Home Assignment 2.

1. In a conducting Medium the magnetic field is given as

$$\bar{H} = xy^2z\hat{x} + 5(x+1)y^2z^2\hat{y} + (x+10)y^2z^2\hat{z}$$ square loop

$y = 2,\ 0 < x < 3,\ 0 < z < 1.$

Ampere's law

$$\bar{J} = \nabla \times H$$

$$\nabla \times \bar{H} = \begin{vmatrix} x & y & z \\ \partial/\partial x & \partial/\partial y & \frac{\partial}{\partial z} \\ xy^2z & 5(x+1)y^2z^2 & (x+10)y^2z^2 \end{vmatrix}$$

$$\nabla \times H = \hat{x}\left(\frac{\partial([x+10]y^2z^2)}{\partial y} - \frac{\partial[5(x+1)y^2z^2]}{\partial z}\right)$$

$$+\hat{y}\left(\partial[xy^2z] - \frac{\partial[(x+10)y^2z^2]}{\partial x}\right) + \hat{z}\left(\frac{\partial[5(x+1)y^2z^2]}{\partial x} - \frac{\partial(xy^2z)}{\partial y}\right).$$

$$J = \left(\left[2(x+10)yz^2\right] - \left[10(x+1)y^2z\right]\right)\hat{x} + \left[xy^2 - y^2z^2\right]\hat{y} + \left[5y^2z^2 - 2xyz\right]\hat{z}$$

at $(4, 1, -1)$

$$J = \left[(2(4+10)(1)(-1)^2 - 10(4+1)(1)^2(-1)\right]\hat{x} +$$

$$\left[4(1)^2 - (1)^2(-1)^2\right]\hat{y} + \left[5(1)^2(-1)^2 - 2(4)(1)(-1)\right]\hat{z}$$

$$J = 78\hat{x} + 3\hat{y} + 13\hat{z}\ A/m^3$$

2. Find $\iint F \cdot ds$ using divergence theorem, where

$F = 6xyz\,i + 24x\,j + 2yz\,k$ and S is the surface of the cube bounded by $x=0$, $x=1$, $y=0$, $y=1$ and $z=0$, $z=1$

A:- $\iint F \cdot ds = F = (6xyz)\hat{i} + (24x)\hat{j} + (2yz)\hat{k}$

~~$\iint F \cdot ds$~~ $= \iiint v.$

$\iint_S F \cdot ds = \iiint_V (\nabla \cdot F) dv$

$= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$

$= 6yz + 0 + 2y$

$= 6yz + 2y$

$= \int_0^1\int_0^1\int_0^1 (6yz + 2y)\, dx\, dy\, dz$

w.r.t $x = \int_0^1 dx = 1 = \int_0^1\int_0^1 (6yz + 2y)\, dy\, dz$

w.r.t $y = \int_0^1 6yz\, dy + \int_0^1 2y\, dy = 3z + 1$

w.r.t $z = \int_0^1 (3z+1)\, dz = \frac{3}{2} + 1 =$

$= \frac{5}{2}$

3. Find the electric field intensity at a distance of 40cm from a charge of $4 \mu c$ in vacuum

A: Given,

$r = 40cm$

$Q = 4 \mu c$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$E = \frac{4 \times 10^{-6}}{4 \times 3.14 \times 8.854 \times 10^{-2} \times (40 \times 40)^2 cm}$$

$$E = \frac{4 \times 10^{-6} \times (9 \times 10)^9}{(0.40)^2}$$

$$= \frac{36 \times 10^3}{0.16}$$

$$= 225 \times 10^3$$

$$= 2.25 \times 10^5 \ v/m$$

$$\boxed{E = 2.25 \times 10^5 \ v/m}$$