DEPARTMENT OF COMPUTER SCIENCE AND SOFTWARE ENGINEERING
CONCORDIA UNIVERSITY

COMP 6771 Image Processing
Fall 2024
ASSIGNMENT 2
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1. Image Sharpening
   a. If we filtered an image I with the given filter A,B,C respectively we will get:
      i. Filtered image I_A; Which will have sharpen edges in diagonal directions.
      ii. Filtered image I_B; Which will have sharpen edges in both horizontal and vertical directions.
      iii. Filtered image I_C; Which is a isotopic filter , that means it can detect edges in all directions (Horizontal, Vertical and Diagonal).

Comparison: Image I_C will have the most sharpen edges between I_A and I_B. Following I_B will have better edges in both directions than I_A.

   b.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -24 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Above is a 5*5 Laplacian filter with canter element is -24 and others are 1 to ensure the coefficient sum is zero. The filter will produce more sharper image than 1(a) because of the magnitude of the center pixel (-24) resulting in stronger sharpening effect on the edges. The filter will capture more global features because of the size (5*5). However , because of larger mask the effect will become more blurrier than the formal.

   c.
      i. Mask size 3*3: This small filter accounts in smaller region around each pixels, which makes the filter more local focus. It detects small scale sharper edges. (Such as , small objects or text edges)
      ii. Mask size 5*5: Broader neighbourhood, more negative weight in the center; More thicker edges than 3*3; More sensitive to noise.
      iii. Mask size 9*9: It captures broader region, very large center magnitude, Blurry

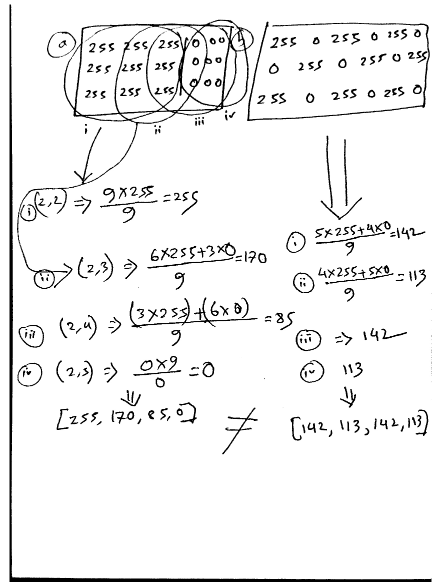
edges. Detects global feature rather than fine details.

iv. Mask size 15*15: More aggressive sharpening effect in wider region, increased sensitivity to noise , blurry details, potentially losing fine details.

2. Box kernel

a.

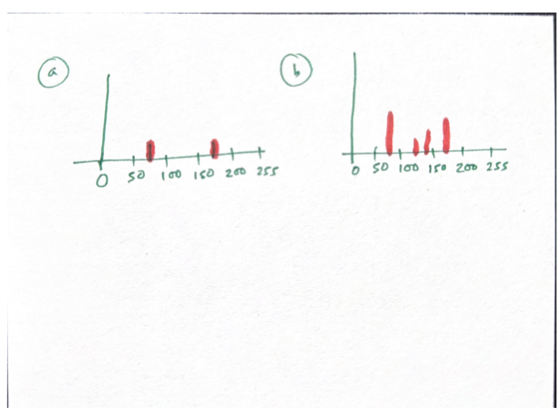
a b



No , the histogram of these images will not be the same after applying 3*3 box kernel.

Before applying the filter the white and black pixel count were the same(50-50). However after the 3*3 blur the image (b) will produce mostly mid grey pixel for averaging. On the other hand, the image (a) will produce near 0 or 255 pixels except the narrow gray boundary zone. Given a small exmaple calculation of their pixel distribution after the filter effect.

b.



We can see histogram of blurred a peaks at black and white pixels and some grey at boundary. The image b has mid grey tone across the entire image.

3. Text restoration

a. The idea is to surround the missing pixels with enough 1 pixels and have a suitable thresholding to cover the gap. We should apply a filter size of min $(2 * 5) + 1 = 11$ pixels to cover gap up to 5 pixels. Thus the minimum filter size would be 11*11. We should also chose a threshold value of ≈ 0.5 to ensure week connections are restored while keeping the background black.

b. Another way is to first find the connected white pixels. Then we find the gaps between these connected white pixels(black pixels). Then we check if the black pixels are in small region or ideal separation between two characters. We skip the large gaps and only fill in the small gaps with white pixels. In this method, we only turn the small black patches into white.

4. Fourier Transform

a.

$$F(\mu) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi\mu t}\, dt$$

$$= \int_{0}^{W} A\, e^{-j2\pi\mu t}\, dt$$

$$= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]_0^W$$

$$= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu W} - 1\right]$$

$$= \frac{A}{j2\pi\mu}\left[e^{j\pi\mu W} - e^{-j\pi\mu W}\right] e^{-j\pi\mu W}$$

$$= \frac{A}{\pi\mu} \sin(\pi\mu W)\, e^{-j\pi\mu W}$$

$$= AW\left[\frac{\sin(\pi\mu W)}{\pi\mu W}\right] e^{-j\pi\mu W}$$

My Result

$A = W = 1 \Rightarrow F(\mu) = \underbrace{\frac{\sin(\pi\mu)}{\pi\mu}}_{\text{Magnitude (Sinc Shape)}} \underbrace{e^{-j\pi\mu}}_{\text{Phase}}$

Text book:

$$F(\mu) = AW \cdot \mathrm{Sinc}(\pi\mu W)$$

The magnitude from of the book example and my result is the same, but there is no phase shift in the book example.

b. Given ,a tent function is the convolution of two box equal box function.

From 4.1:

$$F(\mu) = AW \frac{\sin(\pi\mu W)}{\pi\mu W}$$

Fourier of the tent is

$$F(\mu)\, F(\mu) = (AW)^2 \frac{\sin^2(\pi\mu W)}{(\pi\mu W)^2}$$

c.

Linearity:

$$a f_1(x,y) + b f_2(x,y) \Leftrightarrow a F_1(u,v) + b F_2(u,v)$$

proof $F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{M-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{M}\right)}$

$$\sum_{x=0}^{M-1}\sum_{y=0}^{M-1} \left[a f_1(x,y) + b f_2(x,y)\right] e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{M}\right)}$$

$$= a \sum f_1(x,y) e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{M}\right)} + b \sum f_2(x,y) e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{M}\right)}$$

$$= a F_1(u,v) + b F_2(u,v)$$

Translation:

Multiplication by $e^{j2\pi\left(\frac{u_0 x}{M}+\frac{v_0 y}{M}\right)}$ in spatial domain,

$$f(x-x_0, y-y_0) \Leftrightarrow F(u,v)\, e^{-j2\pi\left(\frac{u_0 x_0}{M}+\frac{v_0 y_0}{M}\right)}$$

5.

a.

(5)(a) $F\left(\frac{df}{dx}\right) = j2\pi u\, F(u,v)$

$F\left(\frac{df}{dx}\right) = j2\pi \cancel{u}\, F(u,v)$

F(u,v) is the fourier of f(x,y)

$F[g_x] = (j2\pi u)F(u,v)$

$F(g_y) = (j2\pi v)F(u,v)$

$$F\left[M(x,y)\right] = \sqrt{(j2\pi u F(u,v))^2 + (j2\pi v F(u,v))^2}$$

$$= \sqrt{(2\pi u)^2 + (2\pi v)^2} \cdot |F(u,v)|$$

$$= 2\pi\sqrt{u^2+v^2} \cdot |F(u,v)|$$

5b $\tilde{F}(\mu) = \int_{-\infty}^{\infty} \tilde{f}(t)\, e^{-j2\pi\mu t}\, dt$

$\tilde{f}(t) = \sum_{n=-\infty}^{\infty} f(t)\, \delta(t - n\Delta t)$

$\tilde{F}(\mu) = \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{\infty} f(t)\, \delta(t - n\Delta t)\, e^{-j2\pi\mu t}\, dt$

Use shifting.

$\tilde{F}(\mu) = \sum_{-\infty}^{+\infty} f(n\Delta T)\, e^{-j2\pi\mu n\Delta t}$

$\Downarrow$

Fourier series expansion of periodic func

So, $\tilde{F}(\mu)$ repeats with $\frac{1}{\Delta T}$

b.