

Q6.i.

Goal: Find p such that $pW - (1-p)L = 0$

$$pW - (1-p)L = 0$$

$$\Rightarrow pW - L + pL = 0$$

$$\Rightarrow p(W+L) = L$$

$$\Rightarrow p = \frac{L}{W+L}$$

Q6.ii. Note that for this problem, A and A^c are predefined.

Don't assume $A \cup A^c = S$.

We have A w/ odds 3/1

A^c w/ odds 1/1

$$\text{By i. } p = \frac{L}{W+L} = \frac{1}{3+1} = \frac{1}{4}$$

$$q = \frac{L}{W+L} = \frac{1}{1+1} = \frac{1}{2}$$

$$P(A \cup A^c) = P(A) + P(A^c) \quad (\text{axiom 3})$$

$$= \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

$$\text{But } P(A \cup A^c) = P(S) \quad (\text{set theory})$$

$$= 1 \quad (\text{axiom 2})$$

Thus, we have contradiction,
and does not satisfy axioms.

Q6.iii.

Recall that if you correctly bet
on A (odds $3/1$), then you get
the original amount you put in (L)
plus your betting rewards ($\text{odds} \times L$)
since W scales proportionally to L .

If you incorrectly bet, then
you lose the original amount
you put in (L).

Example: For event B odds are $5/2$
If you bet \$4 on B and it occurs
then you get original amount (4)
+ rewards $(\frac{5}{2})L = 10$, so you get \$14. (\$10 profit)
If B^c occurs, you lose \$4.

Let b_A be the value you bet into event A .

b_{A^c} be the value you bet into event A^c .

Note: to get a winning profit each time,
you must put money into A and A^c .

Consider $b_A = 2, b_{A^c} = 3$

We want $b_A + b_{A^c} < W + L$

Regardless of outcome, you bet \$5.

If A occurs, recall you bet \$2 into A . ($L=2$)

Then $W + L = \frac{3}{1}L + L$ (We showed on winning case W scales by L)
 $= 3 \cdot 2 + 2$
 $= 8$

Note: No profits are made from A^c , so you lose the \$3 you put into it.

If A^c occurs, recall you bet \$3 into A^c ($L=3$)

Then $W + L = \frac{1}{2}L + L$ (W scales by L)
 $= 2L$
 $= 6$

Note: No profits are made from A , so you lose the \$2 you put into it.

Summary

Essentially, you put money into both A and A^c , and since exactly one of the events will win, you would only need to account for one case's profits on each outcome. In both cases the amount inputted is strictly \$5.

In the case that A occurs, you win \$8, and in the case A^c occurs, you win \$6. Thus, we can conclude that we will get a profit.