MEXANIK TEBRANISHLAR VA TO`LQINLAR

Masalalar

51-masala. Moddiy nuqtaning tebranishi $x = 0{,}25\sin\left(\pi t + \frac{\pi}{2}\right)m$ qonun bo`yicha bajariladi. Tebranish amplitudasi A, davri T, siklik chastotasi $\omega$, boshlang`ich fazasi φ_0 maksimal tezligi $\upsilon_{\max}$ va maksimal tezlanishi $a_{\max}$ topilsin.

Berilgan: $x = 0{,}25\sin\left(\pi t + \frac{\pi}{2}\right)m$

A~? T~? ω~? φ_0~? $\upsilon_{\max}$~? $a_{\max}$~?

Echish. Topilishi kerak bo`lgan kattaliklarni aniqlash uchun tebranishning tenglamasini garmonik tebranishning umumiy ko`rinishidagi tenglamasi bilan solishtiramiz:

$$x = 0{,}25\sin\left(\pi t + \frac{\pi}{2}\right)m$$

$$x = A\sin\left(\frac{2\pi}{T}t + \varphi_0\right)m$$

Bu ikki tenglama taqqoslanishidan quyidagi kelib chiqadi: tebranishning amplitudasi A=0,25m; davri $\frac{2\pi}{T}t = \pi t$, bundan T=2s,; siklik chastotasi $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = 3{,}14 rad/s$; boshlang`ich fazasi $\varphi_0 = \frac{\pi}{2}$:

Tebranishning tezligi υ va tezlanishi *a* mos ravishda siljish funksiyasining birinchi va ikkinchi tartibli hosilasidan iborat bo`lgani uchun:

$\upsilon = \frac{dx}{dt} = 0{,}25\pi\cos\left(\pi t + \frac{\pi}{2}\right)$ bo`lib, $\upsilon_{\max} = 0{,}25\pi m/s = 0{,}785 m/s$. SHunday qilib, $\upsilon_{\max} = 0{,}785 m/s$, $a = \frac{d^2x}{dt^2} = \frac{d\upsilon}{dt} = -0{,}25\pi^2\sin\left(\pi t + \frac{\pi}{2}\right)$ bo`lib, $a = -0{,}25\pi^2 m/s^2$ $a = -0{,}25\cdot 3{,}14^2 m/s^2 = -2{,}46 m/s^2$. SHunday qilib $a = -2{,}46 m/s^2$.

52-masala. *m*=5 g massali moddiy nuqta $\nu = 0{,}5Hz$ chastota bilan garmonik tebranadi. Tebranish amplitudasi *A=3sm*. 1) nuqtaning siljishi *x*=1,5 sm bo`lgan vaqtdagi tezligi $\upsilon$; 2) nuqtaga ta'sir etuvchi maksimal kuch $F_{\max}$; 3) tebranayotgan nuqtaning to`liq energiyasi *W* aniqlansin.

Berilgan: $\nu = 0{,}5Hz$, m=5g$= 5\cdot 10^{-3} kg$, A=3sm $3\cdot 10^{-2} m$

$x = 1{,}5sm = 1{,}510^{-2} m$

υ~?, $F_{\max}$~?, W~?

Echish. 1) garmonik tebranish tenglamasi quyidagi ko`rinishga ega

$$x = A\cos(\omega t + \varphi) \qquad (1)$$

Tezlik formulasini esa siljishidan vaqt bo`yicha birinchi tartibli hosila olib topamiz:

$$\upsilon = \frac{dx}{dt} = -A\omega\sin(\omega t + \varphi) \qquad (2)$$

Tezlikni siljish orqali ifodalash uchun (1) va (2) tenglamalardan vaqtni yo`qotish kerak. Buning uchun har ikkala tenglamani kvadratga ko`tarib, birinchisini A ga, ikkinchisini $A^2\omega^2$ ga bo`lamiz va ularni qo`shamiz:

$$\frac{x^2}{A^2} + \frac{\upsilon^2}{A^2\omega^2} = 1 \quad yoki \quad \frac{x^2}{A^2} + \frac{\upsilon^2}{4\pi^2\nu^2A^2} = 1$$

Oxirgi tenglamani υ ga nisbatan yechib, quyidagini topamiz:

$$\upsilon = \pm 2\pi\nu\sqrt{A^2 - x^2}$$

SHu formula bo`yicha hisoblashni bajarsak

$$\upsilon = \pm 8{,}2 sm/s$$

2) nuqtaga ta’sir etuvchi kuchni Nyutonning ikkinchi qonuniga binoan topamiz

$$F = ma \qquad (3)$$

Bunda a – nuqtaning tezligidan vaqt bo`yicha hosila olib topiladigan tezlanishi

$$a = \frac{d\upsilon}{dt} = -A\omega\cos(\omega t + \varphi) \qquad yoki \qquad a = -4\pi^2\nu^2A\cos(\omega t + \varphi)$$

tezlanishning ifodasini (3) formulaga qo`ysak:

$$F = -4\pi^2\nu^2mA\cos(\omega t + \varphi)$$

Bundan kuchni maksimal qiymati.

$$F_{\max} = 4\pi^2\nu^2mA$$

Bu tenglamaga $\pi, \nu, m \quad va \quad A$ kattaliklarning qiymatlarini qo`ysak,

$$F_{\max} = 1{,}49mN$$

3) tebranayotgan nuqtaning to`liq energiyasi istalgan vaqt oralig`i uchun kinetik va potensial energiyalarning yig`indisiga tengdir.

To`liq energiyani hisoblashning eng sodda yo`li uni kinetik energiya potensial energiya maksimal qiymatga erishganda hisoblashdir. Bu vaqtda potensial energiya nolga teng bo`ladi (yoki kinetik energiya). SHuning uchun ham tebranayotgan nuqtaning to`liq energiyasi W maksimal kinetik energiya W_{kmax} ga teng bo`ladi:

$$W = W_{k\max} = \frac{1}{2}m\upsilon_{\max}^2 \qquad (4)$$

Maksimal tezlik (2) formulaga asosan $\sin(\omega t + \varphi) = -1$ qo`yib

$$\upsilon_{\max} = 2\pi\nu A$$

Tezlikning ifodasini (4) formulaga qo`ysak

$$W = 2\pi^2m\nu^2A^2$$

Kattaliklarning qiymatlarini bu formulaga qo`yib hisoblaymiz:

$$W = 2\cdot(3{,}14)^2\cdot5\cdot10^{-3}\cdot(0{,}5)^2\cdot(3\cdot10^{-2})^2 J = 22{,}1\cdot10^{-6} J = 22{,}1 mkJ$$

53-masala. $x_1 = A_1\cos\omega(t+\tau_1); x_2 = A_2\cos\omega(t+\tau_2)$ tenglamalar bilan ifodalanadigan, bir xil yo`nalishli ikkita tebranish qo`shiladi. Bunda A_1=1 sm, A_2=2sm

$$\tau_1 = \frac{1}{6}s,\ \ \tau_2 = \frac{1}{2}s,\ \omega = \pi s^{-1}$$

1) qo`shiluvchi tebarnishlarning boshlang`ich fazalari φ_1 va $\varphi_2$lar aniqlansin; 2) natijaviy tebranishning amplitudasi A va boshlang`ich fazasi φ topilsin. Natijaviy tebranishning tenglamasi yozilsin.

Berilgan: A_1=1 sm$=1\cdot10^{-2}m$, A_2=2sm $=2\cdot10^{-2}m$,

$$\tau_1 = \frac{1}{6}s,\ \ \tau_2 = \frac{1}{2}s,\ \omega = \pi s^{-1}$$

$\varphi_1 \sim ?,\ \varphi_2 \sim ?\ \varphi \sim ?\ A \sim ?$

Echish. 1. Garmonik tebranishning tenglamasi

$$x = A\cos(\omega t + \varphi) \qquad (1)$$

ko`rinishga ega. Masala shartida berilgan tenglamalarni (1) ko`rinishga keltiramiz

$$x_1 = A_1\cos\omega(t+\tau_1); x_2 = A_2\cos\omega(t+\tau_2) \qquad (2)$$

(2)ifodadan (1) tenglik bilan solishtirishdan birinchi va ikkinchi tebranishlarning boshlang`ich fazalarini topamiz:

$$\varphi_1 = \omega\tau_1 = \frac{\pi}{6}rad \text{ va } \varphi_2\omega = \tau_2 = \frac{\pi}{2}rad$$

2) natijaviy tebranishning amplitudasi A ni aniqlash uchun kosinuslar teoremasidan foydalanamiz (14,4-§)

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\Delta\varphi} \qquad (3)$$

bu yerda $\Delta\varphi$-qo`shiluvchi tebranishlarning fazalar farqi $\Delta\varphi = \varphi_2 - \varphi_1$bo`lganligidan, φ_2 *va* φ_1 larning topilgan qiymatlarini o`rniga qo`ysak,

$$\Delta\varphi = \frac{\pi}{3}rad$$

A_1, A_2 va φ_1, φ_2 larning qiymatlarini (3) formulaga qo`yib hisoblasak

A=2,65 sm

Natijaviy tebranishning boshlang`ich fazasi tangensini (14,4-§) dagi 14,7 rasmdan aniqlaymiz

$tg\varphi = \frac{A_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2}$ bundan boshlang`ich faza

$$\varphi = arctg\frac{A_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2}$$

A_1, A_2 , φ_1 va φ_2 larning qiymatlarini qo`yamiz va hisoblaymiz:

$$\varphi = arctg\left(\frac{5}{\sqrt{3}}\right) = 70{,}9 = 0{,}394\pi rad$$

54-masala. Moddiy nuqta bir paytning o`zida tenglamalari

$$x = A_1 \cos\omega t \quad (1)$$

$$y = A_2 \cos\frac{\omega}{2} t \quad (2)$$

ko`rinishda bo`lgan ikkita o`zaro tik garmonik tebranishda ishtirok etadi. Bunda A_1=1sm, A_2=2sm, $\omega = \pi s^{-1}$. Nuqta traektoriyasining tenglamasi topilsin.

Berilgan: $A_1 = 1sm = 1\cdot 10^{-2} m,\ A_2 = 2sm = 2\cdot 10^{-2} m,\ \omega = \pi s^{-1}$

Traektoriya tenglamasi ~?

Echish. Nuqta traektoriyasining tenglamasini topish uchun, berilgan (1) va (2) tenglamalaridan t vaqtni yo`qotamiz. Bu maqsadda $\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1}{2}(1+\cos\alpha)}$ formulasidan foydalanamiz. U holda $\alpha = \omega t$, shuning uchun

$$y = A_2 \cos\frac{\omega}{2} t = A_2\sqrt{\frac{1}{2}(1+\cos\omega t)}$$

(1) formulaga binoan $\cos\omega t = \frac{x}{A_1}$ ekanligidan, traektori tenglamasi

$$y = A_2\sqrt{\frac{1}{2}(1+\frac{x}{A_1})} \quad (3)$$

hosil bo`lgan ifoda o`qi OX o`qi bilan mos keluvchi parabola tenglamasidir.

Mustaqil yechish uchun masalalar

157.Amplitudasi A=0,1m, davri T=4 s va boshlang`ich fazasi nolga teng bo`lgan garmonik tebranma harakat tenglamasini yozing. (x=0,1Sin0,5πt m).

158.Garmonik tebranish amplitudasi A=5sm, davri T=4 s ga teng. Tebranayotgan nuqtaning maksimal tezligi va uning maksimal tezlanishini toping. (υ_{max}=7,85·10^{-2}m/s; a_{max}=12,3·10^{-2}m/s^2).

159.Garmonik tebranishning boshlang`ich fazasi nolga teng. Nuqta muvozanat vaziyatidan $x_1$=2,4 sm siljiganda nuqtaning tezligi $\upsilon_1$=3sm/s bo`ladi, x_2=2,8 sm siljiganda esa nuqtaning tezligi υ_2=2sm/s bo`ladi. SHu tebranishning amplitudasi va davri topilsin. (A=3,1·10^{-2}m; T=4,1s).

160.Garmonik tebranma harakat qilayotgan jismning to`la energiyasi W=3·10$^{-5}$J, jismga ta’sir etuvchi maksimal kuch F=1,5·10$^{-3}$N ga teng. Tebranish davri T=2 s va boshlang`ich faza φ=60º bo`lsa, bu jismning harakat tenglamasini yozing. $\left[x = 0{,}04Sin\left(\pi t + \frac{\pi}{3}\right)m\right]$.

161.Prujinaga R=98,1N yuk osilgan. Prujina F=9,8 N kuch ta’sirida Δx=1,5sm cho`zilishi ma’lum bo`lsa, yukning vertikal tebranish davrini aniqlang. (T=0,78s).

162.Nuqta garmonik tebranmoqda. Nuqtaning eng katta siljishi x_{max}=10sm, eng katta tezligi υ_{max}=20 sm/s ga teng. Tebranishning doimo

takrorlanib turuvchi sikl chastotasi ω va maksimal tezlanishi topilsin. ($\omega=2s^{-1}$; $a=40sm/s^2$).

163.Ikkita kamerton bir vaqtda ovoz chiqarmoqda. Ularning tebranish chastotalari ν_1 va ν_2 mos ravishda 440 va 440,5 Hnga teng. Tepkili tebranish davri T aniqlansin. (T=2s).

164.Moddiy nuqta bir vaqtning o`zida $x=A_1\cos\omega t$ va $u=A_2\cos 2\omega t$ tenglamalar bilan ifodalanuvchi ikkita o`zaro tik tebranishlarda ishtirok etadi, bunda $A_1=2sm$, $A_2=1sm$. Traektoriya tenglamasi topilsin. ($u=-2(A_2/A_1)\cdot x^2+A_2$; $y=-\frac{1}{2}x^2+1$).

165.Prujinaga osilgan m=250g massali yuk T=1s davr bilan tik yo`nalishda tebranadi. Prujinaning bikrligi K aniqlansin. (K=9,87N/m).

166.Agar qurilmaning xususiy tebranishlar davri T=1s va tebranishning logariflik dekrementi $\lambda=0,628$ bo`lsa, so`nuvchi tebranishlarning davri T topilsin. (T=1,005s).

167.Tebranish tizimi $\nu=1000$ Hn chastotali so`nuvchi tebranishlarni bajarmoqda. Agar rezonans chastota $\nu_{rez}=998$ Hn bo`lsa, xususiy tebranishlar chastotasi ν_0 aniqlansin. ($\nu_0=1002Hz$).

168.$\nu_1=400Hn$ va $\nu_2=600Hn$ chastotalarda majburiy garmonik tebranishlar amplitudasi bir-biriga teng. Rezonans chastota ν_{rez} aniqlansin. So`nish hisobga olinmasin. ($\nu_{rez}=510Hz$).

169.R=30sm radiusli bir jinsli disk uning silindrik sirtining tashkil etuvchilaridan biri orqali o`tuvchi gorizontal o`q atrofida tebranmoqda. Uning tebranish davri T qanday. $\left(T=2\pi\sqrt{\frac{3R}{(2g)}}=1,35s\right)$.