EMTL -

Home Assignment - 1

1. The volume charge desity inside a hallow sphere is $\rho = 10e^{-20r}$ C/m³. Find the total charge enclosed with the sphere. Also Find the electric flux density on the surface of the sphere.

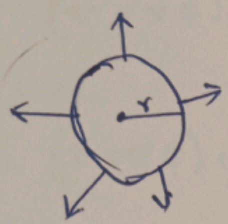
A:- Total charge enclosed :-

$$Q = \int \rho \, dv$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{2} e \, r^2 \sin\theta \, dr \cdot d\theta \cdot d\phi$$

$$= \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta \int_0^{2} (10e^{-20r}) r^2 \, dr$$

$$= \frac{\pi}{100} C$$



electric displacement ~~uniform~~

$$D = \frac{\pi}{100}$$

$$D = \frac{Q}{4\pi r^2} = 0.25 \times 10^{-14} \text{ C/m}^2$$

2) The electric flux density is given as $\bar{D} = x^2 \bar{x} + xy\bar{y} + x^2y^2\bar{z}$ Find the charge density inside a cube of side 4meter Placed centered at ~~ver~~ the origin with its side along the Co-ordinate axes.

$$\nabla \cdot \bar{D} = \rho$$

$$\frac{\delta D_x}{\delta x} + \frac{\delta D_y}{\delta y} + \frac{\delta D_z}{\delta z} = \rho$$

$$\frac{\delta(x^2)}{\delta x} + \frac{\delta}{\delta y}(xy) + \frac{\delta}{\delta z}(x^2y^2) = \rho$$

$$2x + x = \rho$$

$$3x = \rho$$

$\therefore$ cube has 4 sides

$x = 0$ to 4

$$Q = \int_0^4 \int_0^4 \int_0^4 3x \, dx \, dy \, dz$$

$$Q = \left[\frac{3x^2}{2}\right]_0^4 \cdot [y]_0^4 \; [z]_0^4$$

$$= \left[\frac{3(4)^2}{2}\right] \cdot 4 \cdot 4$$

$$= \left[\frac{3 \times 16}{2} \times 4 \times 4\right]$$

$$= 24 \times 4 \times 4$$

$$= 24 \times 16$$

$$= 384.$$

$x = -2$ to 2

$$Q = \int_{-2}^{2} \int_{-2}^{2} \int_{-2}^{2} 3x \, dx \, dy \, dz$$

$$Q = \left[\frac{3x^2}{2}\right]_{-2}^{2} [y]_{2}^{2} [x]_{2}^{2}$$

$$Q = 0.$$

(3) Explain curl of gradient is zero.

The curl of gradient any scalarfeild is always zero. This is a fundamental property in vector calculus and can be expressed as

$$\nabla \times (\nabla \phi) = 0.$$

Gradient of Scalar feild:

$$\nabla \phi = \left(\frac{d\phi}{dx}, \frac{d\phi}{dy}, \frac{d\phi}{dz} \right)$$

Curl of Vector feild $F = (F_x, F_y, F_z)$

$$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ F_x & F_y & F_z \end{vmatrix}$$

$$\nabla \times \nabla \phi = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ \frac{d\phi}{dx} & \frac{d\phi}{dy} & \frac{\partial\phi}{\partial z} \end{vmatrix}$$

Curl of gradient of ϕ

$$(\nabla \times \nabla \phi)_x = \left(\frac{d^2\phi}{dydz} - \frac{\partial^2}{dzdy} \right) = 0$$

$$(\nabla \times \nabla \phi)_y = \left(\frac{d^2\phi}{dzdx} - \frac{\partial^2\phi}{\partial x\, \partial z} \right) = 0$$

$$(\nabla \times \nabla \phi)_z = \left(\frac{\partial^2\phi}{\partial x \partial y} - \frac{\partial^2\phi}{\partial y \partial x} \right) = 0$$

$$\nabla \times \nabla \phi = 0.$$

10/02/25