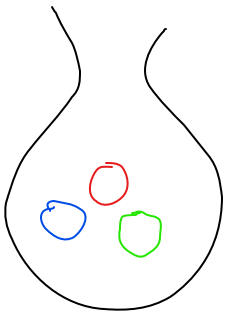
Counting

Recap

Typically 4 scenarios

Consider an urn with 3 balls, R, G, B.



Total Outcomes when choosing 2 balls?

Case 1

Drawing with replacement (Ordered Result)

$S = \{RR, RG, RB, GR, GG, GB, BR, BG, BB\}$ # Outcomes

$|S| = 3 \cdot 3 = 3^2$

(Multiplication)

Case 2

Drawing w/o replacement (Ordered Result)

$S = \{RG, RB, GR, GB, BR, BG\}$

$|S| = 3 \cdot 2 = P_2^3 = 6$

(Permutation)

Case 3

Drawing w/o Replacement (Unordered Result)

$S = \{\{R,G\}, \{R,B\}, \{B,G\}\}$

$|S| = \binom{3}{2} = 3$

(Combination)

Case 4 (Don't worry about for quiz)

Drawing with replacement (Unordered Result)

$S = \{\{R,R\}, \{R,G\}, \{R,B\}, \{B,B\}, \{B,G\}, \{G,G\}\}$

$|S| = \binom{2+3-1}{2} = 6$

(Stars & Bars Argument)

Stars & Bars (continued)

Consider

m stars (m – # of balls selected)

$n-1$ bars (n – # of balls in urn)

$$|S| = \binom{m+n-1}{n-1}$$

$m = 2$ (# Red, # Green) $n = 3$ (# Blue) $\binom{m \text{ stars}}{n-1 \text{ bars}}$

$*\,|\,*\,|$ corresponds to $\{R, G\}$

$|\,*\,|\,*$ corresponds to $\{G, B\}$

$|\,|\,*\,*$ corresponds to $\{B, B\}$

The bars act as arbitrary partitions

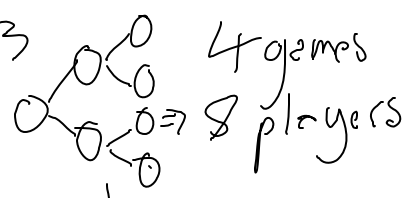
(In this case, seperate blue from red from green)

* The partitions are for us to make sense of the problem

We are essentially counting the number of ways we can reorder the stars and bars sequence

m stars, $n-1$ bars, so

we have $|S| = \binom{m+n-1}{n-1}$

PS Q3 $n=3$  4 games, 8 players

i. n rounds

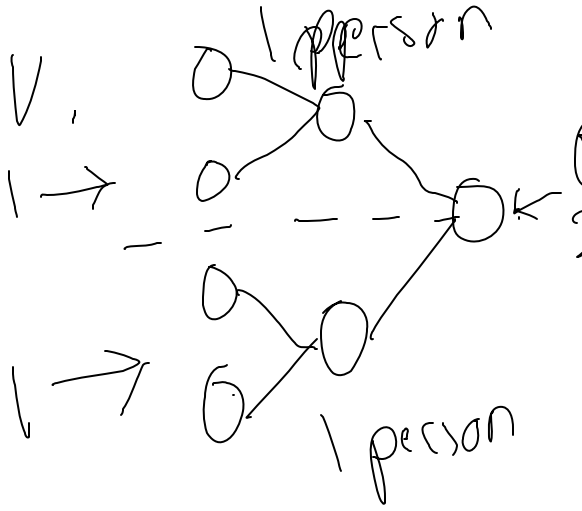
ii. i-th round: 2^{n-i} games $\quad i=1,2,\dots,n$

$$\sum_{i=1}^{n} 2^{n-i} = \sum_{j=0}^{n-1} 2^j \quad (j=n-i)$$

$$= \frac{2^n-1}{2-1}$$

$$= 2^n-1$$

iv.



A : event of winning players meeting in finals

1st round → 2^n players

$$P(A) = \frac{|A|}{|S|} \qquad |A| = \frac{2^n}{2} = 2^{n-1}$$

(since you fix one winning player on one half, the other winning player must be on other half)

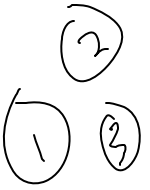
$$|S| = 2^n - 1$$

(One start position is used by first player)

$$P(A) = \frac{2^{n-1}}{2^n-1}$$

WS

4.



$2 \times 2 \times 2 \times 2 = 16$