Neg Image → $S = L - 1 - r$: L = max Pixel, r = curr intensity

Contrast streching → $s = \frac{r - r_{min}}{r_{max} - r_{min}}(s_2 - s_1) + s_1$ $s_1 = 0$, $s_2 = 255$

Omer Sayem (40226505)

Gamma Correction → $T(r) = c \cdot r^{\gamma}$ ← dark → darker, bright → Brighten; < 1 → Bri- Dark region

Bit-plane Slicing → $(200)_{10} = (11001000)_2$ MSB, LSB; = 1, γ > 1 → dark bright region

Histogram Equalization →

① $P_r(r_k) = \frac{n_k}{n}$ ← Total pixel

○ 2^B counter with 0
○ Loop pixel x, y
○ increment

② CDF(0) = $P_r(0)$
CDF(1) = $P_r(1)$ + CDF(0)
CDF(2) = $P_r(2)$ + CDF(1) …

③ $S_k = ((L-1)\, \text{CDF}(r_k))$ Round ← Grey Level

Histogram Matching → (A, B)
PDF(A), PDF(B) → CDF(A), CDF(B)
Match the closet to CDF

CDFA	CDFB
0 (0.50)	0 (0.10)
1 (0.20)	1 (0.50)

Multi Image Avg: → $\underbrace{g_k(x,y)}_{\text{noise Img}} = \underbrace{f(x,y)}_{\text{True Img}} + \underbrace{n_k(x,y)}_{\text{noise}}$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}}$ ← σ = noise variance; N ← Number of Image; $\sigma_{\bar{x}}$ = Desired Noise variance

Fourier transform → $F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-j\omega t} dx$ $\boxed{\omega = 2\pi f}$

IFT → $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$

$F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi\omega x} dx$

Linear Transform

$e^{j\omega t} = \underbrace{\cos(\omega t)}_{\text{even}} + j\underbrace{\sin(\omega t)}_{\text{odd}}$ — R ↓, Imagin ↓: $a + jb$

$A = \sqrt{a^2 + b^2}$

$\theta = \tan^{-1}\frac{b}{a}$

Nyquist Limit → $f \geq 2 f_{max}$

Aliasing → Under sampling

orthogonal → Different basis Vector are independent

Normal → Each basis vector len 1.

Discrete fourier → $f_n = \frac{1}{N}\sum_{k=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$ ← discrete | $\omega = 2\pi f = \frac{2\pi}{N}kn$

Convolution → Requires flipping

correlation → doesn't.

Convolving two Gaussian result in wider because new variance is the sum of original.

Convolution Theorem → $\underbrace{F[f(x) * h(x)]}_{\text{Fourier conv}} \Leftrightarrow \underbrace{H(x) \cdot F(\omega)}_{\text{mul in freq domain}}$

compute 2D DFT → Direct Method → N^4, Row/col decomposition → $2N^3$ add $2N^3$ Multiplication.

FFT → $N^2 \log_2 N$

Under Sample → moiré-like pattern

Log Transform: → $s = c * \log(1+r)$ [stretch low brightness values)

Lowpass filter → blurring Image, Highpass filter → sharpen Img

Larger kernel → Smoother image [Avg more values. reduce sharpness]

Small kernel → sharpen image.

Box filter → $\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$ ← Basic Smoothing

Weighted filter → $\frac{1}{16}\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix}$ ← Smoothing + preserve Edges

Sharpening filter → $\frac{df}{dx} = f(x+1, y) - f(x,y)$ ← Horizontal changes

$\frac{df}{dx} = f(x, y+1) - f(x,y)$ ← vertical changes.

- Laplacian (highpass) → Edge detection → sharpen Image $\begin{bmatrix}1&1&1\\1&-8&1\\1&1&1\end{bmatrix}$ should be 0, ← Isotropic
- Sobel → Edge detection in specific direction
- Box → Smoothing with random noise
- Median → Impulse noise reduction (salt & pepper)
- Unsharp mask → Sharpening → $g_{mask}(x,y) = f(x,y) - \bar{f}(x,y)$ ← blurred version; $g(x,y) = f(x,y) + k\, g_{mask}(x,y)$
- Gamma correction → contrast Enhancement; Halo for excess contrast
- Gradient magnitude → combine sobel-x and sobel-y

$\underbrace{\begin{bmatrix}-1&0&1\\-2&0&2\\-1&0&1\end{bmatrix}}_{\text{vertical}}$ $\underbrace{\begin{bmatrix}-1&-2&-1\\0&0&0\\1&2&1\end{bmatrix}}_{\text{horizontal}}$

Original → AVG mask → Blurred Img → Median thresholding → Sharp Image

order stat filter → Non-linear filter

correlation → $g(x,y) = \sum f(x+m, y+n)\, w(m,n)$

convolution → $g(x,y) = \sum f(x+m, y+n)\, w(-m,-n)$

phase → structural information (edges, contours)

Magnitude → contains intensity (brightness, contrast and smoothness)

Shifting affects the phase, not magnitude.

Wider spatial Gaussian ↔ narrow freq Gaussian

Inverse relationship between width in space and width in freq.

No effects on the phase → $\frac{\text{imaginary of } b}{\text{imaginary of } a} = 0$

offset filter → doesn't remove all the low freq.

Steps: $\boxed{\text{padding } N{\times}N \to 2N{\times}2N}$ → $\boxed{f_p(x,y) \times (-1)^{x+y}}$ (shift in center) → $\boxed{\text{DFT}}$ → $\boxed{G(u,v) = H(u,v) F(u,v)}$ (conv theorem) → $g_p(x,y) \leftarrow \{\text{Real}[F^{-1}(G(u,v))]\} \times (-1)^{x+y}$

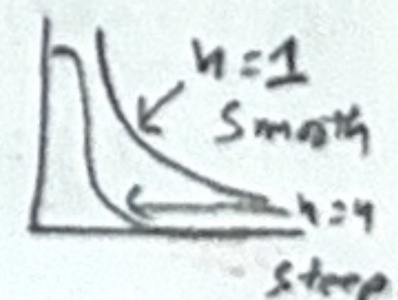
c > 1 → freq. otherwise: spatial

Ideal low pass filter: $H(u,v) = \begin{cases} 1, & \text{if } D(u,v) \leq D_0 \\ 0, & \text{if } D(u,v) > D_0 \end{cases}$ ← cutoff

cutoff R ≥ more details preserved

have ringing effect in freq domain

Butterworth LPF → $\frac{1}{1 + [D(u,v)/D_0]^{2n}}$ ← sol



Gaussian filter → $H(u) = Ae^{-u^2/2\sigma^2}$ $\sigma^2 >$ more smooth, $D_0 >$ more smooth

☐ Bilateral filter: Smoothes homogeneous region while keeping edges sharp.

☐ HPF = 1 - LPF [Sub a low pass from org]

☐ Difference of Gaussian (DoG) → Substract two Gaussian [remove more Low freq] LPF

☐ Homomorphic filter → $f(x,y) = i(x,y)\, r(x,y)$

Affects Low and high differently — Low Freq — high Freq

☐

$f(x,y) \rightarrow \boxed{\ln} \rightarrow \boxed{DFT} \rightarrow \boxed{H(u,v)} \rightarrow (DFT)^{-1} \rightarrow \exp \rightarrow g(x,y)$

Suppressed low freq, high freq enhanced

$H(u,v) = (\gamma_H - \gamma_L)\left(1 - e^{-cD^2(u,v)/D_0^2}\right) + \gamma_L$

☐ Band-pass filter: Keep only mid range freq while removing both very low. $H_{BP}(u,v) = 1 - H_{BR}(u,v)$

☐ Notch filter → Targets specific frequency.
Removes the Periodic noise that appears as bright spots in the freq domain. [must be symmetric around origin, zero-phase shift]

☐ Fourier transform of a real image is symmetric.

☐ Why freq domain? → Spatial → Element wise mul $O(MNmn)$.
Freq → $2MN\log_2(MN)$ [FFT]

⊙ Easier implementation of complex filters [Band pass, homomorphic]

⊙ customizable filter → Butterworth, Gaussian.

$C_s(m) = \frac{2M^2m^2}{2M^2\log_2 M^2}$ $C_n(m) > 1$ use Freq, other spatial | Large m → freq, Small m → spatial

☐ Vector Norms :→ $\cos\theta = \frac{x^T y}{\|x\|\,\|y\|}$

☐ Eigen value :→ $me_i = \lambda_i e_i$

☐ Bayes theorem → $P(B|A) = \frac{P(A|B)\,P(B)}{P(A)}$ ← prior

☐ $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \det(M - \lambda I) = 0$ $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\det(M - \lambda I) = (a-\lambda)(d-\lambda) - bc$; $\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

☐ Joint probability:

Independent → $P(AB) = P(A) \times P(B)$

dependent → $P(A \cap B) = P(A) \times P(B|A)$

☐ Conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

☐ Gaussian (Normal) distribution:

$$N(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\}$$

☐ Erlang distribution:

Mean $= \frac{k}{\lambda}$, Variance $= \frac{k}{\lambda^2}$

k = shape
λ = rate

☐ Rayleigh distribution:

$f(x;\sigma) = \frac{x}{\sigma^2}\exp\left(-\frac{x^2}{2\sigma^2}\right)$, $x \geq 0$. mean $= \sigma\sqrt{\frac{\pi}{2}}$, $\text{var}[x] = \frac{4-\pi}{2}\sigma^2$

PDF

☐ Linear Systems → $g(x) = H[f(x)]$ → System operator

☐ check for Linearity:

⊙ Additivity → $H[x_1(t) + x_2(t)] = H[x_1(t)] + H[x_2(t)]$

⊙ Homogeneity → $H[c * x(t)] = c * H[x(t)]$

☐ Check for time Invariance → System behavior doesn't change over time or space.

$$H[x(t-t_0)] = y(t-t_0)$$

☐ Check for causality; Output depends only on the present and past input, not the future. $f(x) = 0$ for $x < x_0 \Rightarrow g(x) = H[f(x)] = 0$, $x < x_0$

☐ Check for stability → Bounded input produces a bounded output; $|x(t)| < k \Rightarrow |y(t)| < M$

☐ Unit impulse function → $f(x) = \int_{-\infty}^{\infty} f(a)\,\delta(x-a)\,da$ — input — How System responds for Impulse

☐ Multivariate Gaussian →

$$P(x) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right)$$

x = data, Σ = Covariance, Σ^{-1} = Inverse of covariance

$|\Sigma| = \det$

Image Compression;

☐ Img x → $\boxed{y = T(x)}$ → $\boxed{q = Q(y)}$ → $\boxed{c = C(q)}$

reconstruct Img ← $\boxed{\hat{x} = T^{-1}(\hat{y})}$ ← $\boxed{\hat{y} = Q^{-1}(q)}$ ← $\boxed{q = C^{-1}(c)}$

☐ Image degradation → $g(x,y) = \underbrace{h(x,y)}_{\text{blur}} * f(x,y) + \underbrace{\eta(x,y)}_{\text{noise}}$

☐ Number of pixels ≠ Resolution

☐ Monty hall problem → Always switch!

☐ Sobel filter in Spatial ⟺ High pass in Freq

$f(x,y) - f(x-1,y) \Leftrightarrow 1 - e^{-j2\pi u/M}$

$f(x,y) - f(x,y-1) \Leftrightarrow 1 - e^{-j2\pi v/N}$

☐ Avg filter in Spatial ⟺ Low pass in Freq

$\frac{1}{4}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \Leftrightarrow H(u,v) = \frac{1}{2}(\cos(2\pi u) + \cos(2\pi v))$

☐ High freq emphasis ⟹ Histogram Equalization

☐ Fourier of 1D box → $F(u) = AW\,\text{sinc}(uW)$

☐ Fourier of 2D box → $F(u) = ATZ\,\text{sinc}(vZ)$