4-ma’ruza.Tartiblangan to'plamlar. Dekart ko'paytma

Kardinal son. Sanoqli va kontinual to’plamlar.Tartiblangan juftlik tushunchasi. Juftliklar tengligi. Kortej tushunchasi. Kortej uzunligi. To'plamlarning dekart ko'paytmasi. To'plamlarning dekart ko'paytmasining xossalari.

Chekli to‘plamlarning asosiy xarakteristikasi bu ulardagi elementlar sonidir. A chekli to‘plamdagi elementlar sonini $n(A)$ yoki $|A|$ kabi belgilanadi va A **to‘plamning tartibi** yoki **quvvati** deb ham yuritiladi.

Misol. A={a,b,c,d} $\Rightarrow$ $n(A) = 4$, B={ Ø} $\Rightarrow$ $n(B) = 0$.

Ikkita to‘plam yigindisidan iborat to‘plam elementlarini topishda quyidagi asosiy formuladan foydalaniladi:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) \qquad (1)$$

Haqiqatdan ham $n(A) + n(B)$ son A va B to‘plamlardag elementlar soni, lekin ulardagi umumiy elementlar soni ikki marta qoshilgani uchun umumiy elementlari sonini bir marta ayiramiz. (1) formuladan quyidagi tenglikka ega bo‘lamiz

$$n(A) + n(B) = n(A \cup B) + n(A \cap B)$$

(1) formuladan ixtiyoriy sondagi to‘plamlar birlashmasidagi elementlar sonini topish formulasini keltirib chiqarish mumkin.

A, B, C $\in$ U to‘plamlar uchun

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C) \qquad (2)$$

Ixtiyoriy n ta $A_1, A_2, \ldots, A_n \in U$ to‘plam uchun

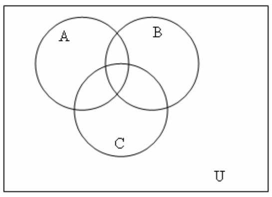
$$n(A_1 \cup A_2 \cup \ldots \cup A_n) = \sum_{i=1}^{n} n(A_i) - \sum_{i \neq j = 1}^{n} n(A_i \cap A_j) + \sum_{i \neq j \neq k = 1}^{n} n(A_i \cap A_j \cap A_k) - \ldots (-1)^{n-1} n(A_1 \cap A_2 \cap \ldots \cap A_n)$$

Misol. 100 ta talaba sessiya topshirishdi. Tarixni 48 kishi, falsafani 42 kishi, matematikani 37 kishi topshirdi. Tarix va falsafani 76 kishi, tarix va matematikani ham 76 kishi, falsafa va matematikani 66 kishi topshirdi. Hamma imtihonlarni 5 kishi topshirdi. Necha kishi bittadan, ikkitadan imtixon topshirgan, necha kishi birorta ham imtixon topshira olmagan?

Yechish: A={Tarixni topshirganlar},
B={falsafani topshirganlar},
C={matyematikani topshirganlar}

$$n(A) = 48, \quad n(B) = 42, \quad n(C) = 37$$

$$n(A \cup B) = 76, \quad n(A \cup C) = 76, \quad n(B \cup C) = 66, \quad n(A \cap B \cap C) = 5$$



$n(A \cap B) = n(A) + n(B) - n(A \cup B) = 48 + 42 - 76 = 14$ kishi

$n(A \cap C) = n(A) + n(C) - n(A \cup C) = 48 + 37 - 76 = 11$ kishi

$n(B \cap C) = n(B) + n(C) - n(B \cup C) = 42 + 37 - 66 = 13$ kishi

Faqat ikkitadan fanni topshirganlar

$$n(A \cap B \cap \overline{C}) = n(A \cap B \setminus A \cap B \cap C) = n(A \cap B) - n(A \cap B \cap C) = 14 - 5 = 9\, kishi$$

faqat tarix va falsafani,

$$n(A \cap \overline{B} \cap C) = n(A \cap C \setminus A \cap B \cap C) = n(A \cap C) - n(A \cap B \cap C) = 11 - 5 = 6\, kishi$$

faqat tarix va matematikani,

$$n(\overline{A} \cap B \cap C) = n(B \cap C \setminus A \cap B \cap C) = n(B \cap C) - n(A \cap B \cap C) = 13 - 5 = 8\, kishi$$

faqat falsafa va matematikani topshirishgan.

Faqat bitta fanni topshirganlar:

$n(A\cap \overline{B}\cap \overline{C}) = n(A \setminus A\cap B \setminus A\cap \overline{B}\cap C) = n(A) - n(A\cap B) - n(A\cap \overline{B}\cap C) = 48-14-6=28\,kishi$ faqat tarixni topshirishgan,
$n(\overline{A}\cap B\cap \overline{C}) = n(B\setminus A\cap B\setminus \overline{A}\cap B\cap C) = n(B)-n(A\cap B)-n(\overline{A}\cap B\cap C)=42-14-8=20\,kishi$ faqat falsafani topshirishgan,
$n(\overline{A}\cap\overline{B}\cap C)=n(C\setminus A\cap C\setminus \overline{A}\cap B\cap C)=n(C)-n(A\cap C)-n(\overline{A}\cap B\cap C)=37-11-8=18\,kishi$ faqat matematikani topshirishgan.
Umuman topshirmaganlar:

$$n(\overline{A\cup B\cup C}) = n(U\setminus(A\cup B\cup C)) = n(U) - n(A\cup B\cup C) =$$
$$= n(U) - (n(A)+n(B)+n(C)-n(A\cap B)-n(A\cap C)-n(B\cap C)+n(A\cap B\cap C)) =$$

=100-(48+42+37-14-11-13+5)=100-94=6 kishi umuman imtixon topshira olmagan.

Biror bir universal to‘plamning barcha to‘plam ostilari to‘plami va 1-21 xossalarni qanoatlantiruvchi unda kiritilgan yig‘indi, kesishma, va to‘ldiruvchi amallari BUL ALGEBRASINI tashkil qiladi.

To‘plar ustida kiritilgan amallar yetarlimi degan savol tug‘iladi.

Teorema. A va B ixtiyoriy to‘plamlar bo‘lsin, u holda

$$A\cup B = (A\Delta B)\Delta(A\cap B), \qquad A\setminus B = A\Delta(A\cap B)$$

Yig’indi va ayirmani simmetrik ayirma va kesishmalar orqali ifodalash mumkin. Bunday yondoshish matematikaning turli sohalarida fundamental tadbiqini topdi. Bunday yondoshishning rivojlanishiga asos bo‘lib to‘plamlar halqasi tushunchasi xizmat qildi.

Ta’rif 1. Bo‘sh bo‘lmagan C to‘plamlar tizimi to‘plamlar halqasi deyiladi, agar u kesishma va simmetrik ayirma amallariga nisbatan yopiq bo‘lsa, ya’ni,agar $A, B\in C$ $\Rightarrow A\Delta B\in C \quad A\cap B\in C$ bo‘lsa.

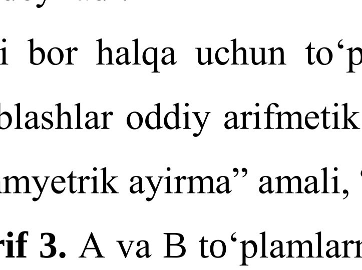
To‘plamlar halqasi assotsiativ, kommutativ bo‘lib uning noli bo‘lib bo‘sh to‘plam Ø xizmat qiladi. Halqada 1 ham mavjud bo‘lishi mumkin.

Ta’rif 2. Agar ixtiyoriy $A\in C$ uchun $A\cap E = A$ bo‘lsa, $E\in C$ to‘plam **halqaning biri** deyiladi.

Biri bor halqa uchun to‘plamlar algebrasi tushunchasi kiritilgan. Halqalarda algebraik hisoblashlar oddiy arifmetik qoidalarga o‘xshab amalga oshiriladi. Bunda “yig‘indi” rolini “simmyetrik ayirma” amali, “ko‘paytma”rolini “kesishma”amali bajaradi.

Ta’rif 3. A va B to‘plamlarning **dekart ko‘paytmasi** deb, barcha tartiblashtirilgan **$<a_i, b_j>$** juftliklar to‘plamiga aytiladi va $A\times B$ kabi belgilanadi, bu yerda $a_i\in A$ va $b_j\in B$. Shunday qilib

$$A\times B = \{<a_i, b_j>, a_i\in A, b_j\in B\}$$



Misol. $A=\{a_1,a_2\}$ va $B=\{b_1,b_2,b_3\}$ bo‘lsa, $A\times B$-?

$$A\times B = \{<a_i, b_j>, a_i\in A, b_j\in B\} =$$
$$=\{<a_1,b_1>,<a_1,b_2>,<a_1,b_3>,<a_2,b_1>,<a_2,b_2>,<a_2,b_3>\}$$