Concordia University
Department of Computer Science
& Software Engineering

COMP 478/6771 Image Processing

Assignment#2

Due Date: February 25, 2024

1. Image Sharpening

a) Consider the following three Laplacian filters A, B, and C:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -4 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

If we filter a given image I by using filters A, B, and C respectively to obtain the sharpened images I_A, I_B, and I_C. What can you say about these filtered images? (Hint: compare the sharpness of the images).

b) Write down a 5 by 5 Laplacian like filter with the centre element equal to -24. Apply this Laplacian like filter to an image I, do we get a sharper image compared to the images in part a)? Explain your answer.

c) How does this type of filter behave as a function of mask size, for example: 3x3, 5x5, 9x9, and 15x15?

2. The two images shown in the following figure are quite different, but their histograms are the same. Suppose that each image is blurred using a 3x3 box kernel.

(a) Would the histograms of the blurred images still be equal? Explain.

(b) If your answer is no, either sketch the two histograms or give two tables detailing the histogram components.

3. In character recognition, text pages are reduced to binary form then followed by a thinning process that will reduce the characters to strings of binary 1s on a background of 0s. Due to noise the binarization and thinning processes could result

in broken strings of 1s with gaps ranging from 1 to 5 pixels. The aim is to repair it so that there are no gaps in the strings of 1s. This is done as follows:

a) Blur the binary image by an averaging filter then apply a thresholding method to convert it back to binary form. For this approach give the minimum size of the blurring mask and the minimum value of the threshold to accomplish the task.

b) Without using the averaging filter and thresholding, can you design another method to repair the gaps?

4. Fourier Transform:

a) Study Example 4.1 [Obtaining the Fourier transform of a simple continuous function] on page 211 of the textbook, then follow the steps in that example to find the Fourier Transform of the function f(t) = A for 0 ≤ t ≤ W and f(t) = 0 otherwise; where both A and W are constants. Then, explain the differences between your result and the result in Example 4.1. Now, consider the case where A = W = 1, what is the Fourier Transform of f(t) in this case?

b) Use the result of Example 4.1 to find the Fourier Transform of the tent function. The tent function is the function shown in Figure 4.6(a) on page 218 of the Text.

Hint: The tent unction is the convolution of two box functions.

c) Prove the linearity and Translation properties of DFT as follows:

Linearity	$a f_1(x,y) + b f_2(x,y) \Leftrightarrow aF_1(u,v) + bF_2(u,v)$
Translation (general)	$f(x,y)e^{j2\pi(u_0x/M + v_0y/N)} \Leftrightarrow F(u-u_0, v-v_0)$ $f(x-x_0, y-y_0) \Leftrightarrow F(u,v)e^{-j2\pi(ux_0/M + vy_0/N)}$

5. a) Can you think of a way to use the Fourier transform to compute (or partially compute) the magnitude of the gradient [Eq. (3-58)] for use in image differentiation? If your answer is yes, give a method to do it. If your answer is no, explain why?

$$M(x,y) = \|\nabla f\| = \text{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2} \qquad (3\text{-}58)$$

b) Show that $\tilde{F}(\mu)$ in Eq. (4-40) is infinitely periodic in both directions, with period $1/\Delta T$.

$$\begin{aligned}
\tilde{F}(\mu) &= \int_{-\infty}^{\infty} \tilde{f}(t)e^{-j2\pi\mu t}\,dt = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(t)\delta(t-n\Delta T)e^{-j2\pi\mu t}\,dt \\
&= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f(t)\delta(t-n\Delta T)e^{-j2\pi\mu t}\,dt \qquad (4\text{-}40) \\
&= \sum_{n=-\infty}^{\infty} f_n e^{-j2\pi\mu n\Delta T}
\end{aligned}$$