Introduction to Show Beamer Theme

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19. Let X be the number of working pumps befor the third non-working pump is found. Then, $X \sim \text{Neg}(r = 3, p = 0.2)$.
Let T be the total time (in minute) for testing and repairing the pumps. Then $T = 10X + 3(30) = 10X + 90$. Thus,

$$
\begin{aligned}
E(T) &= E(10X + 90) = 10E(X) + 90 \\
&= 10\left(\frac{r(1-p)}{p}\right) + 90 = 10\left(\frac{3(0.8)}{0.2}\right) + 90 = \cdots \\
V(T) &= V(10X + 90) = 10^2 V(X) \\
&= 100 \times \frac{r(1-p)}{p^2} = 100\left(\frac{3(0.8)}{(0.2)^2}\right) = \cdots
\end{aligned}
$$

23. Let X be the number of failed diodes among the 200 diodes. Then, $X \sim \text{Bin}(n = 200, p = 0.01)$
   - a. $E(X) = np = (200)(0.01) = 2$ and $\sigma_X = \sqrt{npq} = \sqrt{(200)(0.01)(0.99)} = \cdots$
   - b. Since n is large, p is small and $np = 2 < 5$ then X is approximately Poisson distributed. That is $X \sim \text{Poi}(\lambda = \mu_X = 2)$.

$$P(X \geq 4) = 1 - P(X < 4) \approx 1 - \sum_{x=0}^{3} \frac{e^{-2} 2^x}{x!} = \cdots$$

- c Let Y be the number of boards among the five selected boards that works. Then, $Y \sim \text{Bin}(n = 5, p = P(X = 0))$. Since

$$p = P(X = 0) \approx \frac{e^{-2}2^0}{0!} = e^{-2} \approx 0.1353$$
$$q = 1 - p = 1 - e^{-2} \approx 0.8647$$

Then, $P(Y = y) = \binom{5}{y}(0.1353)^y(0.8647)^{5-y}$ and

$$P(Y \geq 4) = P(Y = 4) + P(Y = 5) = \cdots$$

24 The set of possible values of X is $D = \{x \in \mathbb{N} : x \geq 0\}$ and $a > 0$.

$$\begin{aligned}\sum_{x\geq 0} xP(X=x) &= \sum_{0\leq x<a} xP(X=x) + \sum_{x\geq a} xP(X=x)\\ &\geq \sum_{x\geq a} xP(X=x)\end{aligned}$$

But $\sum_{x\geq a} xP(X=x) \geq \sum_{x\geq a} aP(X=x)$ then

$$\sum_{x\geq 0} xP(X=x) \geq \sum_{x\geq a} aP(X=x)$$
$$E(X) \geq aP(X \geq a)$$

Therefore, $P(X \geq a) \leq \dfrac{E(X)}{a}$.

25 We have $a > 0$.

$$\begin{aligned} P(|X - E(X)| \geq a) &= P([X - E(X)]^2 \geq a^2) \\ &\leq \frac{E([X - E(X)]^2)}{a^2} \\ &= \frac{E[(X - \mu_X)^2]}{a^2} \\ &= \frac{V(X)}{a^2} \end{aligned}$$

Thus, $P(|X - E(X)| \geq a) \leq \dfrac{V(X)}{a^2}$.

26 We know that Y is the number of customers per day at a sales counter. The distribution of Y is not known.

$$\begin{aligned} P(16 < Y < 24) &= P(-4 < Y - 20 < 4) \\ &= P(|Y - 20| < 4) \\ &= 1 - P(|Y - 20| \geq 4) \\ &\geq 1 - \frac{V(Y)}{4^2} \\ &= 1 - \frac{2^2}{4^2} = 0.75 \end{aligned}$$

Thus, we infer that $P(16 < Y < 24) \geq 0.75$.

27 We know that X is random with $\mu_X = 11$ and $V(X) = 9$.

a Find a lower bound for $P(6 < X < 16)$

$$\begin{aligned} P(6 < X < 16) &= P(-5 < X - 11 < 5) = P(|X - \mu_X| < 5) \\ &= 1 - P(|X - \mu_X| \geq 5) \geq 1 - \frac{V(X)}{5^2} \\ &= 1 - \frac{9}{5^2} = \frac{16}{25} \end{aligned}$$

Thus, a lower bound of $P(6 < X < 16)$ is $\frac{16}{25}$.

b Find value of C such that $P(|X - 11| \geq C) \leq 0.09$. We know that

$$P(|X - 11| \geq C) \leq \frac{V(X)}{C^2} = \left(\frac{3}{C}\right)^2$$

To have $P(|X - 11| \geq C) \leq 0.09$, we let $\left(\frac{3}{C}\right)^2 \leq 0.09$.

That is $C \geq 10$.