Q2) Sketchbook : Time complexity Analysis for Recurrences

size = n
Basic Operation = BiRec(floor(n/2)) +1

$T(n) = T(n/2) + 1$
$= T(n/2^2) + 1 + 1$ $\quad \therefore T(n/2) = T(n/2/2) + 1 = T(n/2^2) + 1$

$= T(n/2^2) + 2$ $\quad \therefore T(n/2^2) = T(n/2^2/2) + 1 = T(n/2^3) + 1$

$= T(n/2^3) + 3$

$= T(n/2^k) + k$ $\quad \therefore n/2^k$

$\cancel{= T(0) +}$ $\quad \therefore k = \log_2 N$

$= T(1) + \log_2 n$ $\quad \therefore$ for least value

$= O(\log n)$

$n/n = 1$
$\therefore 2^k = n$
$\therefore k = \log_2 n$

(2.1) $T(N) = 4T(N/2) + N$

$$= 4\left[4T(N/2^2) + N/2\right] + N$$

$$= (4)^2 T(N/2^2) + 4\frac{N}{2} + N$$

$$= (4)^2 T(N/2^2) + 2N + N$$

$$= (4)^2 T(N/2^2) + 3N$$

$\vdots$

$$= (4)^k T(N/2^k) + (k+1)N$$

$\cancel{4}^k = 2^k = N$

$\therefore k = \log_2 N$

$$= (4)^{\log_2 N} T(\cancel{N/}1) + (\log_2 N + 1) N$$

$$= N^{\log_2 4} + N\log_2 N + N$$

$$= N^2 + N\log_2 N + N \quad \because \log_2 4 = 2$$

$$= \Theta(N^2)$$

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(2.2) $T(N) = 2T(N/2) + N$

$= 2\left[2T(N/2^2) + N/2\right] + N$

$= (2)^2 T(N/2^2) + \frac{2N}{2} + N$

$= (2)^2 T(N/2^2) + N + N$

$= (2)^2 T(N/2^2) + 2N$

$\vdots$

$= (2)^k T(N/2^k) + kN$

$2^k = N \therefore k = \log_2 N$

$= (2)^{\log_2 N} T(N/N) + (\log_2 N) N$

$= N^{\log_2 2} + N(\log_2 N)$

$= N + N \log_2 N$

$= \Theta(N \log_2 N)$

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(2.3) $T(N) = 2T(N/2) + N^2$

$$= 2\left[2T\left(N/2^2\right) + \left(N/2\right)^2\right] + N^2$$

$$= (2)^2 T\left(N/2^2\right) + \frac{2N^2}{4} + N^2$$

$$= (2)^2 T\left(N/(2)^2\right) + \frac{N^2}{2} + \cancel{2}N^2$$

$$= (2)^2 T\left(N/2^2\right) + 3N^2$$

$$\vdots$$

$$= (2)^k T\left(N/2^k\right) + (k+1)N^2$$

$$2^k = N \quad \therefore \quad k = \log_2 N$$

$$= \left(\log_2 N\right) + \left[\left(\log_2 N\right) + 1\right] N^2$$

$$= \log_2 N + N^2 \log_2 N + N^2$$

$$= \Theta(N^2)$$