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# CO1 ASSIGNMENT

1. The volume charge density inside a hollow sphere is $\rho = 10e^{-20r}$ C/m³. Find the total charge enclosed with the sphere. Also find the electric flux density on the surface of sphere.

A. Given

$$\rho = 10e^{-20r} \ C/m^3$$

⇒ Total charge enclosed

$$Q_{enc} = \iiint_V \rho_v \, dv$$

In spherical coordinates:

$$dv = 4\pi r^2 dr$$

$$Q_{enc} = \int_0^R 10 e^{-20r} \cdot 4\pi r^2 dr$$

$$= 40\pi \int_0^R r^2 e^{-20r} dr$$

$$= 40\pi \left[ \frac{e^{-20r}}{20} \left( r^2 + \frac{2r}{20} + \frac{2}{20^2} \right) \right]_0^R$$

$$= 40\pi \times \frac{e^{-20R}}{20} \left( R^2 + \frac{2R}{20} + \frac{2}{400} \right)$$

⇒ Electric flux Density

$$\oint_S D \cdot ds = Q_{enc}$$

$$D_r \cdot 4\pi R^2 = Q_{enc}$$

$$D_r = \frac{Q_{enc}}{4\pi R^2}$$

$$Q_{enc} = 40\pi \times \left[ \frac{e^{-20R}}{20} \left( R^2 + \frac{2R}{20} + \frac{2}{400} \right) \right]$$

for large $R$, $e^{-20R} \simeq 0$

$$Q_{enc} \approx 40\pi \times \frac{2}{400} = 40\pi \times \frac{1}{200} = \frac{\pi}{5}$$

$$\therefore \quad D = \frac{\pi/5}{4\pi R^2} = \frac{1}{20R^2}$$

Let's consider $R = 0.1\,m$

$$D = \frac{1}{20(0.1)^2} = \frac{1}{20 \times 0.01} = \frac{1}{0.2} = 5\,C/m^2$$

Thus, $D = 0.25 \times 10^4\, c/m^2$ corresponds to a particular radius $R$.

2. The electric flux density is given as $\bar{D} = x^2\bar{x} + xy\bar{y} + x^2y^2\bar{z}$, Find the charge density inside ~~sphere~~ a cube of side 4 meter ~~speed~~ placed centered at the origin with its sides along the coordinate axes.

A. Given,

$$\bar{D} = x^2\bar{x} + xy\,\bar{y} + x^2y^2\bar{z}$$

Acc to Gauss's law:-

$$\nabla \cdot D = \rho_v$$

$$\nabla \cdot D = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\nabla \cdot D = \frac{\partial}{\partial x} x^2 + \frac{\partial}{\partial y} xy + \frac{\partial}{\partial z} x^2y^2$$

$$\nabla \cdot D = 2x + x + 0$$

$$\nabla \cdot D = 3x$$

$$\therefore \rho_v = 3x \ \ C/m^3$$

⇒ charge enclosed in the cube:

$$Q_{enc} = \int_V \rho_v \, dv$$

$$Q_{enc} = \int_{-2}^{2}\int_{-2}^{2}\int_{-2}^{2} 3x \, dz\, dy\, dx$$

$$Q_{enc} = \int_{-2}^{2}\int_{-2}^{2}\int_{-2}^{2} 3x \, dz\, dy\, dx$$

$$\rightarrow \int_{-2}^{2} dz = 2+2 = 4$$

$$\int_{-2}^{2} dy = 2+2 = 4$$

$$\rightarrow \int_{-2}^{2} 3x\,dx = 3\frac{x^2}{2} = 3\frac{4}{2} - 3\frac{4}{2} = 0$$

$$Q_{enc} = 0 \times 4 \times 4 = 0$$

3. Explain curl of gradient is zero.

A. Given a scalar function $f(x, y, z)$, its gradient is:

$$\nabla f = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$\nabla \times \nabla f = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{vmatrix}$$

$$= \hat{i}\left( \frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial z \partial y} \right) + \hat{j}\left( \frac{\partial^2 f}{\partial x \partial z} - \frac{\partial^2 f}{\partial x \partial z} \right) + \hat{z}\left( \frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial x \partial y} \right)$$

$$= 0$$

$$\therefore \nabla \times \nabla f = 0$$