MAE 5010 | DATA ASSIMILATION

FINAL PROJECT

(Due: April 30th class time)

Feel free to use any computer language you like and you can use available packages (i.e., you do not have to write standard tools from scratch, you can use built-in packages).

Please report your findings clearly and concisely, and return via hard copy (you can embed code snaps into your report if you wish, or preferably you can provide GitHub links for your codes if it is easier for you).

This is an open-ended project by comparing at least two data assimilation method for a system governed by PDEs (sparse ODEs in discrete sense). Here is a brief definition of our final project:

We will apply and compare at least two different DA methods (could be, for example, 4D-VAR, FSM, and ENKF or any other variants) for a PDE system. Every student will select a different PDE problem from the following list, and make a final report and presentation (10+2 minutes) at the end of the course. Potential candidate examples are:

1-) 2D shallow water equations on Cartesian grid
2-) 2D shallow water equations on spherical grid
3-) 2D Navier-Stokes equations on Cartesian grid
4-) 3D Navier-Stokes equations on Cartesian grid
5-) 1D Burgers equation
6-) 2D Burgers equation
7-) 1D Kuramoto–Sivashinsky equation
8-) 1D convection-diffusion-reaction equation
9-) 2D convection-diffusion-reaction equation
10-) Any other time-space PDE that you will offer (e.g., 1D or 2D Euler equations for gas dynamics, Boussinesq equations, Kawahara equation etc..)

A general framework
a-) You need to write/develop a forward solver (a solver for the PDE): any space-time discretization/numerics will be OK.
b-) Assume some initial condition and run your model forward in time, then record some snapshots at certain time in the time evolution (may be 10 snapshots, 20 snapshots, 50 snapshots, 100 snapshots).
c-) From snapshots obtained in (b), downsample your data, e.g., let's say you are running your forward model on 1000 grid points, you can decide to collect data of 10% (100 grid pints), 20% (200 grid points), 50% (half grid points), or 100% (all grid points). This data will constitute clean input for your observations.
d-) Add some noise to your observation data using N(0, sigma2). Finally this data will be your observation data.
e-) Now, forget about the initial condition you introduce/put in (a). Start from any arbitrary initial guess

(may be 20% off data from the correct initial condition), and try to recover the correct initial condition, from the observation data provided in (d).
f-) Report your results for comparing/documenting at least two DA algorithms. Include demonstration of various sigma2 etc (a hand written report due to the last day of class). Report will include the definition of problem, numerics, DA models, problem set-ups, results, and conclusions.
g-) On the last day of the class (April 30th), everyone will make a ppt/beamer presentation (10+2 minutes) based on the data you provided in your report.

Note: If corona virus spread continues, there will be only final report.

A suggested template for report:

Title (4DVAR and EnKF for application to the Burgers equations with nonlinear observations)

Author/Affiliation (Name Lastname, Oklahoma State University)

Abstract (Max 200 worlds)

1. **Introduction** (About 4 paragraphs, (i)brief history of DA, (ii) some literature on Model 1, (iii) some literature on Model 2, (iv) key paragraph to highlight the present study.)
2. **Mathematical Models**
   2.1 PDE Model (present in continuum form)
   2.2 PDE Model (present in discrete form, presentation of underlying numerical methods)
   2.3 DA Model 1 (presentation and mathematical derivation of model 1)
   2.4 DA Model 2 (presentation and mathematical derivation of model 2)
   2.5 DA Model 3 (presentation and mathematical derivation of model 3, optional)
   2.6 … (optional)
3. **Supplementary Methods/Toolbox** (details of optimization or other methods used in this study)
4. **Problem Statement** (details of parameters (physical, numerical, statistical), number of observations, and any problem relevant definitions, definition of the cases examined)
5. **Results** (figures, tables, etc. Please explain each figure and table in the text)
6. **Concluding Remarks**
   References