Assignment 2
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Question 1)

Assignment 2

Question 1 - a)

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -4 & 0 \\ 1 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

A is a basic laplacian Filter, B is similar to A but the non-central weights are distributed diagonally, there fore; it consider more surronding pixels → enhancing edges more than A.
C: Since the sum of coefficients is stronger, this filter is expected to enhance edges the most.

Comparing sharpness:
I-A: will show moderate sharpening with some edge enhancement.
I-B: will provide slightly stronger sharpening due to its inclusion of diagonal neighbors.
I-C: will give the strongest sharpening effect because the center weight is the most negative, making edges stand out the most. However if an image has fine details, this may introduce more noise due to excessive enhancement, in this situation, A provides a more balanced sharpening.

Question 1 - b)

$$\begin{bmatrix} 0 & 0 & 2 & 0 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 2 & 2 & -24 & 2 & 2 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix}$$

yes, the edges appear even sharper compared to filters A, B, C.
- A broader neighborhood is taking into account → stronger edge detection, and more noise.
- The center weight (-24) is more extreme than the -4 or -8.

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Question 1-c)

Laplacian filters of different sizes behave differently due to the increasing size of their kernel.

3x3: detect fine details and sharp edges but it is sensitive to noise. It works well for high-frequency details in images, such as detecting small objects or text edges.

5x5: It captures a larger neighborhood → stronger sharpening and edge detection. It is suitable for general image sharpening while balancing noise amplification.

9x9: It captures broader region → emphasize large-scale edges rather than fine details. It will be used for detecting larger structures like objects in satelite images or medical scans.

15x15: It detects extremely strong edges with a tendency to lose fine textures. It is suitable in large-scale image processing. It will blur finer details and emphasize only the most significant edges, potentialy introducing artifacts

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Question 2)

Question 2 - a)

NO, the histograms of the blured images will not be equal.

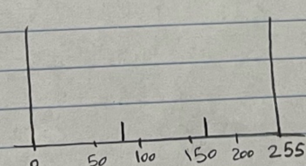
At first both have the same histogram because they contain the same number of black and white pixels. However after applying a 3x3 box filter the histogram will change.

First image  after bluring the transition between black and white becomes a smooth gradient.
The histogram will shift some pixelvalues from extreme black/white to middle grayscale value, but the change is localized to the transition region.

second image  Bluring will mix black and white pixels uniformly across the entire image, leading to more mid-gray tones every-where.
The histogram will have a more even distribution of pixel intensities from black to white rather than just affecting a transition region.

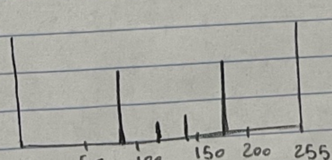
Question 2 - b)

First image:



The histogram peaks at black and white but introduce some mid-gray values at the boundry where bluring occures.

second image:



This image has wider range of mid-gray tones across entire image. It has multiple peaks instead of 2. representing various shades of gray due to mixed pixels.

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Question 3)

Question 3 _ a)

To repair the gaps, we should apply a 5x5 averaging filter to smooth out missing pixels, then use thresholding to restore the binary form.

minimum filter size : 5x5 which covers gaps up to 5 pixels.

Threshold value: ~50 which ensures weak connections are restored while keeping the background black.

This method effectively fills small gaps making the text continous.

Question 3 _ b)

One way is to use the method below:

1- identify white pixels regions (foreground text pixels)
2- check for gaps between white pixels.
→ if the gap contains only a few black pixels (small noise or missing part), fill it in.
→ if the gap is too large, leave it as it is (to avoid merging seperate characters)
3- fill the small gaps by changing black pixels to white.

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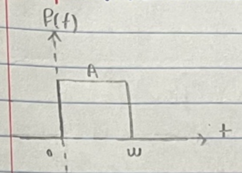
Question 4)

Question 4 - a) $f(t) = A$ for $0 \leq t \leq W$ and $f(t) = 0$ otherwise



$$F(\mu) = \int_{-\infty}^{+\infty} f(t)\, e^{-j2\pi\mu t}\, dt = \int_0^W A e^{-j2\pi\mu t}\, dt = \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]_0^W$$

$$= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu W} - 1\right] = \frac{A}{j2\pi\mu}\left[1 - e^{-j2\pi\mu W}\right]$$

$e^{-jx} = \cos(x) - j\sin(x) \Rightarrow F(\mu) = AW \cdot \frac{\sin(\pi\mu W)}{\pi\mu W}\, e^{-j\pi\mu W}$

$F(\mu) = AW \cdot \text{sinc}(\pi\mu W)\, e^{-j\pi\mu W}$

Textbook result is $F(\mu) = AW \cdot \text{sinc}(\pi\mu W)$

There is no phase shift because the function is symmetric around $t=0$
my result has a phase shift $e^{-j\pi\mu W}$ because shifting a function in time results in multiplication by a phase term in the frequency domain.

$A = W = 1$ 8

$F(\mu) = 1 \times 1 \times \text{sinc}(\pi\mu 1)\, e^{-j\pi\mu 1} = \text{sinc}(\pi\mu)\, e^{-j\pi\mu}$

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Question 4-b)

a tent function is the convolution of 2 box functions.

Tent (t) = Box (t) * Box (t)

By the convolution theorem in fourier transform:

$$F_{tent}(\mu) = F_{Box}(\mu) \cdot F_{Box}(\mu)$$

$\Rightarrow F_{Box}(\mu) = AW \cdot \text{sinc}(\pi \mu W) \rightarrow F_{Tent}(\mu) = (AW)^2 \cdot \text{sinc}^2(\pi \mu W)$

which decays faster than the single sinc function.

Question 4-c)

linearity:

DFT is linear $\rightarrow$ $af_1(x,y) + bf_2(x,y) \leftrightarrow aF_1(u,v) + bF_2(u,v)$

proof: $F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$

applying linearity: $\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [af_1(x,y) + bf_2(x,y)]\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$

$= a \sum f_1(x,y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)} + b \sum f_2(x,y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$

$= aF_1(u,v) + bF_2(u,v)$

Translation:

multiplication by $e^{j2\pi\left(\frac{u_0 x}{M} + \frac{v_0 y}{N}\right)}$ in spatial domain shifts the frequency response.

shifting in spatial domain introduces a phase shift in the frequency domain. $f(x-x_0, y-y_0) \leftrightarrow F(u,v)\, e^{-j2\pi\left(\frac{u_0 x_0}{M} + \frac{v_0 y_0}{N}\right)}$

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Question 5)

Question 5 - a)

yes, It is possible

$F\left[\frac{\partial f}{\partial x}\right] = (j2\pi u)\, F(u,v)$

$F\left\{\frac{\partial f}{\partial y}\right\} = (j2\pi v)\, F(u,v)$

$F(u,v)$ is the fourier transform of $f(x,y)$

→ we obtain

$F[g_x] = (j2\pi u)\, F(u,v)$
$F[g_y] = (j2\pi v) F(u,v)$

$$F[M(x,y)] = \sqrt{(j2\pi u F(u,v))^2 + (j2\pi v F(u,v))^2}$$

$$= \sqrt{(2\pi u)^2 + (2\pi v)^2} \cdot |F(u,v)|$$

$$= 2\pi \sqrt{u^2 + v^2} \cdot |F(u,v)|$$

Therefore method to compute Gradient magnitude using fourier transform is:

1- compute fourier transform of the image
2- multiply $F(u,v)$ by $2\pi\sqrt{u^2+v^2}$
3- compute the inverse transform to obtain $M(x,y)$

Thus it is possible to use fourier transform to compute the Gradient magnitude in the frequency domain without explicit differentiation in the spatial domain.

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Question 5-b)

$$\tilde{F}(\mu) = \int_{-\infty}^{+\infty} \tilde{f}(t)\, e^{-j2\pi\mu t}\, dt$$

→ use sampling representation:

$$\tilde{f}(t) = \sum_{n=-\infty}^{+\infty} f(t)\,\delta(t - n\Delta T)$$

$$\tilde{F}(\mu) = \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} f(t)\,\delta(t - n\Delta T)\, e^{-j2\pi\mu t}\, dt$$

→ use shifting property of delta function:

$$\tilde{F}(\mu) = \sum_{n=-\infty}^{\infty} f(n\Delta T)\, e^{-j2\pi\mu n\Delta T}$$

which is a Fourier Series expansion of a periodic function.

Thus $\tilde{F}(\mu)$ repeats with period $\frac{1}{\Delta T}$