DIGITAL COMMUNICATION

⇒ sampling:-

Any real time signal is in the form of continuous signals. To convert a continuous signal into digital signal, we need to follow the below steps:-

continuous signal
↓
Sampling
↓ Discrete
Quantization
↓
Encoder
↓
Digital signal.

⇒ continuous signal:-

The signal which is defined at every point of time.

⇒ sampling:-

It is a process in which we store the magnitude of signal after regular interval of time.

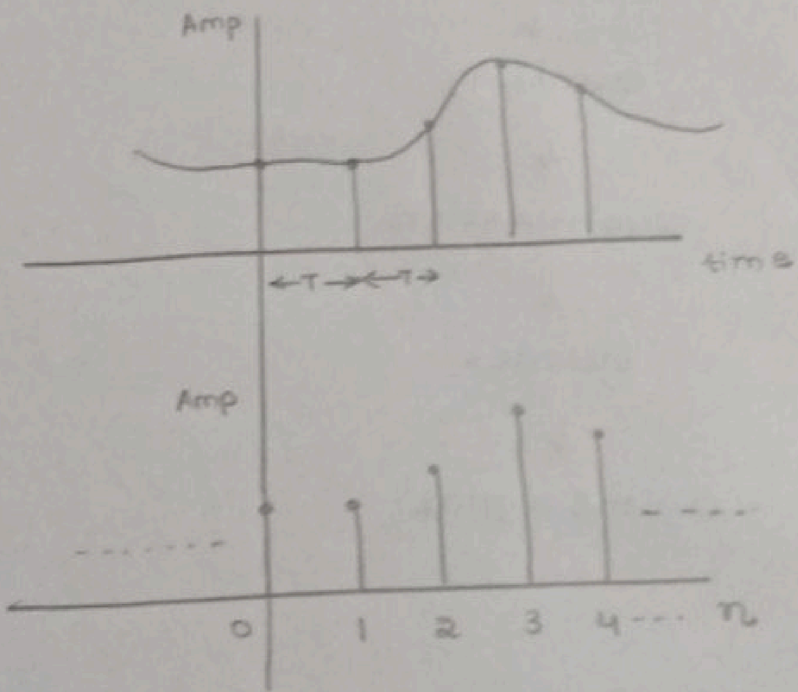
→ Discrete signal:

The signal which can take any value of amplitude

→ Digital signal:

The signal in which the magnitude is pre-defined or the pre-defined levels.

Ex: For suppose we have 5 bits, then we can take any level of the $2^5 = 32$ levels.



⇒ Sampling theorem:

It states that in order to construct the same signal without loosing any information, the message signal must be sampled at Nyquist rate.

$$\boxed{f_s > 2f_m}$$

Nyquist rate $= f_s$

Nyquist interval $= 1/f_s = T_s$

→ composite signal:-

The signal which is constructed out of different frequencies.

Let us consider the message signal is a composite signal which is made up of f_1, f_2, f_3 then,

$$f_1 < f_2 < f_3$$

f_m is the highest frequency which is f_3.

→ Determine sampling frequency of 40 sin(200πt)

we know that $A_m \sin(2\pi f_m t)$

$$f_s > 2f_m$$

By comparing $f_m = 100$

∴ sampling frequency should be greater than 200.

→ Determine sampling frequency of

$$30 \sin(100\pi t) + 60 \sin(300\pi t)$$

$$30 \sin(2\pi F_1 t) + 60 \sin(2\pi F_2 t)$$

By comparing

$$f_1 = 50, f_2 = 150$$

It is a composite signal so we have to consider higher frequency.

$$f_m = 150$$

∴ Sampling frequency > 300.

→ Determine sampling frequency of $30 \sin^2(100\pi t)$.

$$30 \sin^2(100\pi t)$$

$$\because \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\therefore f_s > 200$$

→ Determine sampling frequency of $30 \sin(100\pi t) . 60 \sin(300\pi t)$

$$\cos\left(\frac{A+B}{2}\right) - \cos\left(\frac{A-B}{2}\right)$$

$$\cos\left(\frac{100+300}{2}\right) - \cos\left(\frac{100-300}{2}\right)$$

wrong

$$\cos(200\pi t) - \cos(-100\pi t)$$

$$\cos(200\pi t) - \cos(100\pi t)$$

$$f_1 = 2$$

→ Determine sampling frequency of

$$30 \sin(100\pi t) . 60 \sin(300\pi t)$$

$$\sin a . \sin b = 1800 . \frac{\cos(100-300\pi t)}{2} - \frac{\cos(100\pi t + 300\pi t)}{2}$$

$$= 1800 . \cos(-100\pi t) - \cos(200\pi t)$$

$$f_1 = 100, f_2 = 200$$

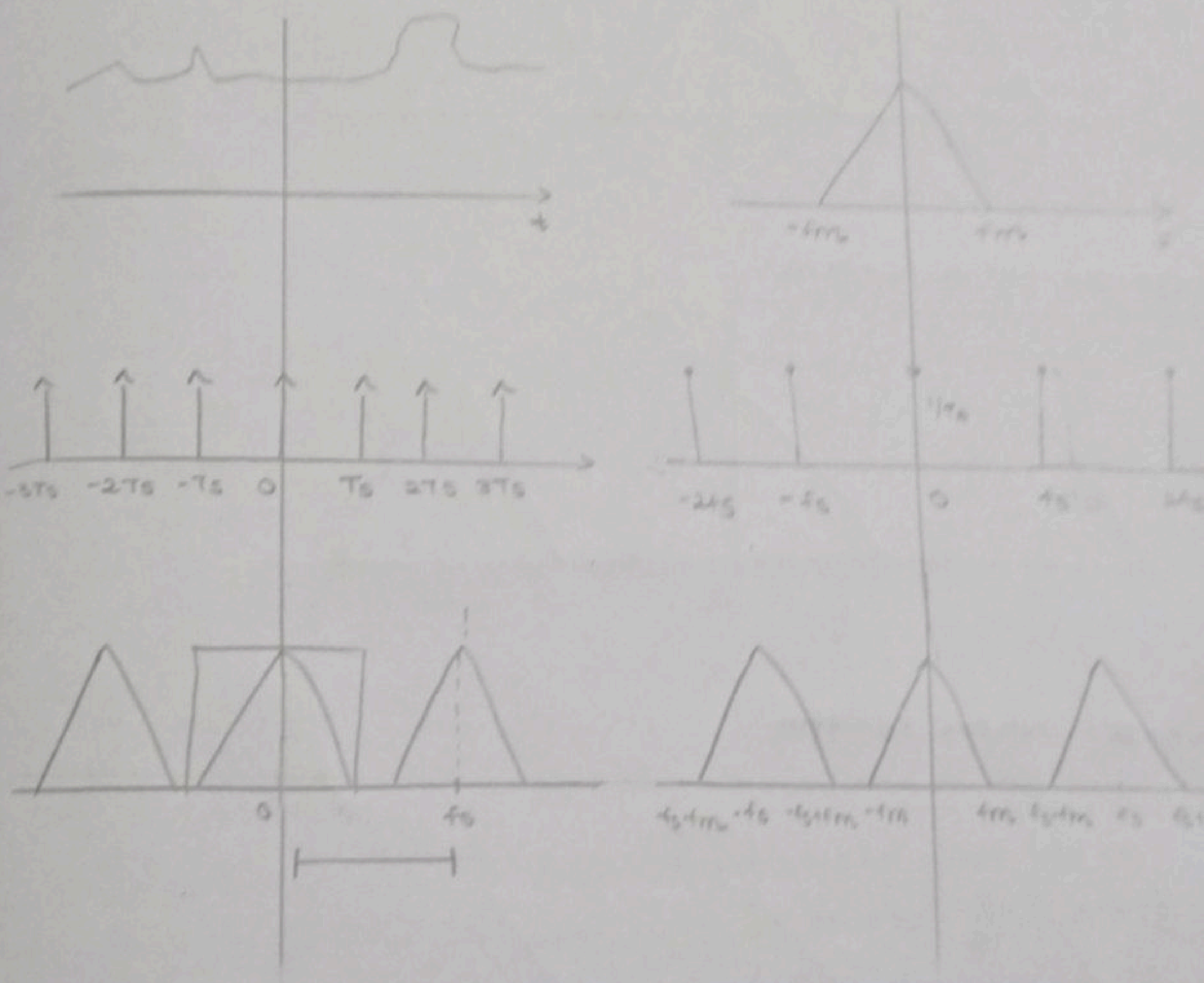
$$\therefore f_m = 200$$

$$f_s > 2 . f_m$$

$$f_s > 400$$

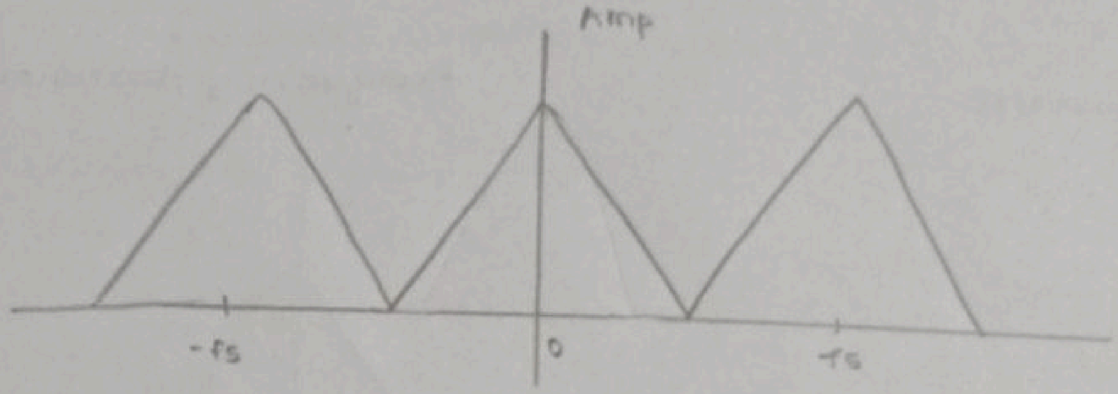
⇒ Proof of sampling theorem:

Time domain   frequency domain

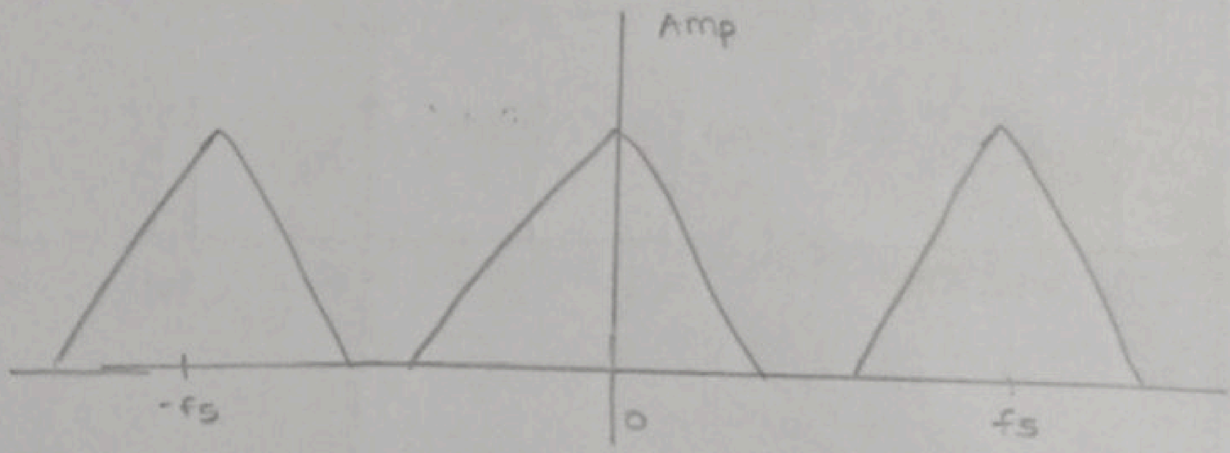


If we multiply in time domain then we need to convolve in frequency domain.

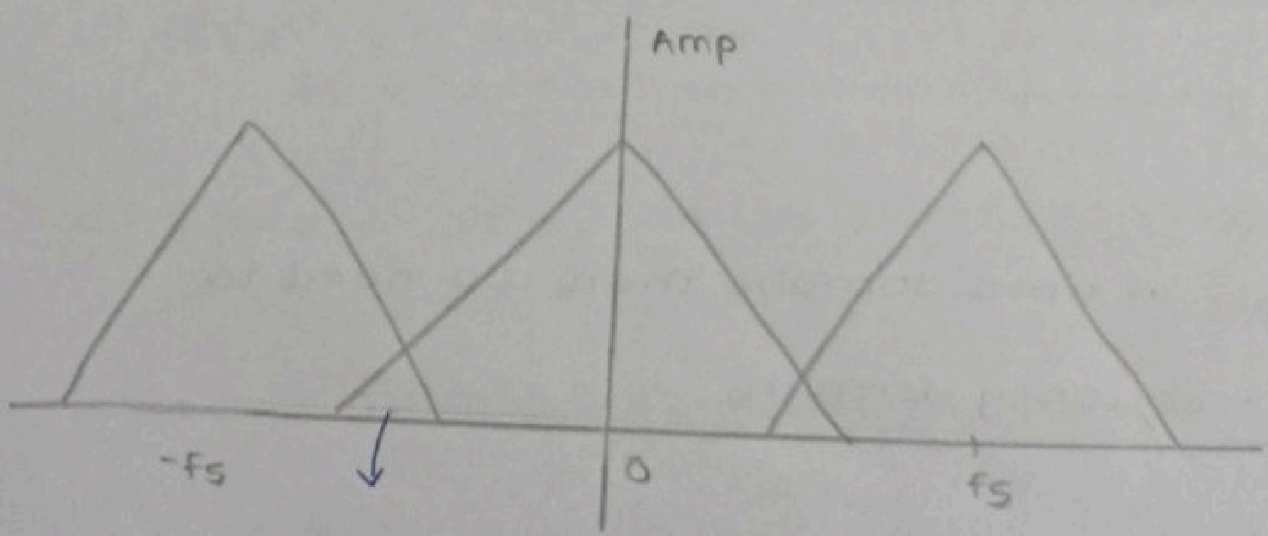
case-I :- When $f_s = 2f_m$



case-II :- When $f_s > 2f_m$

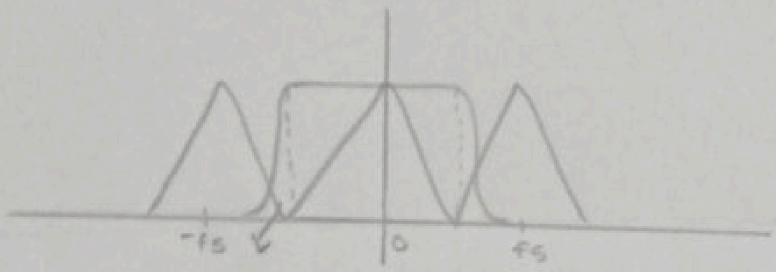


case-III :- When $f_s < 2f_m$



Aliasing ($f_s < 2f_m$)

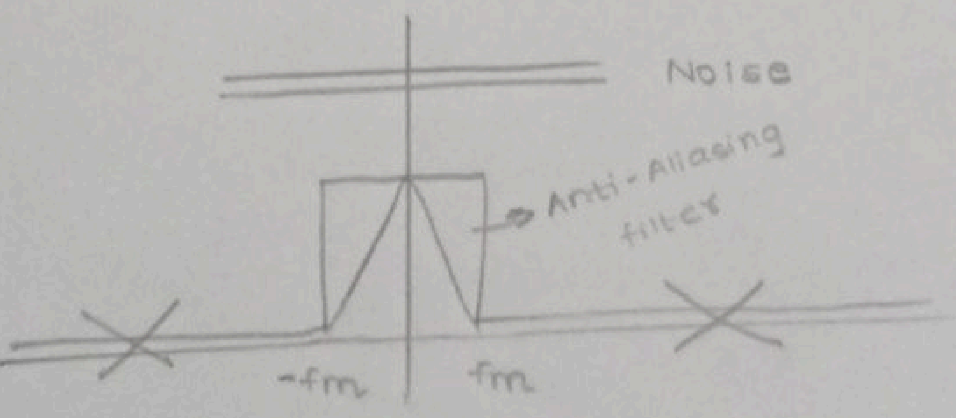
→ When we are considering a sampling frequency we should consider it too much greater than twice of fm because when we apply we can get back the original signal. While in other cases, we can't get back our original signal as there will be some part of both sides in the filter.



Due to this part
we can't get back
our original signal

⇒ Anti-Aliasing filter:-

It is a low pass filter which limits the signal. When we construct a signal the noise will gets added to it and makes the signal from $-\infty$ to $+\infty$. If the signal is from $-\infty$ to $+\infty$ then we can't sample the signal. In order to sample the signal we use a LPF which cancels the higher frequency.



⇒ sampling

over sampling | critical sampling | under sampling
$f_s > 2f_m$ | $f_s = 2f_m$ | $f_s < 2f_m$

⇒ Quantization:-

It is a process in which sampled value is mapped to some quantized value or quantized letter.

Types of quantization:-

→ uniform

→ Non-uniform

uniform Quantization:-

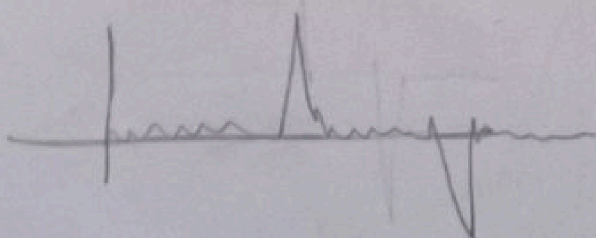
When we divide the range into equal parts then it is known as uniform quantization.

Ex:- $4 \sin(8\pi t)$

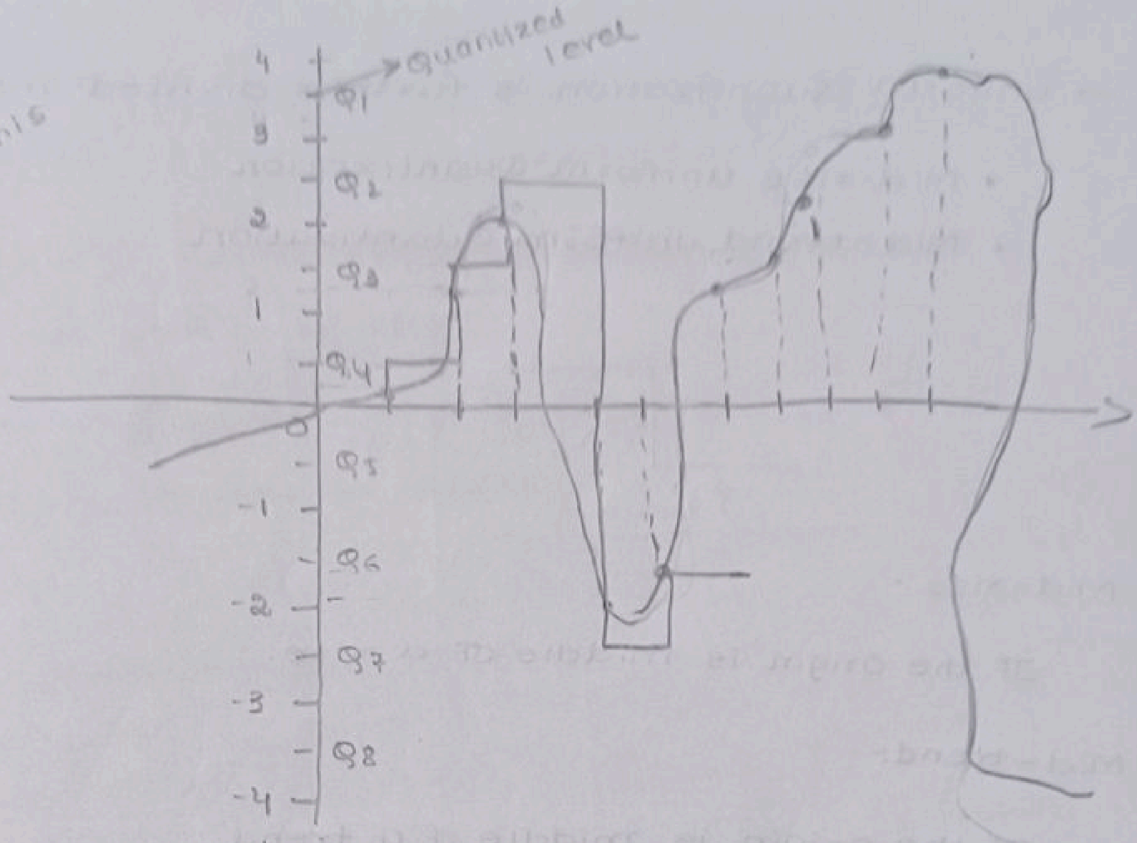
−4 8 4

Non-uniform Quantization:-

When we divide the range into non-equal parts then it is known as non-uniform quantization.



Each sample must be stored with 3 bits in this case.



→ Any signal which comes within 0 to 1 will be mapped to Q4.

SV = 0.2	SV = 1.3	SV = 2.1	SV = -1.9	SV = -1.6	SV = 1.4
QV = 0.5	QV = 1.5	QV = 2.5	QV = -1.5	QV = -1.5	QV = 1.5

SV = 1.8	SV = 2.3	SV = 3.1	SV = 4.6
QV = 1.5	QV = 2.5	QV = 3.5	QV = 3.5

→ Quantized Error $= SV - QV|$

$= SV - 0.5$

$= 0.2 - 0.5$

$= -0.3$

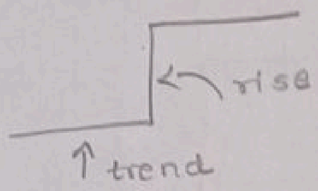
This error should be as small as possible

To reduce this error we need to increase the levels.

so that when we decode the signal we get the desired signal.

= → uniform Quantization is further divided into:-

- Mid-rise uniform Quantization
- Mid-trend uniform Quantization

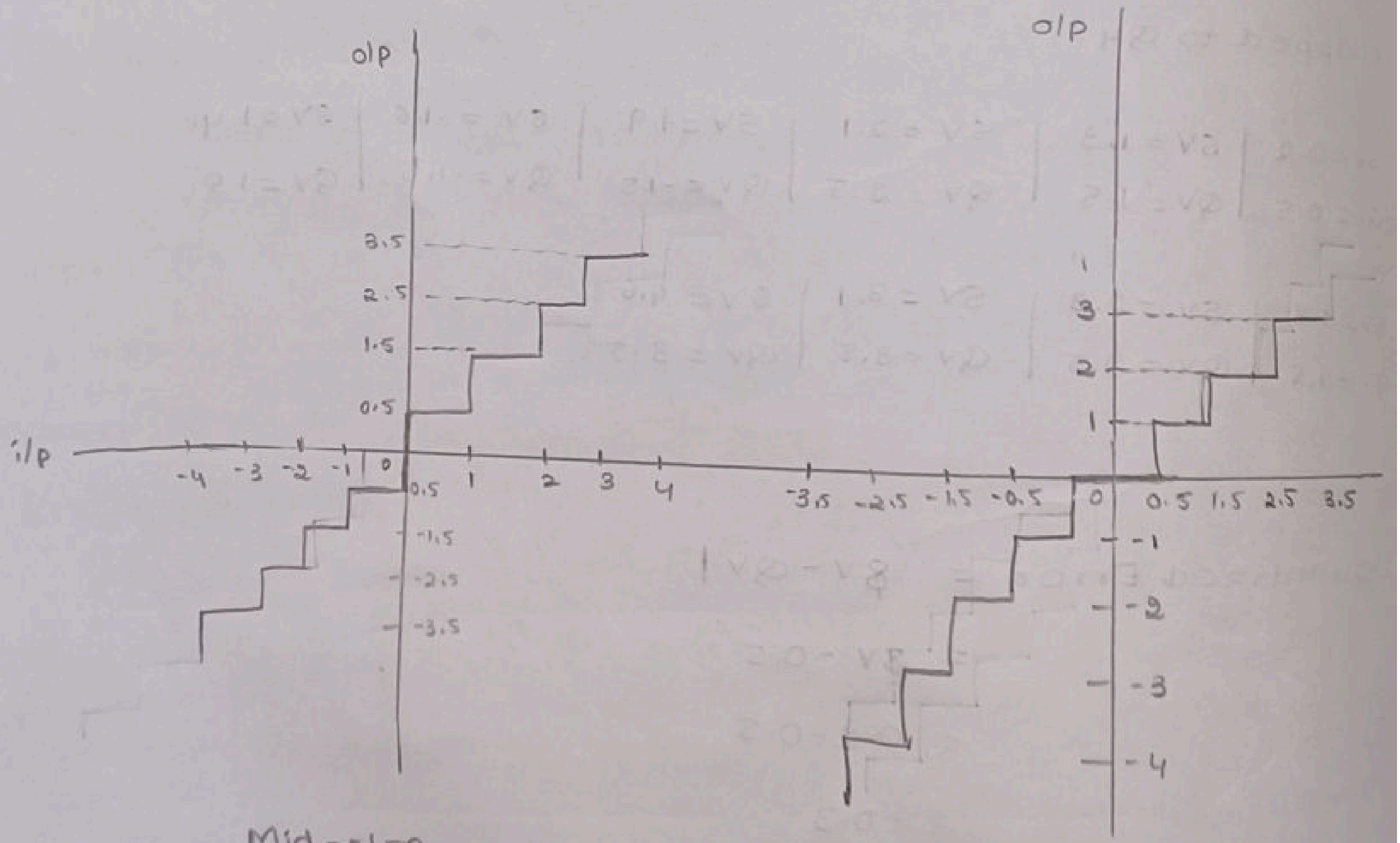


Mid-rise:-

= If the origin is middle of a rise.

Mid-trend:-

If the origin is middle of a trend.



Mid-rise

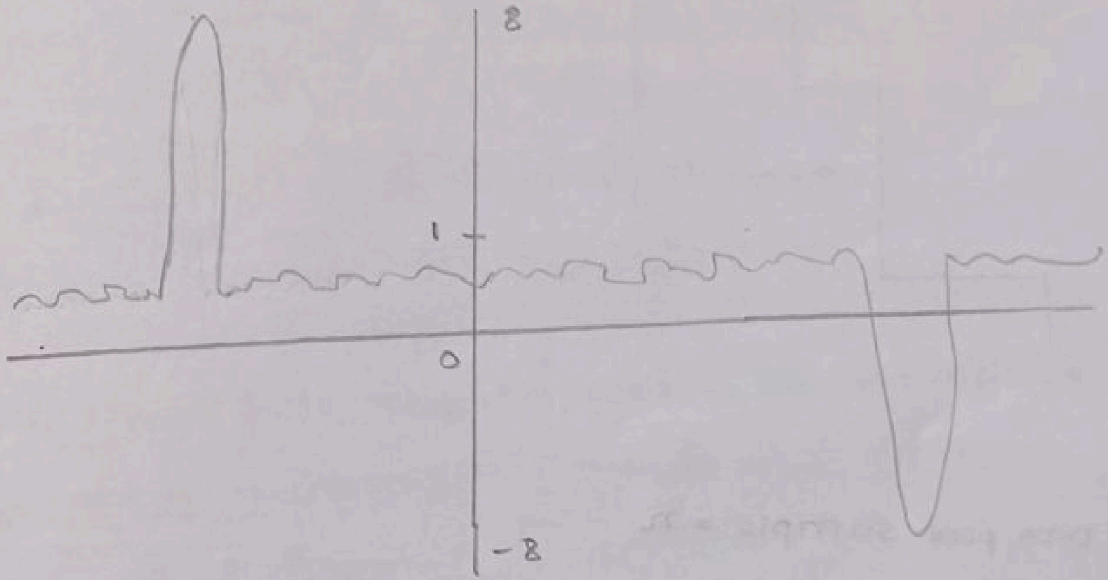
Mid-trend

→ In mid-trend, zero is one of the quantized value whereas In mid-rise, zero is not a quantized value.

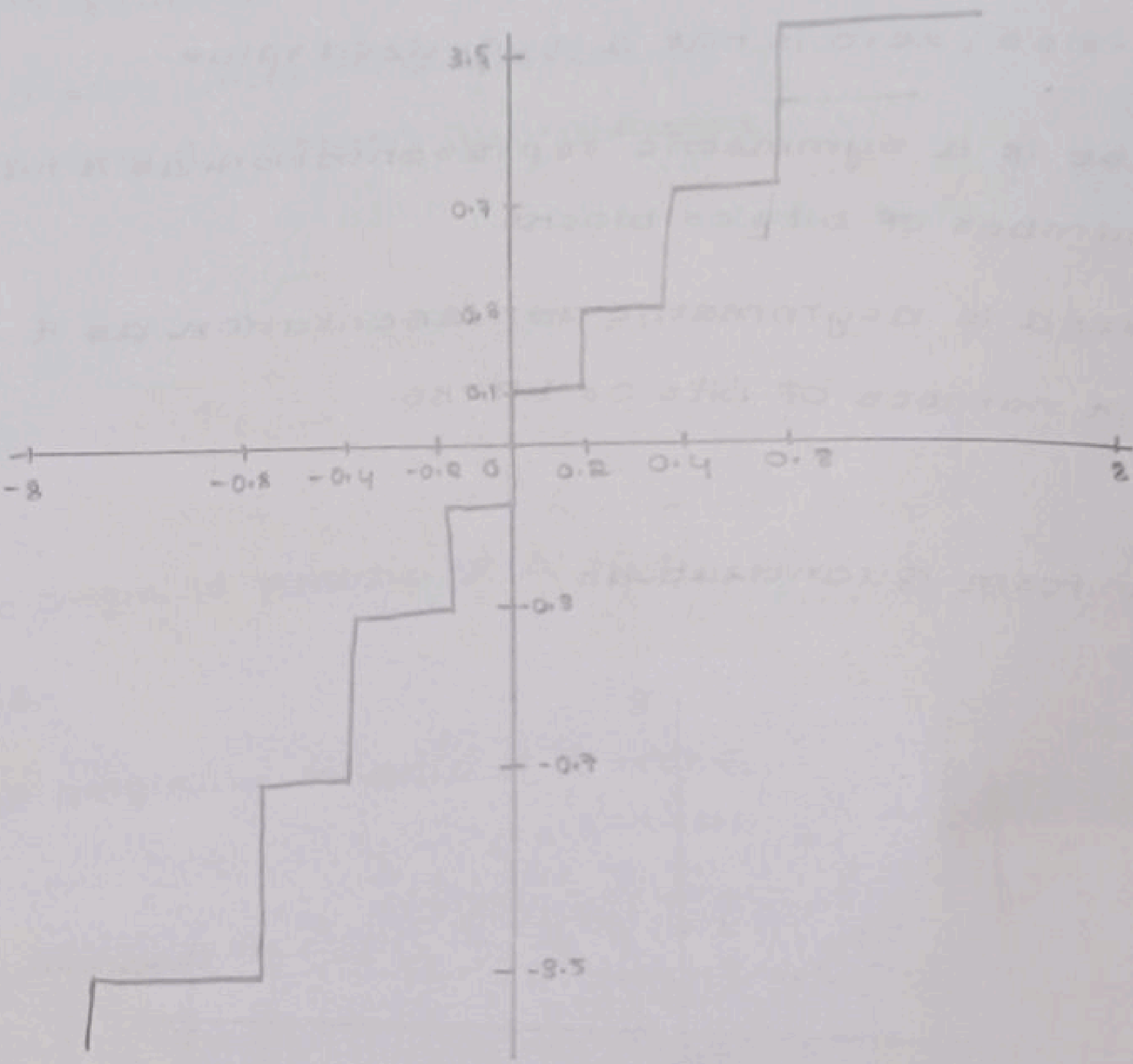
→ Mid-rise is a symmetric representation as it has same number of bits or blocks.

→ Mid-trend is asymmetric representation as it has different number of bits or blocks.

→ Non-uniform Quantization:-



In this case, all the sampled values will be mapped to sampled quantized value in case of uniform quantization, then we may not get the desired signal.



→ Let no. of bits per sample = n

no. of quantization levels = 2^n

→ Length of each quantization level / segment / step size

$$= \Delta = \frac{\text{Range of the signal}}{2^n}$$

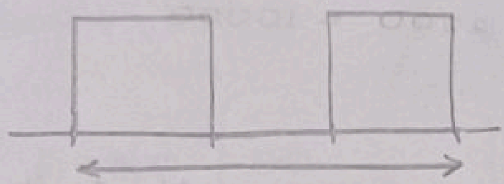
$$\therefore \Delta = \frac{\text{Range of the signal}}{2^n} = \frac{V_{max} - V_{min}}{2^n}$$

→ sampling frequency is the no.of frequency of a sample per sec.

→ Bit duration " T_b " = $\frac{T_s}{n}$

where

n is no.of bits required to represent one sample.



$T_s = 1/f_s$

→ Bit rate = $\frac{\text{bits}}{\text{sample}} \times \frac{\text{sample}}{\text{sec}} = nf_s$

No.of bits per second is called bit rate.

Bit rate is represented by "R_b".

→ Bandwidth is the frequency band or the space which is required to transmit the signal

→ ~~Be~~ max Bandwidth = R_b

Min Bandwidth = $R_b/2$.

Ex: A message signal of $10 \sin 2\pi 10^3 t$ is transmitted through 5 bit quantization level system. Find all the parameters of the system.

A. No.of bits per sample = $n = 5$

No.of quantization levels $= 2^n = 2^5 = 32$

Length = $\frac{\text{Range}}{2^n} = \frac{10-(-10)}{32} = \frac{20}{32} = 0.625$

→ Bit duration = $\frac{T_s}{n}$

$T_s = 1/f_s = 1/2\times10^3 = 1/2000 = 0.0005$

Bit duration = $\frac{0.0005}{5} = 0.0001$

→ Bit rate = $n \cdot f_s = 5 \times 2000 = 10000$

Ex:- A message signal of $4 \sin 2\pi 10^2 t$ is transmitted through 3 bit system. If the sample value is 2.3 1.6 3.9 3.1 2.8 -- 2.3 - 3.6 -1.1 and encoded output and quantization error and quantization value. Determine all the parameters if sampling frequency is 150% the nyquist rate.

A. Given message signal

$4 \sin 2\pi 10^2 t$

→ $f_m = 10^2 = 100$

$f_s = 2f_m = 2\times100 = 200$

→ sampling frequency = $200 + \frac{150}{100} \times 200 = 200 + 300 = 500$

→ no. of bits per sample = $n = 3$

→ no. of quantization levels = $2^n = 8$

→ Length = $\frac{4-(-4)}{8} = \frac{8}{8} = 1$

→ Bit duration = $\frac{T_s}{n} = \frac{0.002}{3} = 0.0006$

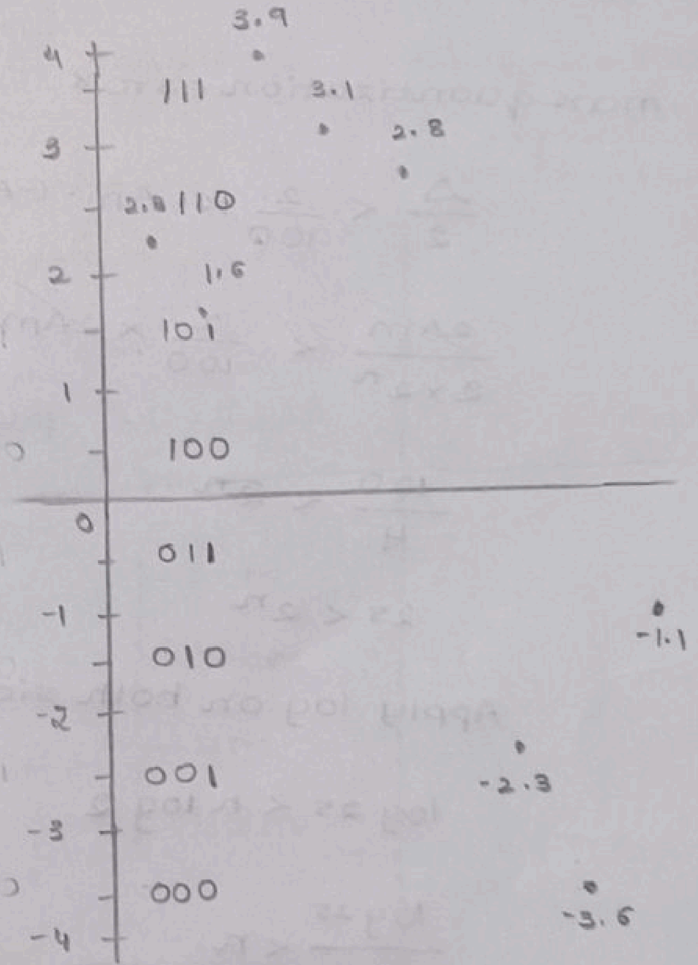
$T_s = 1/f_s = 1/500 = 0.002$

→ Bit rate = $3 \cdot 500 = 1500$

→ max Bandwidth = 1500

→ min Bandwidth = 750

SV	QV	QE	EO
2.3	2.5	-0.2	110
1.6	1.5	0.1	101
3.9	3.5	0.4	111
3.1	3.5	-0.4	111
2.8	2.5	0.3	110
-2.3	-2.5	0.2	001
-3.6	-3.5	-0.1	000
-1.1	-1.5	0.4	010



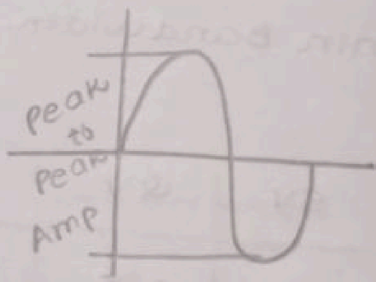
max quantization error = 0.49999.. ≈ 0.5

★ → max quantization error = $\pm \frac{\Delta}{2}$

→ Quantization error cannot be recovered using any kind of algorithm.

Ex:- A sinusoidal message signal is transmitted through a system such that max quantization error should be 2% of peak-to-peak amplitude of the message signal. Find minimum no. of bits required to represent each sample.

A. Let's consider peak is Am



max quantization error

$$\frac{\Delta}{2} < \frac{2}{100} \times Am - (-Am)$$

$$\frac{\cancel{2Am}}{2 \times 2^n} < \frac{2}{100} \times \cancel{2Am}$$

$$\frac{100}{4} < 2^n$$

$$25 < 2^n$$

Apply log on both sides

$$\log 25 < n \log 2$$

$$\frac{\log 25}{\log 2} < n$$

→

Modulation

- Analog
  - Amp
    - Amp
    - DSBSC
    - SSBSC
  - Angle
    - FM
    - PM
- Digital
  - Baseband mod
    - Pulse analog mod
      - PAM
      - PPM
      - PWM
    - Pulse digital mod
      - PCM
      - DPCM
      - DM
      - ADM
  - Bandpass mod
    - 1-bit signaling
      - ASK
      - PSK
      - FSK
    - M-array signaling
      - M-ASK
      - M-PSK
      - M-FSK

⇒ Signal to quantization noise power ratio (SQNR):-

Let $m(t) = A_m \cos 2\pi f_m t$

noise → unwanted entity

$$\boxed{SQNR = \frac{P_S}{P_{Qe}}}$$

$$\Rightarrow P_S = \frac{\left(\frac{A_m}{\sqrt{2}}\right)^2}{R} = \frac{A_m^2}{2R}$$

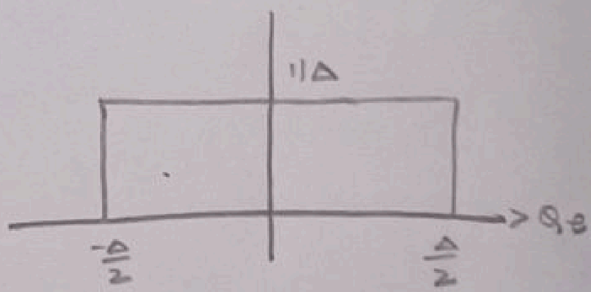
$$P_S = \frac{A_m^2}{2} \text{ when } R = 1\Omega$$

$$\Rightarrow P_{Qe} = \text{Power}(Qe) \quad \{Qe \Rightarrow \text{Random variables}\}$$

$$= MSQ(Qe)$$

$$= E(Qe^2)$$

$$= \int_{-\infty}^{+\infty} Qe^2 \, f_{Qe}(Qe) \, dQe$$



$$= \int_{-\frac{\Delta}{2}}^{\Delta/2} Qe^2 \frac{1}{\Delta} \, dQe$$

$$= \frac{1}{\Delta} \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} Qe^2 \, dQe$$

$$= \frac{1}{\Delta} \left. \frac{Qe^3}{3} \right|_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{\Delta} \left[\frac{\left(\frac{\Delta}{2}\right)^3}{3} - \frac{\left(-\frac{\Delta}{2}\right)^3}{3} \right]$$

$$= \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} + \frac{\Delta^3}{8}\right]$$

$$= \frac{1}{3\cancel{\Delta}} \frac{\cancel{2}^{1}\Delta^{\cancel{3}\,2}}{\cancel{8}\,4}$$

$$= \frac{\Delta^2}{12}$$

$$\Delta = \frac{\text{Range}}{2^n} = \frac{2Am}{2^n}$$

$$= \frac{\left(\frac{2Am}{2^n}\right)^2}{12}$$

$$= \frac{\frac{4Am^2}{2^{2n}}}{12}$$

$$= \frac{\cancel{4}^{1}Am^2}{\cancel{12}_{3} \times 2^{2n}}$$

$$= \frac{Am^2}{3 \times 2^{2n}}$$

$$\therefore \; SQNR = \frac{\frac{\cancel{Am^2}}{2}}{\frac{\cancel{Am^2}}{3 \times 2^{2n}}} = \frac{1}{2} \times 3 \times 2^{2n} = \frac{3 \times 2^{2n}}{2}$$

→ SQNR must be as high as possible.