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Assignment : Asymptotic Analysis Of Algorithm.

1) Compute Sums :

a) $\sum_{i=3}^{n+1} 1 = n+1-3+1 \quad \cdots \sum_{i=l}^{u} 1 = u-l+1$

$= n-2+1$

$= n-1 \approx O(n)$

b) $\sum_{i=3}^{n+1} i = \frac{\cancel{n+1}(n+1)(n+1+1)}{2} - 3$

$= \frac{(n+1)(n+2)}{2} - 3$

$= \frac{(n+1)(n+\cancel{1}2) - 6}{2} \approx O(n^2)$

c) $\sum_{i=3}^{n+1} i \times (\cancel{i+1}) = \sum_{i=3}^{n+1} i^2 + \sum_{i=3}^{n+1} i$

$\left[\sum_{i=1}^{n+1} i^2 - \sum_{i=1}^{2} i^2\right] + \sum_{i=3}^{n+1} i$

$= \cancel{2}\left[\frac{n+1(n+2)(2(n+1)+1)}{6} - \frac{(2)\cancel{(2)}3(5)}{6}\right] +$

$\frac{(n+1)(n+2) - 6}{2}$

$= \left[\frac{(n+1)(n+2)(2n+2+1)}{6} \cancel{3} - 5\right] +$

$\frac{(n+1)(n+2) - 6}{2}$

$$= \left[\frac{(n+1)(n+2)(2n+3)}{6} - 5\right] +$$

$$\left[\frac{(n+1)(n+2)}{2} - 6\right]$$

$$= \left[\frac{(n+1)(n+2)(2n+3) - 30}{6}\right] +$$

$$\left[\frac{(n+1)(n+2) - 6}{2}\right] \approx O(n^3)$$

Q1)4) $\sum_{i=3}^{n+1} \frac{1}{i(i+1)} = \sum_{i=3}^{n+1} \frac{1}{i} - \sum_{i=3}^{n+1} \frac{1}{i+1}$

$$= \left[\frac{1}{3} + \cdots + \frac{1}{n+1}\right] - \left[\frac{1}{4} + \cdots + \frac{1}{n+2}\right] \quad \cdots (1)$$

the terms before $i=3$, would be

$$\left[1 + \frac{1}{2}\right] - \left[\frac{1}{2} + \frac{1}{3}\right] = \frac{2}{3} \quad \cdots (2)$$

Equat^n 1 would become

$$= \log(n+1) - \log(n+2) - \frac{2}{3}$$

$$= \cancel{\log\left(\frac{n+1}{n+2}\right)} \approx O(\log(n+1)).$$

Q2)

(a) $T(N) = T(N-1) + 5$ for $n > 1$, $T(1) = 0$

$\therefore T(n-1) = T(N-1-1) + 5$
$= T(N-2) + 5$

$= T(N-2) + 5 + 5$
$= T(N-2) + 2(5)$

$\therefore T(N-2) = T(N-2-1) + 5$
$= T(N-3) + 5$
$= T(N-3) + 3(5)$
$\vdots$
$= T(N-k) + k(5)$

$\therefore k = N-1$ $= T(N-(N-1)) + (N-1)(5)$
$= T(1) + (N-1)(5)$

$= 0 + (N-1)(5)$
$= O(N)$

b) $T(N) = 3T(N-1)$ for $n > 1$, $T(1) = 4$

$T(N-1) = 3T(N-2)$

$= 3[3T(N-2)]$

$\therefore T(N-2) = 3T(N-2-1)$
$= 3T(N-3)$

$T(N) = \cancel{(3)[T}\ (3)^2 + (N-2)$

$= (3)^2 [3T(N-3)]$

$= (3)^3 T(N-3)$

$\vdots$

$= (3)^{k+1} T(N-(k+1))$

$k+1 = N-1$
$k = N-2$

$= (3)^{(N-2)+1} T(N-(N-1))$

$= (3)^{N-1} T(1)$

$= (3)^{N-1} [4]$

$= 4(3^{N-1})$

$= \theta(3^{N-1})$

c) $T(N) = T(N/3) + 1$

$T(N/3) = T(N/(3)^2) + 1$

$= T[N/(3)^2] + 1 + 1$

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$= T\left(N/(3)^2\right) + 2$

$\therefore T\left(N/(3)^2\right) = T\left(N/(3)^3\right) + 1$

$= T\left(N/(3)^3\right)$

$= T\left(N/(3)^3\right) + 3$

$\vdots$

$= T\left(N/(3)^k\right) + k.$

$k = \log_3 N$

$3^k = N$

$= T(1) + \log_3 N$

$= 1 + \log_3 N$

$= O\left(\log_3 N\right)$

Q3)

a) $T(N) = 2T(N/2) + N^4.$

$a = 2 \quad b = 2$

$n^{\log_b a} = n^{\log_2 2} = n$

$\log_2 2 = 1$

$\therefore$ Case 3 it is.

$\therefore T(n) = \Theta(f(n))$
$= \Theta(N^4)$

2) $T(N) = T(9N/10) + N$

~~$a=9 \quad b=10$~~

$\cancel{N^{\log_b^a}} = \cancel{N^{\log_{10}^9}}$

$a=1 \quad b = 10/9$

$\therefore N^{\log_b^a} = N^{\log_{10/9}^1} = N^0 = 1$

Case 3:

$T(n) = \Theta(f(n))$
$= \Theta(N)$

3) $T(N) = 16T(N/4) + N^2$

$\Rightarrow a = 16 \quad b = 4$

$N^{\log_b^a} = N^{\log_4^{16}} = N^2$

Case 2:
$\because f(n) = \Theta(n^{\log_b^a})$

$\therefore T(n) = \Theta(N^2 \log N)$

4) $T(N) = 2T(N/4) + \sqrt{N}$

$a = 2 \quad b = 4$

$\therefore N^{\log_b^a} = N^{\log_4^2} = N^{1/2}$

$\therefore$ Case 2:-

$f(n) = \Theta(N^{\log_b^a})$

$\therefore T(N) = \Theta(N^{1/2} \log N)$

Q3 (e) $T(N) = T(N-1) + N$

$T(N-1) = T(N-2) + N-1$

$\therefore T(N) = T(N-2) + N-1 + N$

$T(N-2) = T(N-3) + N-2$

$\therefore T(N) = T(N-3) + N-2 + N-1 + N$

$\vdots$

$= T(0) + \sum_{k=1}^{N} k$

$= \frac{N(N+1)}{2}$

$= O(N^2)$

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f) $T(N) = T(\sqrt{N}) + 1$

$m = \log N \quad \therefore N = 2^m$

$T(2^m) = T(2^{m/2}) + 1$

$\therefore f(m) = T(2^m)$
$= f(m/2) + 1$

$a = 1 \quad b = 2 \quad f(N) = 1$

$g(N) = N^{\log_2 1} = N^0 = 1$

$f(N) = g(N)$

$\therefore T(N) = T(2^m) = S(m)$
$= \Theta(\log m)$
$= \Theta(\log(\log N))$

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