MAE 5010 | DATA ASSIMILATION

HOMEWORK ASSIGNMENT 8

(Due: April 16th class time)

Feel free to use any computer language you like and you can use available packages (i.e., you do not have to write standard tools from scratch, you can use built-in packages).

Please report your findings clearly and concisely, and return via soft copy to Canvas>Assignments (or by email to osan@okstate.edu, you can embed code snaps into your report if you wish, or preferably you can provide GitHub links for your codes if it is easier for you).

Please study the materials provided in Module 8.4 Ensemble Reduced Rank Filter

https://www.youtube.com/watch?v=RUiv__byUkc

Lecture notes can be found at Canvas. There are 3 filtering methods described in this module: (i) stochastic EnKF (described in slides 6-10, with virtual observations), (ii) deterministic EnKF (slides 17-21, with W), and (iii) RRSQRT filter (based on SVD). Please see lecture notes for (iv) UKF method given by Module-9.3 Unscented Kalman Filter.

Please write a computer program and compare these 4 filters for the Lorenz 96 (L96) problem. Further details of the L96 problem can be found in hand written notes. See also: https://en.wikipedia.org/wiki/Lorenz_96_model

Here are specific parameters: Assume that your state vector has 36 variables and you can conduct all your experiments using forcing F=8.

Time window for testing and assimilation: $0 < t < 20$.

Solver: RK4 (or you can use built-in ode45) with $\Delta t = 0.02$ (1000 steps).

Observations: Generate 100 observations (every 10^{th} time index) at t=0.2, 0.4, …, 20.0. You can use $\sigma^2 = 0.0001$ (or something bigger) for observational noise. Set your observation operator (a linear operator of the state) as follows

(i) Collect 3 observations: x(12), x(24), x(36)
(ii) Collect 6 observations: x(6), x(12), x(18), x(24), x(30), x(36)
(iii) Collect 18 observations: x(2), x(4), …, x(36)

Initial condition: You can start at t=-5 from equilibrium solution (all x(i) =F, i=1,2,..,36), and you can add small perturbation to the 20th variable (i.e., u(20)=F+0.01). Run ODE integrator upto t=0 (using the same Δt) and save the solution at t=0 as your initial condition for your data assimilation cycle. You can perturb initial solution (at t=0) by some error from normal distribution $N(0, s^2)$, where s^2 is variance (i.e., often $s=10\sigma$). You can generate your initial ensemble from the same variance. Therefore, your initial covariance matrix can be computed from the same variance s^2.

Please plot the trajectory of x(36) and compare 4 different filtering methods with 3 different cases as observation operator. You can add observational dots, erroneous and true trajectories as baseline to your figures. Please draw some conclusions from your findings.