Problem Set #1

$\boxed{\text{Q6. i.}}$

Goal: Find p such that $pW-(1-p)L=0$

$$pW-(1-p)L=0$$
$$\Rightarrow pW-L+pL=0$$
$$\Rightarrow p(W+L)=L$$
$$\Rightarrow p=\frac{L}{W+L}$$

$\boxed{\text{Q.6.ii.}}$ Note that for this problem, A and A^c are <u>predefined</u>.

Don't assume $A\cup A^c=S$.

We have A w/ odds $3/1$

A^c w/ odds $1/1$

By i. $p=\frac{L}{W+L}=\frac{1}{3+1}=\frac{1}{4}$

$$q=\frac{L}{W+L}=\frac{1}{1+1}=\frac{1}{2}$$

$$P(A\cup A^c)=P(A)+P(A^c)\quad(\text{axiom } 3)$$
$$=\frac{1}{4}+\frac{1}{2}=\frac{3}{4}$$

But $P(A\cup A^c)=P(S)$ (set theory)

$=1$ (axiom 2)

Thus, we have contradiction, and does not satisfy axioms.

$\boxed{\text{Q6.iii.}}$

<u>Recall</u> that if you correctly bet on A (odds 3/1), then you get the original amount you put in (L) plus your betting rewards (W×L) since W scales proportionally to L.

If you incorrectly bet, then you lose the original amount you put in (L).

Exemple: For event B odds are 5/2
If you bet \$4 on B and it occurs
then you get original amount (4)
+ rewards $(\frac{5}{2})L = 10$, so you get \$14. (\$10 profit)

If B^c occurs, you lose \$4.

Let b_A be the value you bet into event A.
b_{A^c} be the value you bet into event A^c.

Note: to get a winning profit each time, you must put money into A and A^c.

Consider $b_A = 2$, $b_{A^c} = 3$

We want $b_A + b_{A^c} < W + L$

Regardless of outcome, you bet \$5.

If A occurs, recall you bet \$2 into A. ($L=2$)

Then $W+L = \frac{3}{1}(L) + L$ (We showed on winning case W scales by L)

$= 3 \cdot 2 + 2$

$= 8$

Note: No profits are made from A^c, so you lose the \$3 you put into it.

If A^c occurs, recall you bet \$3 into A^c ($L=3$)

Then $W+L = (\frac{1}{1})L + L$ (W scales by L)

$= 2L$

$= 6$

Note: No profits are made from A, so you lose the \$2 you put into it.

Summary

Essentially, you put money into both A and A^c, and since exactly one of the events will win, you would only need to account for one case's profits on each outcome. In both cases the amount inputted is strictly \$5.

In the case that A occurs, you win \$8, and in the case A^c occurs, you win \$6.