL	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
NP	0	5	13	57	100	39	21	12	7	2	0	0	0	0	0	0

a) Plot the histogram of a 1-D image array given in the table above.

b) Let r_k be the GL given in the table, perform histogram equalization by:

i) Calculate s_k from the table.

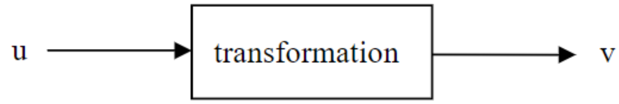
ii) Plot the probability density functions $p_r(r_k)$ and $p_s(s_k)$.

c) Plot the new histogram after performing the histogram equalization.

d) Explain why the discrete histogram equalization technique does not, in general, yield a flat histogram.

e) A digital image is subjected to histogram equalization. Does a second pass of histogram equalization (on the histogram-equalized image) produce a different or the same result as the first pass? Explain your answer.

3. In the class lecture, we have learnt how to derive the equation for mapping an arbitrary input to an output that has the probability density distribution (pdf). In this problem, we want to derive the equation for the cube-root hyperbolic distribution.



We want the pdf of the output, v, to be as the following:

$$p(v) = \begin{cases} \frac{1}{3} . v^{-2/3} \Big/ (v_{\max}^{1/3} - v_{\min}^{1/3}) & \text{if } v_{\min} \le v \le v_{\max} \\ 0 & \text{otherwise} \end{cases}$$

a) Derive the mapping function v = T(u) that maps any arbitrary u to output v with the desired property. Show the derivation process and the final equation.

b) Assume the input, u, has a uniform pdf over [0,1]. Plot the input-output (u-v) mapping curve.

4. In photography, the exposure time for capturing an image often follows a gamma distribution. Suppose the exposure time follows a gamma distribution with shape parameter k=2 and rate parameter λ=0.5. Determine the probability that the exposure time is less than 4 seconds and find the mean and variance of the exposure time.

5. a) Explain the differences between Euclidian norm, city block norm and chessboard norm and visualize them in a cartesian plane.

   b) Write a Matlab code to show the differences and support your explanation. (hint: you may use “bwdist” as in https://www.mathworks.com/help/images/ref/bwdist.html)

   c) Use different centers points and number of shapes (e.g., in the Euclidean distance (metric), change 3 circles to 7 and the centre point to (40,40), ….) and show your visualization.

   d) Using the definition of Euclidian norm, write a Matlab code to produce the following figure (i.e., a black-white image containing two overlapped circles that their centre is (-10, -10) and (10,10)).

NOTE: Attached your script/file code as well as the details of your solution.