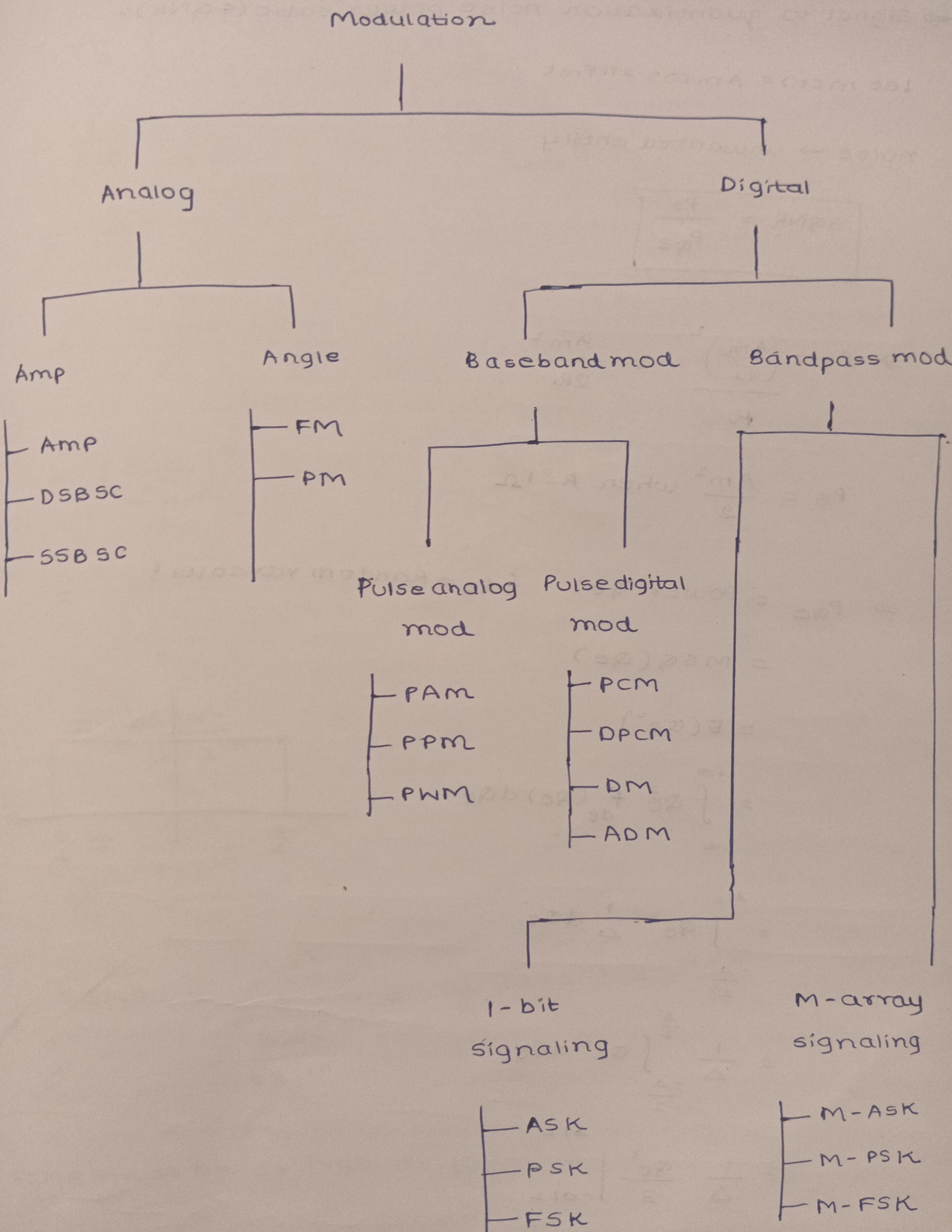

Modulation
Analog
Digital
Amp
Angle
Baseband mod
Bandpass mod
Amp
DSBSC
SSBSC
FM
PM
Pulse analog mod
Pulse digital mod
PAM
PPM
PWM
PCM
DPCM
DM
ADM
1-bit signaling
M-array signaling
ASK
PSK
FSK
M-ASK
M-PSK
M-FSK

⇒ Signal to quantization noise power ratio (SQNR):

Let $m(t) = A_m \cos 2\pi f_m t$

noise → unwanted entity

$$\boxed{SQNR = \frac{P_s}{P_{qe}}}$$

$$\Rightarrow P_s = \frac{\left(\frac{A_m}{\sqrt{2}}\right)^2}{R} = \frac{A_m^2}{2R}$$

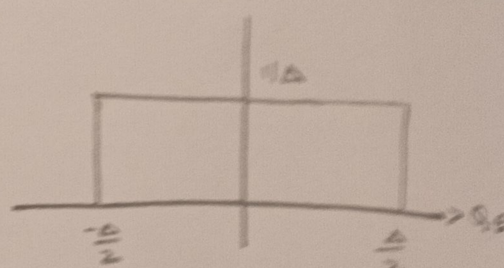
$$P_s = \frac{A_m^2}{2} \text{ when } R = 1\Omega$$

$$\Rightarrow P_{qe} = \text{Power}(q_e) \quad \{ q_e \Rightarrow \text{Random variables} \}$$

$$= MSQ(q_e)$$

$$= E(q_e^2)$$

$$= \int_{-\infty}^{+\infty} q_e^2 \, f_{q_e}(q_e) \, dq_e$$



$$= \int_{-\frac{\Delta}{2}}^{\Delta/2} q_e^2 \frac{1}{\Delta} \, dq_e$$

$$= \frac{1}{\Delta} \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} q_e^2 \, dq_e$$

$$= \frac{1}{\Delta} \left. \frac{q_e^3}{3} \right|_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{\Delta} \left[\frac{\left(\frac{\Delta}{2}\right)^3}{3} - \frac{\left(-\frac{\Delta}{2}\right)^3}{3} \right]$$

$$= \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} + \frac{\Delta^3}{8}\right]$$

$$= \frac{1}{3\cancel{\Delta}} \frac{\cancel{2}^{1}\Delta^{\cancel{3}\,2}}{\cancel{8}\,4}$$

$$= \frac{\Delta^2}{12}$$

$$\Delta = \frac{\text{Range}}{2^n} = \frac{2Am}{2^n}$$

$$= \frac{\left(\frac{2Am}{2^n}\right)^2}{12}$$

$$= \frac{\frac{4Am^2}{2^{2n}}}{12}$$

$$= \frac{\cancel{4}^{1}Am^2}{\cancel{12}_{3} \times 2^{2n}}$$

$$= \frac{Am^2}{3 \times 2^{2n}}$$

$$\therefore \text{SQNR} = \frac{\frac{\cancel{Am^2}}{2}}{\frac{\cancel{Am^2}}{3 \times 2^{2n}}} = \frac{1}{2} \times 3 \times 2^{2n} = \frac{3 \times 2^{2n}}{2}$$

→ SQNR must be as high as possible.

$\Rightarrow$ $SQNR = \frac{3}{2} 2^{2n}$

$$SQNR_{dB} = 10 \log_{10} \left(\frac{3}{2} 2^{2n} \right)$$

$$SQNR_{dB} = 10 \log_{10} \frac{3}{2} + 10 \log_{10} 2^{2n}$$

$$SQNR_{dB} = 10 \log_{10} \frac{3}{2} + 20 n \log_{10} 2$$

$$\boxed{SQNR_{dB} = 1.76 + 6.02 n}$$

$\Rightarrow$

No. of bits	SQNR	
1	7.78 dB	6 dB
2	13.8 dB	6 dB
3	19.82 dB	6 dB
4	25.84 dB	

$\Rightarrow$ $N \rightarrow X$

$N+3 \rightarrow X + 18$ dB

$\Rightarrow$ compander:

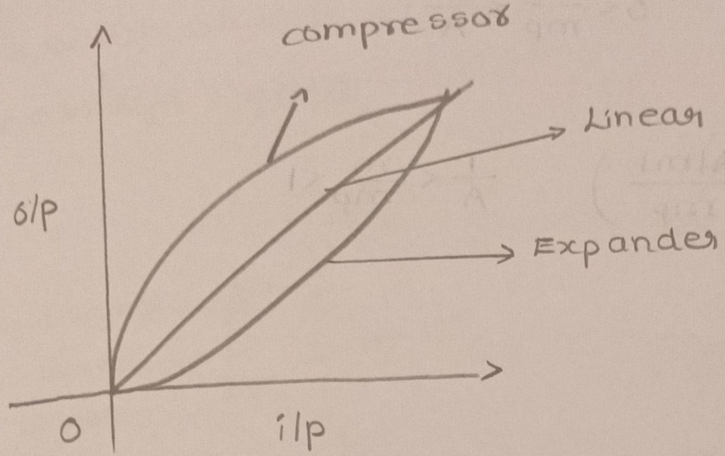
Compressor — Uniform Quantizer — Expander — O/P

→ compressor don't know whether the signal is uniformly or non-uniformly distributed.

→ compander: It is a combination of compressor and expander.

→ Expander exactly performs opposite operation of compressor.

I/P characteristics:



compressor: mapping inputs to outputs in a uniformly distributed

expander: mapping outputs to inputs in a uniformly distributed.

⇒ compander :-

- μ-law
- A-law

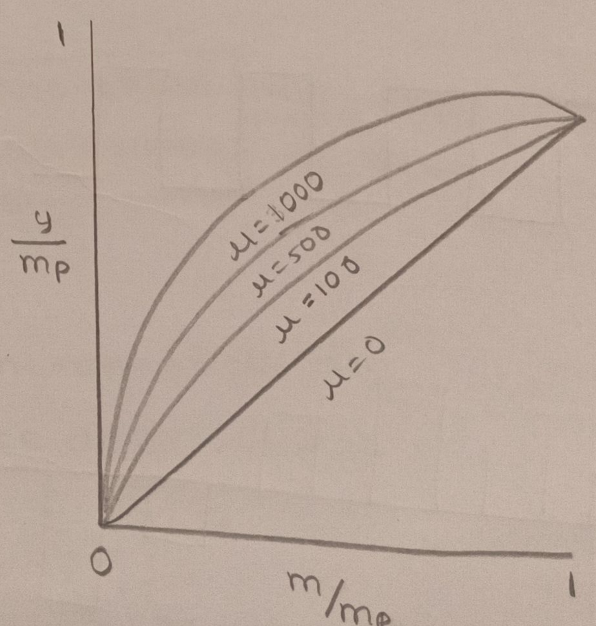
• μ-law:

$$\frac{y}{m_p} = \frac{1}{\ln(1+\mu)} \ln\left(1 + \mu\frac{|m|}{m_p}\right)$$

$0 \le \mu < \infty$

$0 \le \frac{|m|}{m_p} < 1$

$m_p \rightarrow$ max peak



- A-law:

$$\frac{y}{m_p} = \begin{cases} \frac{A}{1+\ln A}\left(\frac{|m|}{m_p}\right) & 0 < \frac{m}{m_p} < \frac{1}{A} \\ \frac{1}{1+\ln A}\left(1+\ln\frac{A|m|}{m_p}\right) & \frac{1}{A} < \frac{m}{m_p} < 1 \end{cases}$$

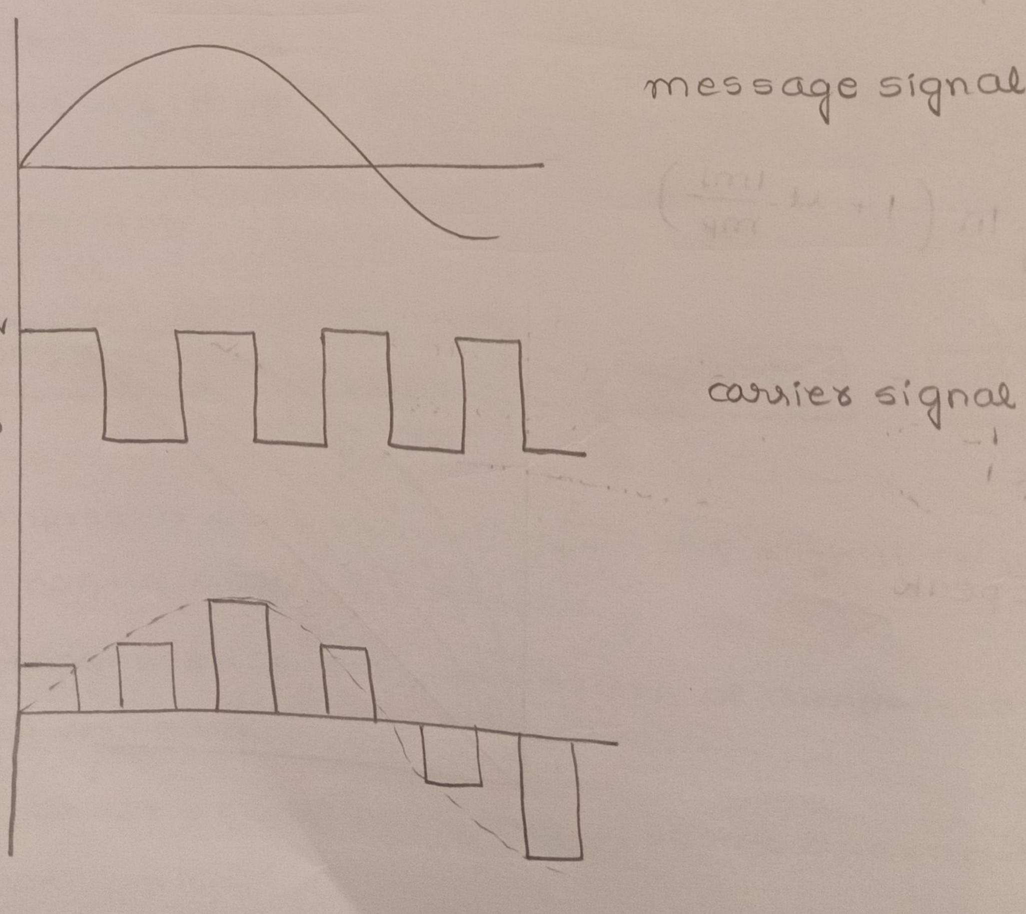
$\therefore\ 0 < A < \infty$

=> Pulse Analog modulation :-

we will use square wave as a carrier signal.

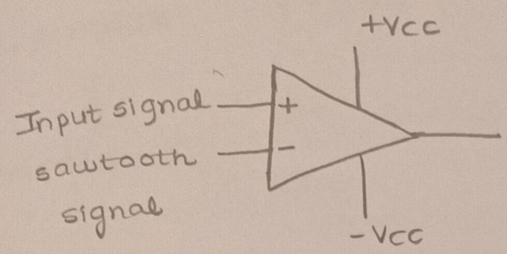
-> Pulse Amplitude modulation :-

The amplitude of the pulse will change according to the message signal.



Pulse width modulation:-

The width of the pulse will change according to the message signal.

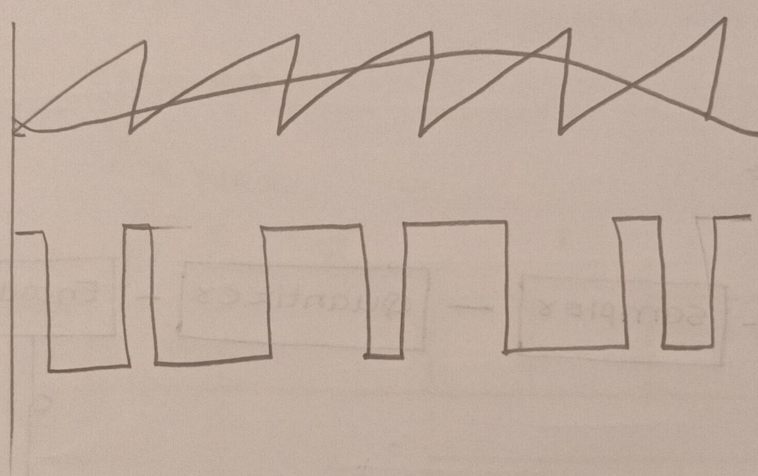


input > sawtooth

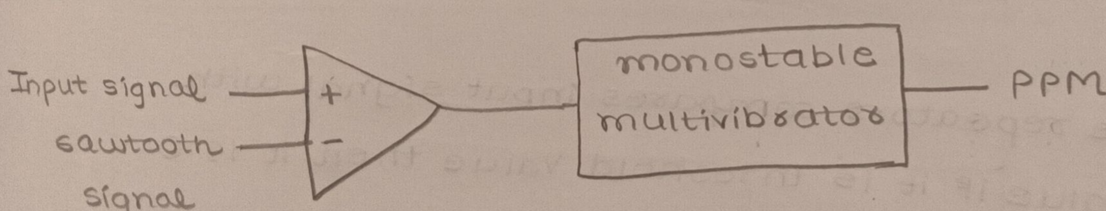
+Vcc

input < sawtooth

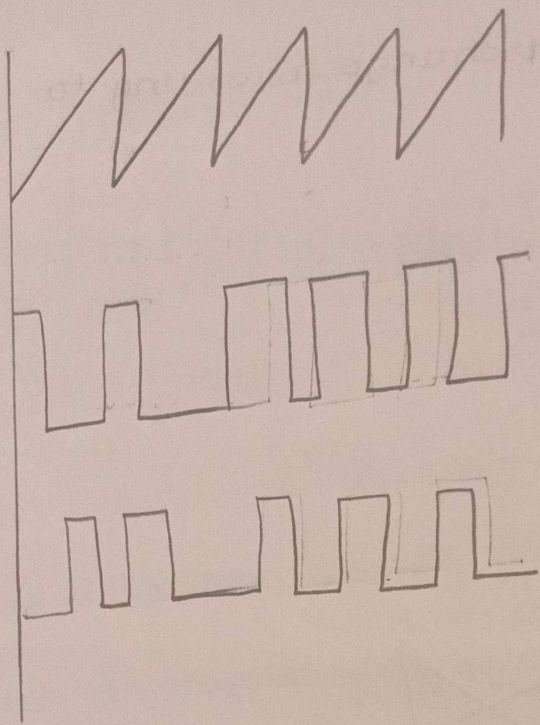
-Vcc



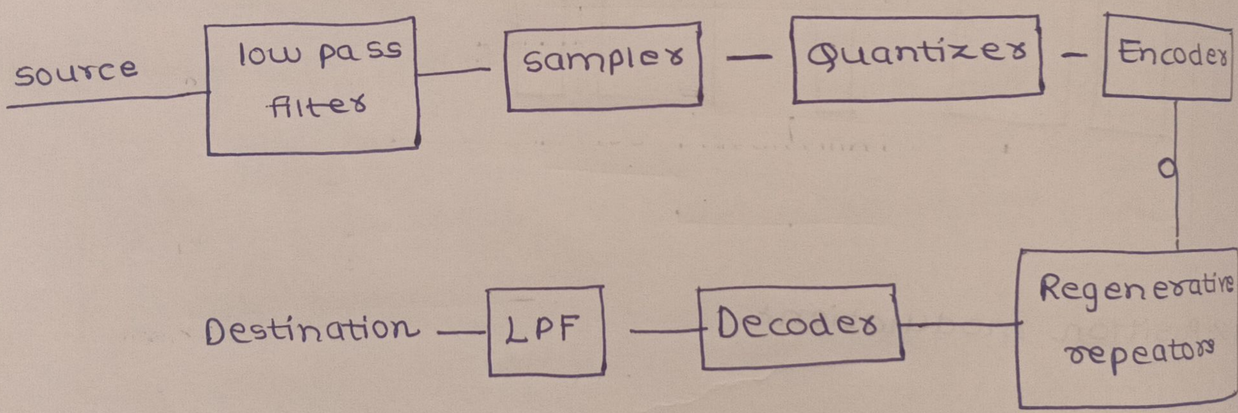
Pulse Position modulation:-



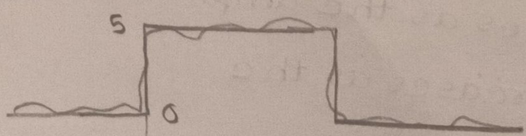
In this case, the distance increases as the amplitude increases and the distance decreases as the amplitude decreases.



⇒ Pulse code modulation:-



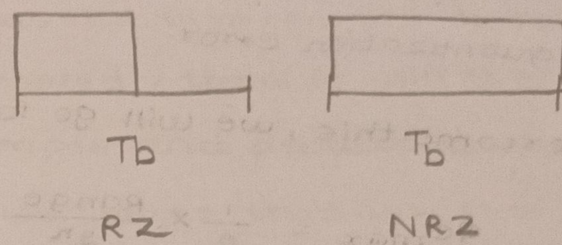
Regenerative repeators compares input signal with threshold value if it is threshold value then it is 0. if not it is 5.



$\frac{5-0}{2} = 2.5 \Rightarrow$ threshold value

⇒ Line coding:

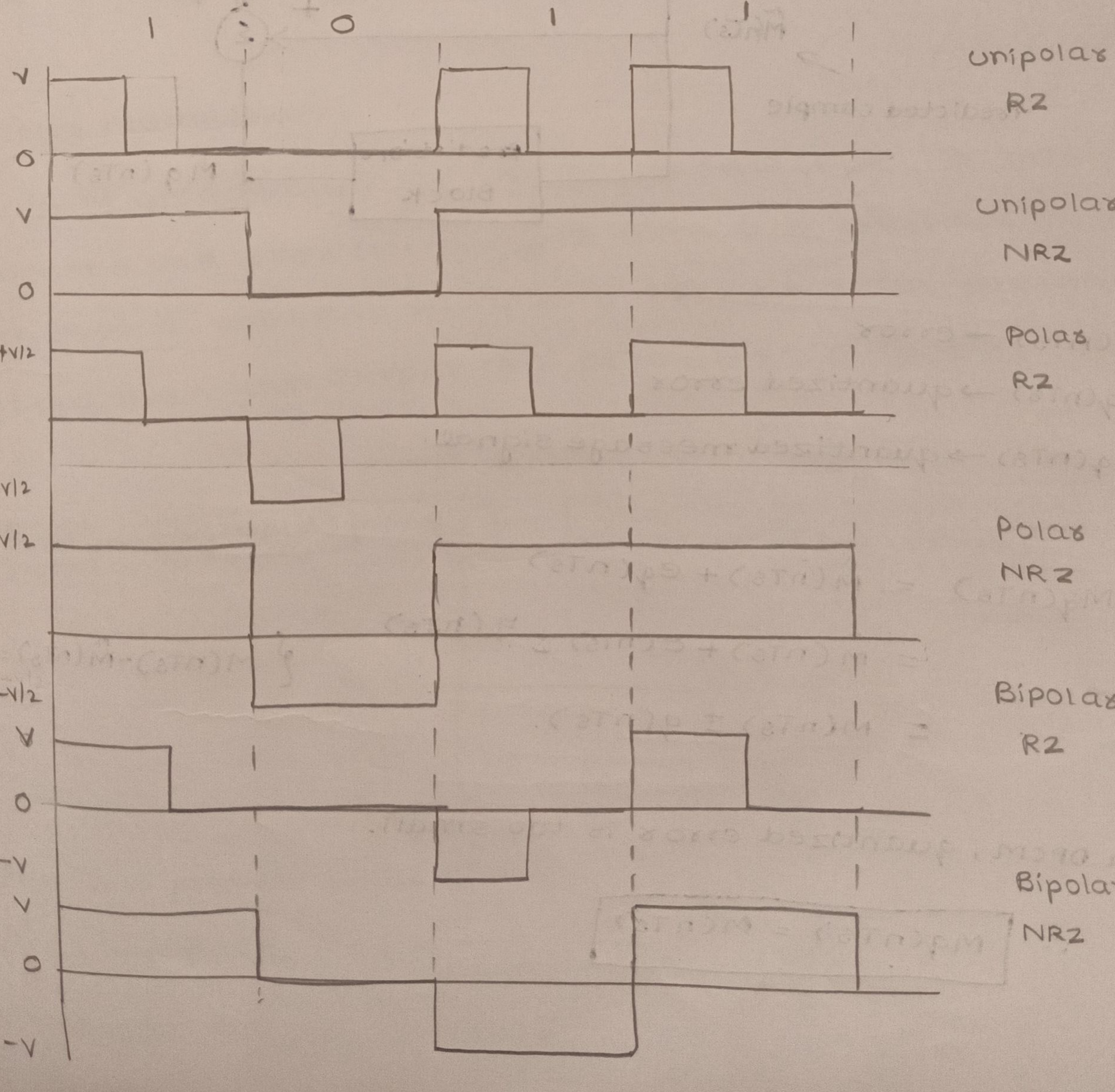
- Unipolar 0 – V
- Polar −V/2 – V/2
- Bipolar −V – 0 – V



Unipolar
- RZ
- NRZ

Polar
- RZ
- NRZ

Bipolar
- RZ
- NRZ



⇒ Differential Pulse code modulation:-

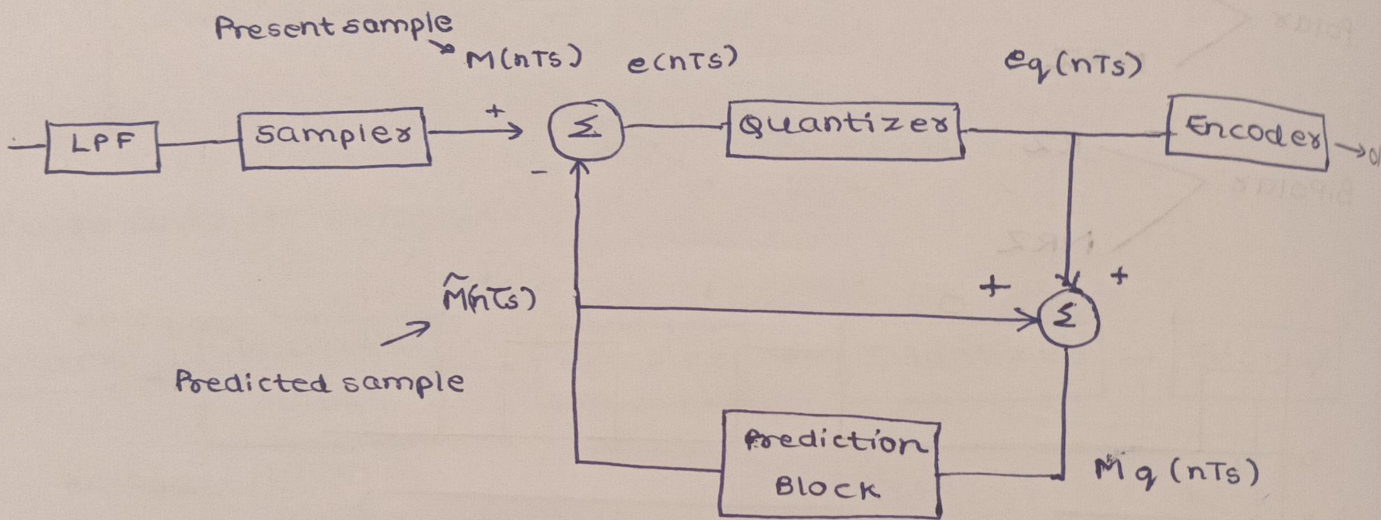
Limitations of PCM:-

- High quantization error

→ To overcome this, we will go for Differential PCM.

$$Qe_{max} = \frac{1}{2} \times \frac{Range}{2^n}$$

→ To decrease Qe, either increase n value or decrease range.



$e(nTs) \rightarrow$ error

$eq(nTs) \rightarrow$ quantized error

$Mq(nTs) \rightarrow$ quantized message signal.

$$\therefore Mq(nTs) = \hat{M}(nTs) + eq(nTs)$$

$$= \hat{M}(nTs) + e(nTs) \pm q(nTs) \qquad \{M(nTs) - \hat{M}(nTs) = e(nTs)\}$$

$$= M(nTs) \pm q(nTs)$$

∵ In DPCM, quantized error is too small.

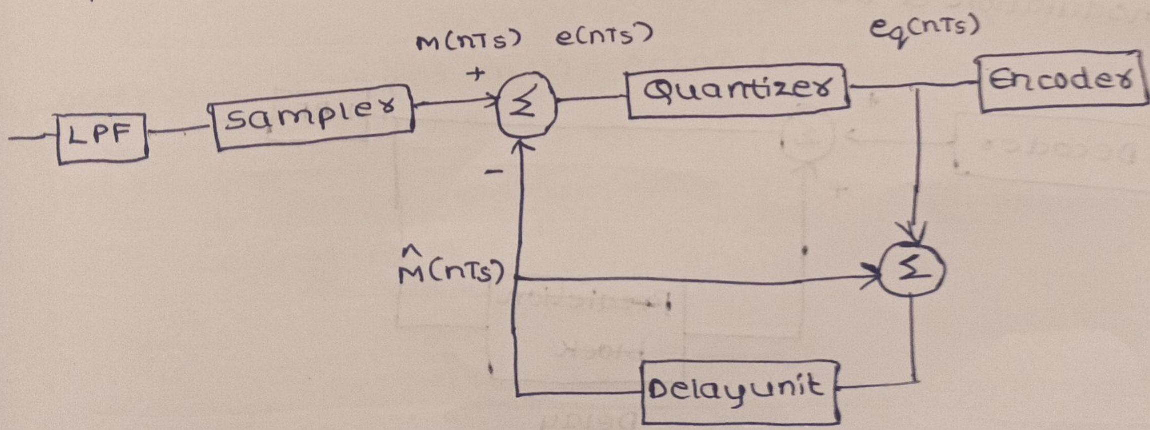
$$\therefore \boxed{Mq(nTs) = M(nTs)}$$

∴ We will give message signal to the prediction block so that it can predict the next coming signal.

→ A continuous analog signal is passed through LPF to remove the noise and then it is sampled, then it will the compare both the present sample and predicted sample and produces error and it will pass through the quantizer and gives quantization error and sent through encoder and produces output. The same quantized error is sent to summation and predicted sample is also sent to summation and the combination of these both is given to prediction block to predict the next signal based on present sample.

⇒ Delta Modulation:-

It is also known as one-bit modulation. It basically compares the present sample with the previous sample such that if present sample is greater than previous sample then the output is '1' otherwise output is '0'.



$M(nTs)$ → present sample

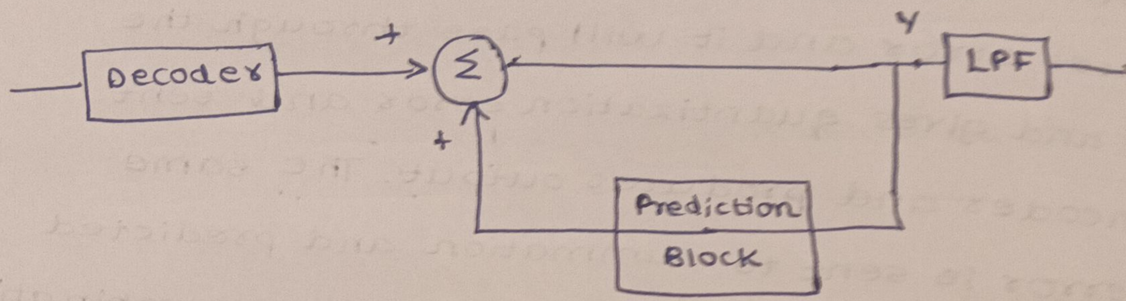
$\hat{M}(nTs)$ → past sample

→ Advantages:- Reduces bandwidth.

→ The delay unit won't work until the next clock is applied.

The delay unit gives the value (input) to the past sample.

⇒ Demodulation of DPCM:



$$y(nT_s) = e_q(nT_s) + \hat{M}(nT_s)$$

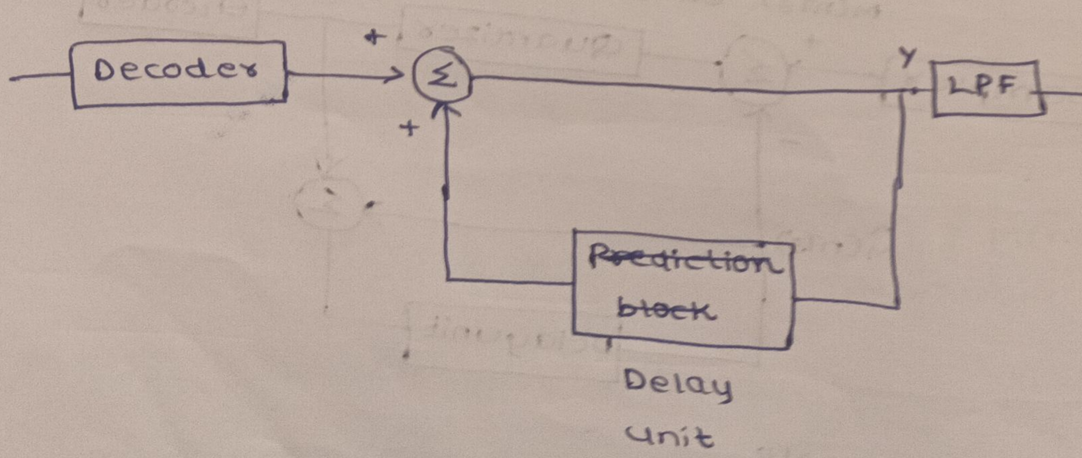
$$= e(nT_s) \pm q(nT_s) + \hat{M}(nT_s)$$

$$= M(nT_s) \pm q(nT_s)$$

∵ quantized error is too small

$$\boxed{y(nT_s) = M(nT_s)}$$

⇒ Demodulation of Delta modulation:



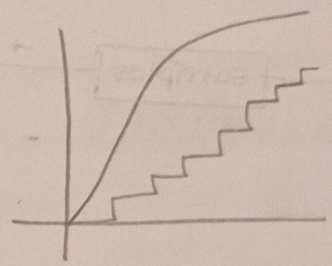
→ In order to reconstruct the original signal then slope of the signal should be equal to the slope of the reconstructed signal.

$$\boxed{\frac{d}{dt} m(t) = \frac{\Delta_{opt}}{T_s}}$$

→ If $\frac{d}{dt} m(t) > \frac{\Delta_{opt}}{T_s}$ then it is known as overload error.

$$\frac{\Delta_{opt}}{\cancel{T_s}} > \frac{\Delta}{\cancel{T_s}}$$

$$\Delta_{opt} > \Delta$$

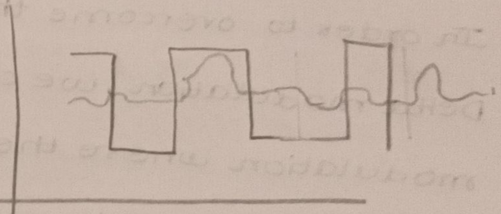


→ When slope of the message signal is less than the slope of the reconstructed signal is granular error.

$$\frac{d}{dt} m(t) < \frac{\Delta}{T_s}$$

$$\frac{\Delta_{opt}}{\cancel{T_s}} < \frac{\Delta}{\cancel{T_s}}$$

$$\Delta_{opt} < \Delta$$



Ex:- Determine Δ_{opt} for $m(t) = A_c \cos(2\pi f t)$

we know that

$$\frac{d}{dt} m(t) = \frac{\Delta_{opt}}{T_s}$$

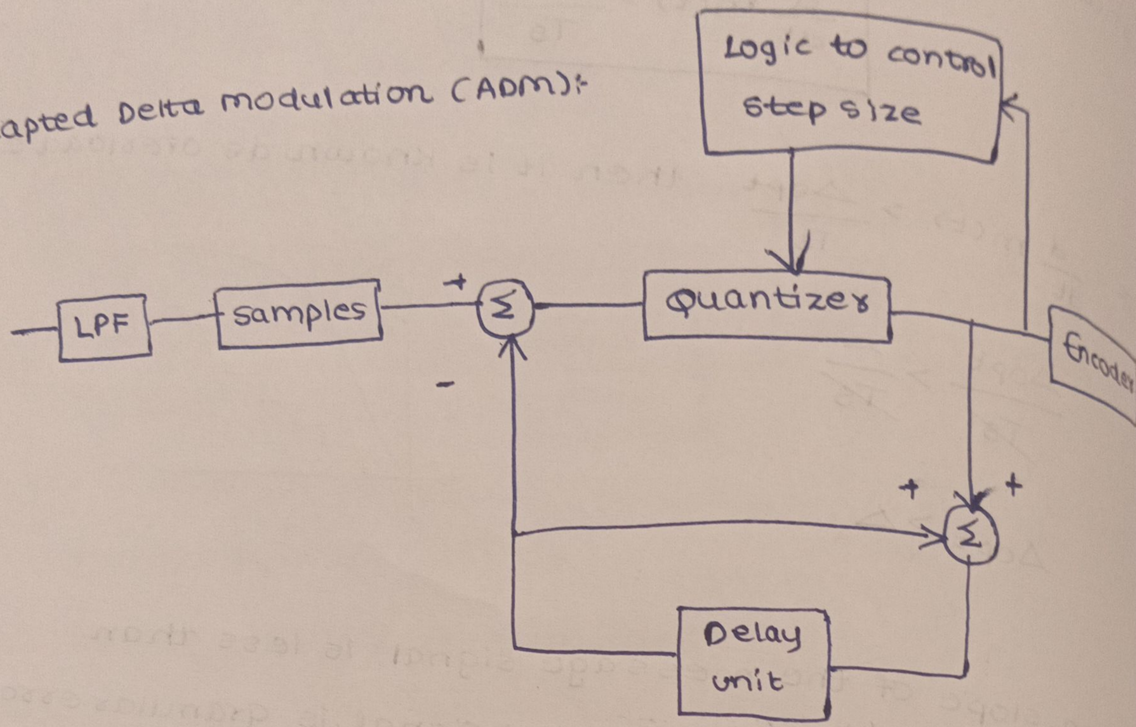
$$\frac{d}{dt}(A_c \cos(2\pi f t)) = \frac{\Delta_{opt}}{T_s}$$

$$A_c \frac{d}{dt} \cos(2\pi f t) = \frac{\Delta_{opt}}{T_s}$$

$$A_c \cdot -\sin(2\pi F t) \cdot 2\pi F = \frac{\Delta_{opt}}{T_s}$$

$$\Delta_{opt} = -2\pi F \cdot T_s \cdot A_c \sin(2\pi F t)$$

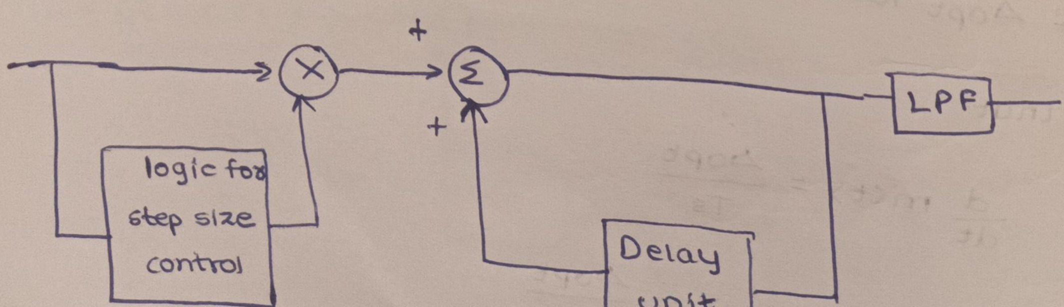
⇒ Adapted Delta modulation (ADM):-



In order to overcome the limitations of Demodulation of Delta modulation, we came up with Adapted Delta modulation where the step size is not constant.

It will the control the stepsize based on error.

At Receiver end:-



→ Advantages:-

Reduces bandwidth

No presence of overload and granular error.

→ Disadvantages:-

Logical block is too complex.

Ex:- A continuous signal of $8\sin(8\pi\times10^3 t)$ is passed through delta modulation whose pulse rate is 4000 pulses/second. Find optimal step size of the receiver.

$$\left|\frac{d}{dt} m(t)\right| = \frac{\Delta opt}{T_s} \qquad f_s = 4000$$

$$\left|\frac{d}{dt} 8\sin(8\pi\times10^3 t)\right| = \frac{\Delta opt}{1/4000}$$

$$\left|8\cos(8\pi\times10^3 t)\times(8\pi\cdot10^3)\right| = \frac{\Delta opt}{1/4000}$$

$$\left|64\cos(8\pi\times10^3 t)\pi10^3\right| = \frac{\Delta opt}{1/4000}$$

$$\left|64\pi\cos(8\pi\times10^3 t)\cdot10^3\right| = 4000\,\Delta opt$$

$$\left|\overset{16}{\cancel{64}}\pi\cos(8\pi\times10^3 t)\cancel{10^3}\right| = \overset{1}{\cancel{4}}\times\cancel{10^3}\,\Delta opt$$

$$\left|16\pi\cos(8\pi\times10^3 t)\right| = \Delta opt$$

$$16\pi = \Delta opt$$

Ex:- If a message is defined as 10t is transmitted through a pulse code modulator which works at 1000 bits/sec, Determine optimal step size.

$$\left| \frac{d}{dt} m(t) \right| = \frac{\Delta opt}{T_s}$$

$$\frac{d}{dt} 10t = \frac{\Delta opt}{1/1000}$$

$$\cancel{10}^{1} \cdot 1 = \Delta_{opt} \times \cancel{1000}^{100}$$

$$\Delta opt = \frac{1}{100}$$

$$\Delta opt = 0.01$$

Ex: A sinusoidal message signal of frequency 'fm' and amplitude 'Am' is passed through delta modulation, whose step size is 0.628 V and sampling rate is 40000 samples/sec. For which of the following delta modulation will be slope overloaded.

a) Am = 3V, Fm = 1K

b) Am = 2V, Fm = 1.5K

c) Am = 2V, Fm = 2.5K

d) Am = 1V, Fm = 2.5K

$$\frac{d}{dt} m(t) > \frac{\Delta}{T_s}$$

$$\frac{d}{dt} \quad Am \sin(2\pi f m t) > \frac{\Delta}{T_s}$$

$$Am \frac{d}{dt} \sin(2\pi f m t) > \frac{\Delta}{T_s}$$

$$\left| Am \cos(2\pi f m t) . 2\pi f m \right| > \frac{0.628}{1/40000}$$

$$2\pi f_m A_m > 0.628 \times 40000$$

$$\Delta = 0.628 > \frac{2\pi f_m A_m}{40000}$$

a) $$\Delta > \frac{2\pi(1000)(3)}{40000} = \frac{18840}{40000} = 0.471$$

b) $$\Delta > \frac{2\pi(1500)(2)}{40000} = \frac{18840}{40000} = 0.471$$

c) $$\Delta > \frac{2\pi(2500)(2)}{40000} = \frac{31400}{40000} = 0.785$$

d) $$\Delta > \frac{2\pi(2500)(1)}{40000} = \frac{15700}{40000} = 0.3925$$

Ex:- A message signal of peak-to-peak of 1.536 V is passed through PCM system having 128 quantization level. Find quantization noise power.

$$\text{Quantization noise power} = \frac{\Delta^2}{12}$$

$$= \frac{\left(\frac{\text{Range}}{2^n}\right)^2}{12}$$

$$= \frac{\left(\frac{1.536}{128}\right)^2}{12}$$

$$= \frac{2.359296}{16384} \times \frac{1}{12}$$

$$= \frac{0.000144}{12}$$

$$= 0.000012$$

Ex:- How many bits per sample must be assign such that SQNR should be greater than 1000.

$$SQNR \geq 1000$$

$$\frac{3}{2} 2^{2n} \geq 1000$$

$$2^{2n} \geq 1000 \times \frac{2}{3}$$

$$2^{2n} \geq \frac{2000}{3}$$

$$2^{2n} \geq 666.6$$

Apply log on B.S

$$\log(2)^{2n} \geq \log(666.6)$$

$$2n > 9.38$$

$$n > \frac{9.38}{2}$$

$$n > 4.69$$

$\therefore$ n should be almost 5 bits.

Ex:- A message signal band limited to 4K is transmitted through 256 level PCM system. Find transmitting or transmitter Bandwidth of the system.

$$\text{Bandwidth} = \frac{nf_s}{2} = \frac{8 \times \cancel{2} \times 4K}{\cancel{2}} = 32\,KHz$$

$(\because f_s = 2f_m)$

Ex:- A message signal sampled at 8K is transmitted through 512 level PCM system. Find SQNR and bit rate.

$$\Rightarrow \quad SQNR = \frac{3}{2} \times 2^{2n}$$

$$= \frac{3}{2} \times (512)^2$$

$$= \frac{3}{2} \times 262144$$

$$= 3 \times 131072$$

$$= 393216$$

$$\Rightarrow \quad \text{Bit rate} = nf_s$$

$$= 9 \times 8K$$

$$= 72K$$

$$\Rightarrow \quad SQNR_{dB} = 1.76 + 6.02n$$

$$= 1.76 + 6.02 \times 9$$

$$= 1.76 + 54.18$$

$$= 55.94 \text{ dB}$$

Ex:- If a message signal is varying between -3V to 5V. is transmitted through a PCM system of stepsize of 1V. Find SQNR in dB.

$$SQNR = 1.76 + 6.02n$$

$$\Delta = \frac{\text{Range}}{2^n}$$

$$1 = \frac{5V - (-3V)}{2^n}$$

$$1 = \frac{8V}{2^n}$$

$$2^n = 8$$

$$2^3 = 8$$

$$\therefore n = 3$$

$$\therefore SQNR = 1.76 + 6.02 \times 3$$

$$= 1.76 + 18.06$$

$$= 19.82$$

Ex:- For a PCM system as the no. of quantization level increases from 2 to 8, then transmitter bandwidth requirement will be

a) increased by 4 times

b) increased by 3 times

c) double

d) no change.

$$2^1 = 2, \; 2^3 = 8$$

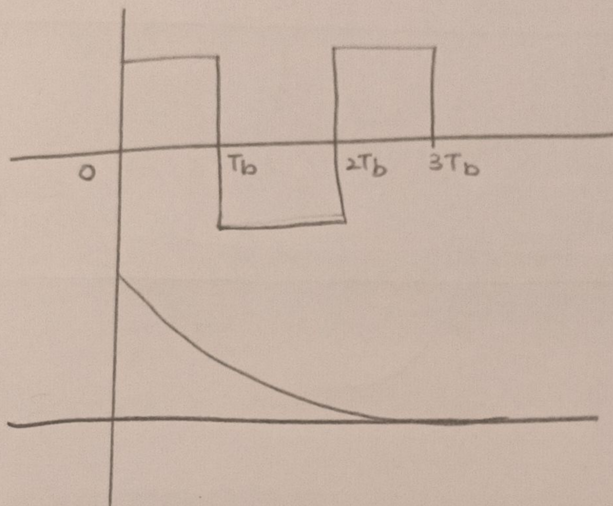
$$BW = \frac{nf_s}{2} = \frac{1.f_s}{2} = \frac{f_s}{2}$$

$$= \frac{3.f_s}{2} = \frac{3f_s}{2}$$

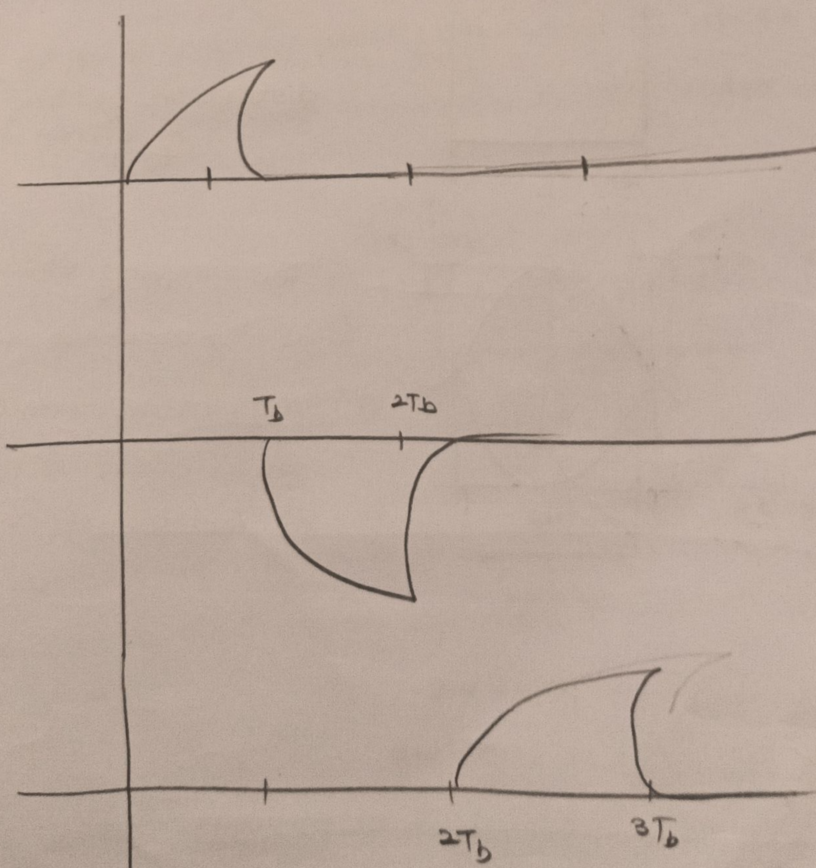
$$\therefore \frac{3\cancel{f_s}}{\cancel{2}} \Big/ \frac{\cancel{f_s}}{\cancel{2}} = \uparrow \text{ by 3 times}$$

⇒ Intersymbol Interference

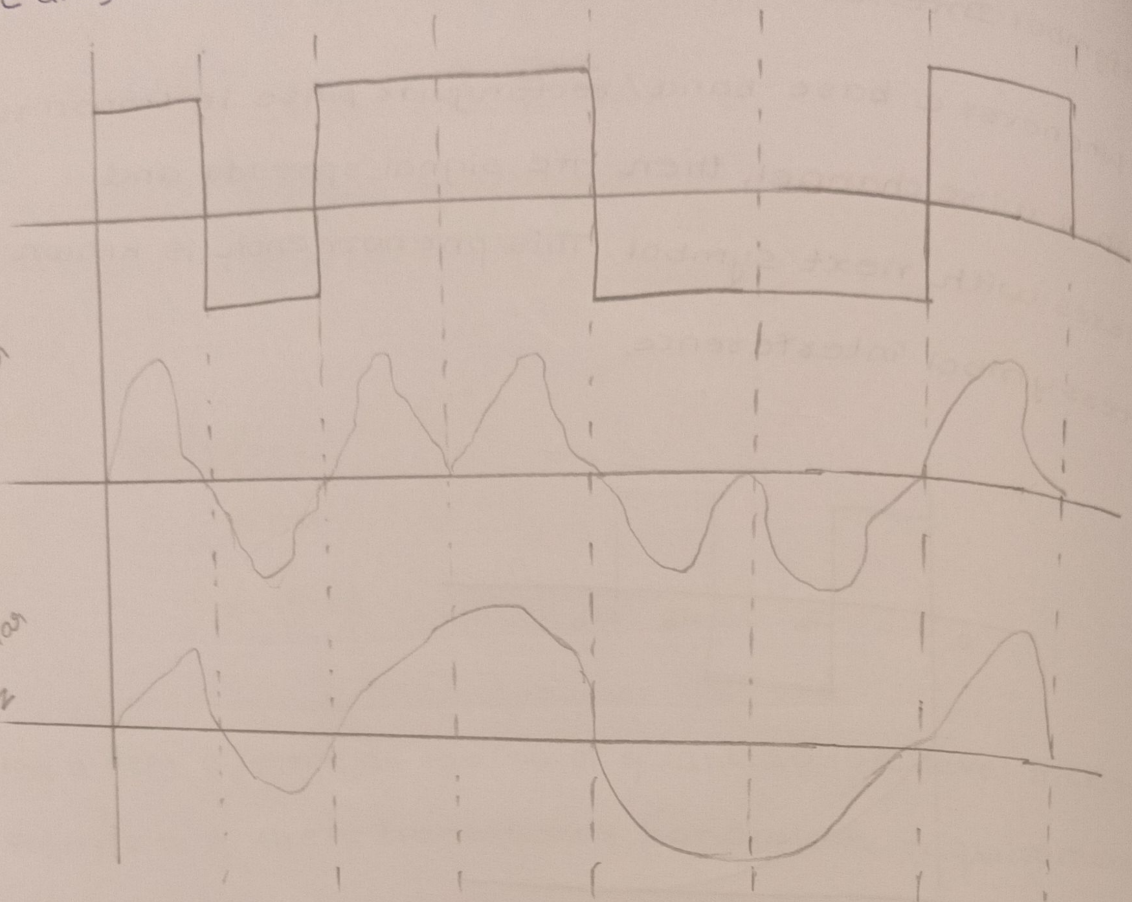
Whenever a base-band/rectangular pulse is transmitted through a wire channel, then the signal spreads and interferes with next symbol. This phenomenon is known as intersymbol interference.

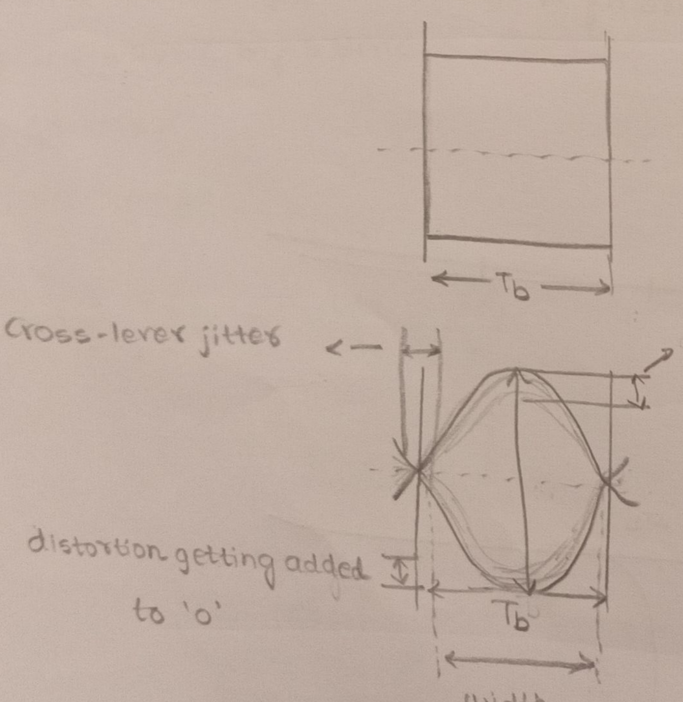


convolve by reversing & then shifting



⇒ Eye diagram :-





The sampling should be done at a instant where there is maximum opening. so that we will receive the same original signal.

→ width should be as high as possible.

→ cross-lever must be as low as possible.

→ Maxmium eye opening must be as high as possible.

1) maximum eye opening

2) sensitivity to time jitter

3) cross-lever jitter

⇒ Power spectrum Density of PAM:-

Power spectrum states that how the power is distributed across the signal.

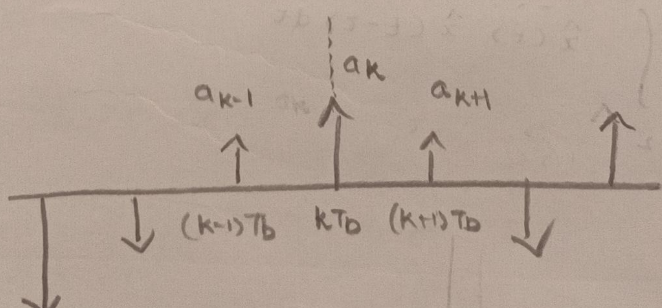
$$x(t) \longrightarrow \boxed{\text{PAM}} \xrightarrow{y(t)} \qquad h(t) = p(t)$$

$$\boxed{S_Y(f) = |P(f)|^2 S_X(f)}$$

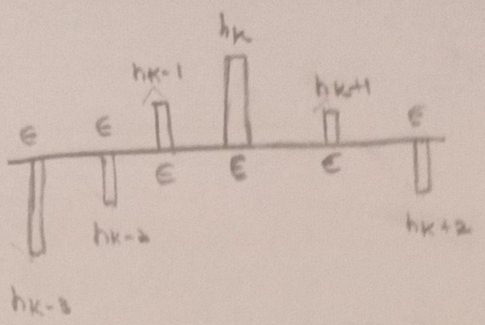
$PSD = FT\{R_X(\tau)\}$

$R_X(\tau) \rightarrow$ Auto correlation.

→ Auto correlation is same as convolution but there will be no reversing of the signal just we have to shift and more and when they overlap, multiply & calculate area.

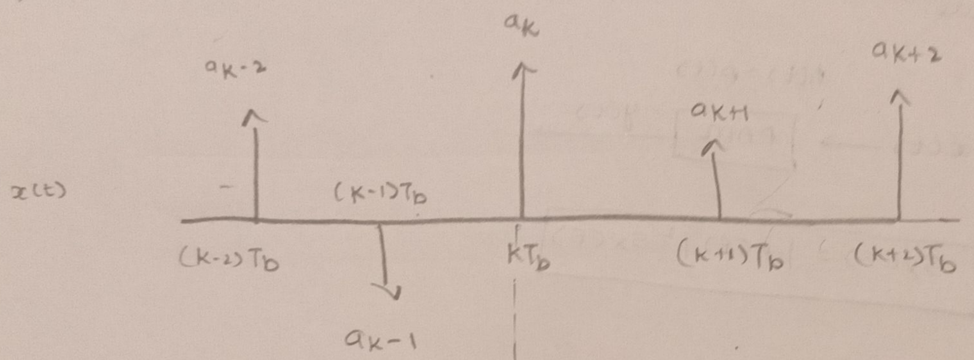


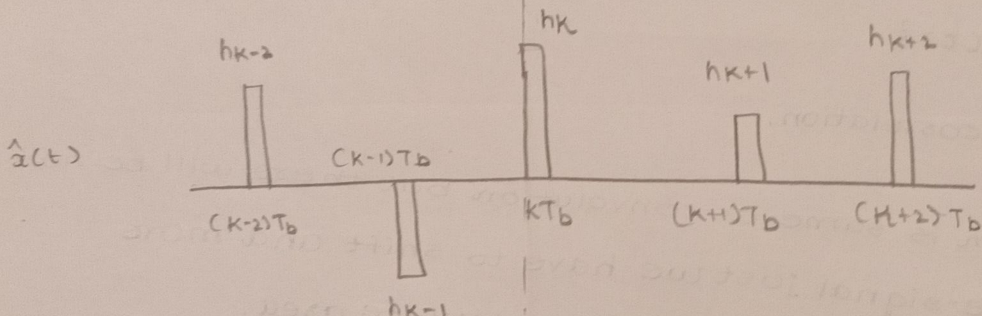
→ A signal whose length is inversely proportional to width of the signal is known as mother signal of the impulse signal.



$a_k = h_k \epsilon_0$

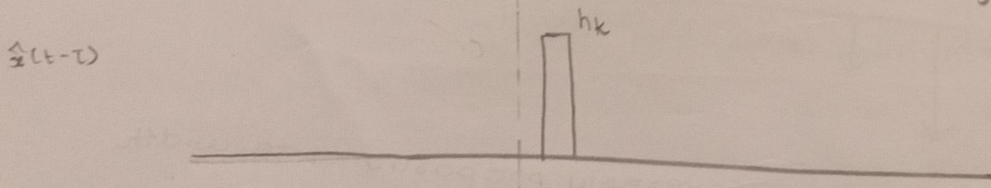
$\Rightarrow$

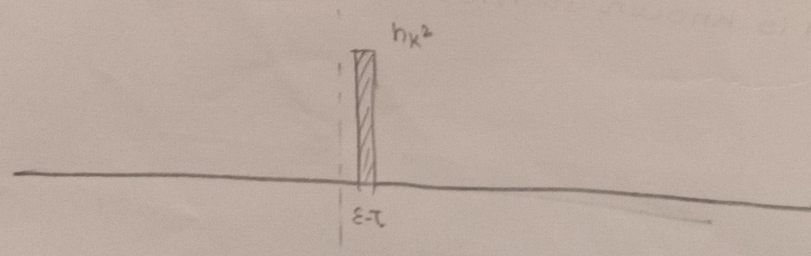




$$R_{\hat{x}}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \hat{x}(t)\, \hat{x}(t-\tau)\, dt$$

multiply





$\tau \to$ By what amount did we shift the signal.

case-1: $\tau < \epsilon$

$$R_{\hat{x}}(\tau) = \lim_{T\to\infty} \frac{1}{T} \sum_k h_k^2 (\epsilon - \tau)$$

$$= \lim_{T\to\infty} \frac{1}{T} \sum_k a_k^2 \left(\frac{\epsilon-\tau}{\epsilon^2}\right)$$

$$= \frac{R_0}{\epsilon T_b}\left(1 - \frac{\tau}{\epsilon}\right)$$

where

$$R_0 = \lim_{T\to\infty} \frac{T_b}{T} \sum_k a_k^2$$

$T \to \infty$ then $N \to \infty$

$$\boxed{N = \frac{T}{T_b}}$$

$$\therefore \boxed{R_0 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k^2}$$

case 2:

$$R_{\hat{x}}(\tau) = \lim_{T\to\infty} \frac{1}{T} \sum_k h_k h_{k+1} (\epsilon - \tau)$$

$$= \lim_{T\to\infty} \frac{1}{T} \sum_k h_k h_{k+1} \left(\frac{\epsilon-\tau}{\epsilon^2}\right)$$

$$= \frac{R_1}{\epsilon T_b}\left(1 - \frac{\tau}{\epsilon}\right)$$

where,

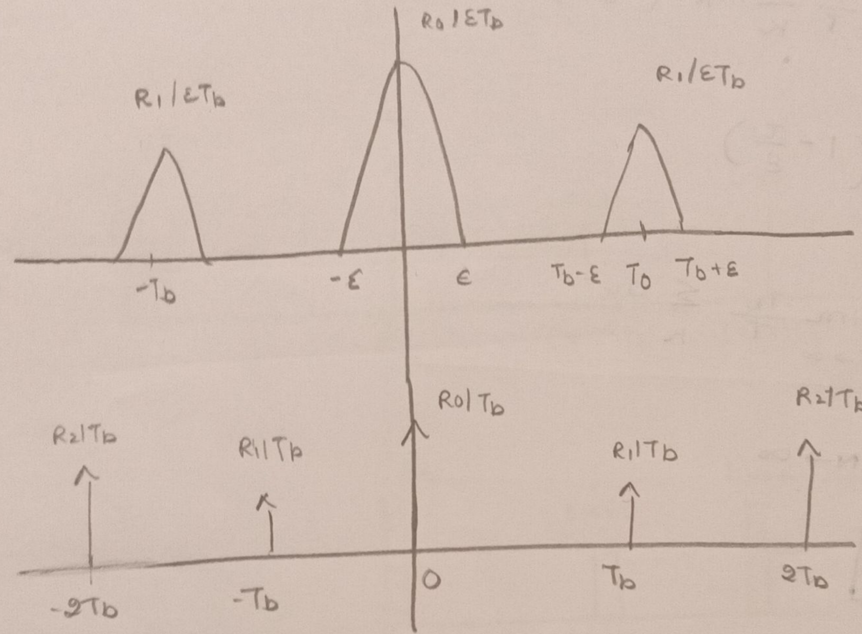
$$R_1 = \lim_{T\to\infty} \frac{T_b}{T} \sum_k h_k h_{k+1}$$

$$\boxed{R_1 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k a_{k+1}}$$

$$\therefore \quad \boxed{R_n = \lim_{N\to\infty} \frac{1}{N} \sum_{K} a_K a_{K+n}}$$

→ Plotting of Autocorrelation

of $z(t)$



Area of Δle $=$

$\frac{1}{2} \times b \times h$

$\frac{1}{2} \times 2\varepsilon \times \frac{R_0}{\varepsilon T_b}$

$= \frac{R_0}{T_b}$

$$\Rightarrow R_x(\tau) = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} R_n \, \delta(\tau - nT_b).$$

$$S_x(f) = FT\{R_x(\tau)\}$$

$$= \int_{-\infty}^{\infty} R_x(\tau) e^{-j\omega\tau} d\tau$$

$$= \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau$$

$$= \int_{-\infty}^{\infty} \frac{1}{T_b} \sum_{n=-\infty}^{\infty} R_n \, \delta(\tau - nT_b) \, e^{-j2\pi f\tau} d\tau$$

$$= \frac{1}{T_b} \sum_{n=-\infty}^{\infty} R_n e^{-j2\pi f n T_b} \underbrace{\int_{-\infty}^{\infty} \delta(\tau - nT_b) d\tau}_{1}$$

$$= \frac{1}{T_b} \sum_{n=-\infty}^{\infty} R_n e^{-j2\pi f n T_b}$$

$$S_x(f) = \frac{1}{T_b}\left[R_0 + 2\sum_{n=1}^{\infty} R_n \cos 2\pi f n T_b\right] \quad \{ e^{-j\theta} = \cos\theta - j\sin\theta \}$$

↓

power spectrum of input signal

$$\Rightarrow S_y(f) = |P(f)|^2 S_x(f)$$

$$\boxed{S_y(f) = \frac{|P(f)|^2}{T_b}\left[R_0 + 2\sum_{n=1}^{\infty} R_n \cos 2\pi f n T_b\right]}$$

$P(f) \rightarrow$ fourier transform of impulse response of a system.

$$\Rightarrow R_0 = \lim_{N\to\infty} \frac{1}{N} \sum_{k} a_k^2 = \lim_{N\to\infty} \frac{1}{N} \times N = 1$$

$$\boxed{\therefore R_0 = 1}$$

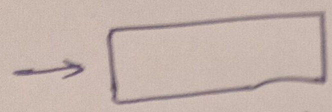
$$R_1 = \lim_{N\to\infty} \frac{1}{N} \sum_{k} a_k a_{k+1} = \lim_{N\to\infty} \frac{1}{N}\left(\frac{N}{2}(1) + \frac{N}{2}(-1)\right) = 0$$

-1	-1	+1	50
-1	1	-1	50
1	-1	-1	chances
1	1	+1	

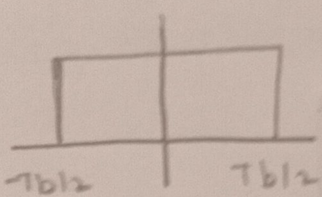
$$\boxed{\therefore R_1 \dots R_n = 0}$$

$$\therefore S_y(f) = \frac{|P(f)|^2}{T_b}[1]$$

$$\boxed{S_y(f) = \frac{|P(f)|^2}{T_b}}$$



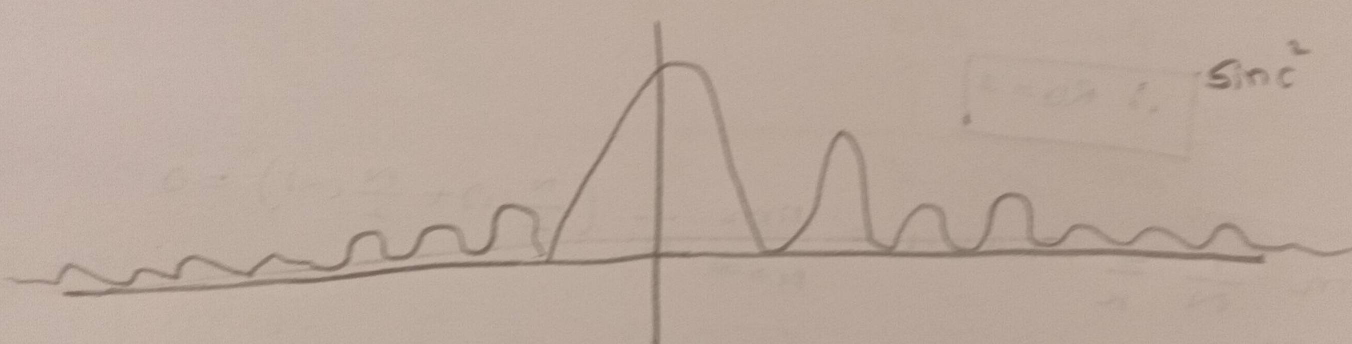
$h(t) = p(t)$



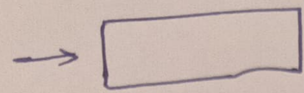
$\longleftrightarrow T_b \operatorname{sinc}(\omega T_b/2)$

$$\therefore \quad S_Y(f) = \frac{|P(f)|^2}{T_b} = \frac{T_b^2}{T_b} \operatorname{sinc}^2(\omega T_b/2)$$

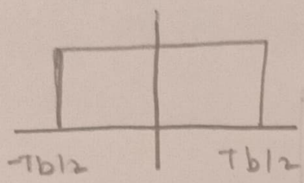
$$\boxed{S_Y(f) = T_b \operatorname{sinc}^2(\omega T_b/2)}$$



→ PSD of a PAM signal stretches from $-\infty$ to ∞.



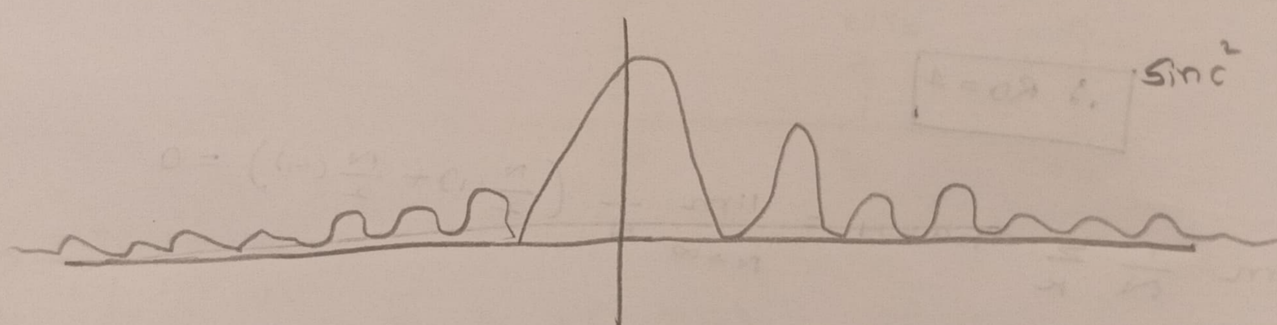
$h(t) = p(t)$



$\longleftrightarrow T_b \operatorname{sinc}(\omega T_b/2)$

$$\therefore \quad S_X(f) = \frac{|P(f)|^2}{T_b} = \frac{T_b^2}{T_b} \operatorname{sinc}^2(\omega T_b/2)$$

$$\boxed{S_X(f) = T_b \operatorname{sinc}^2(\omega T_b/2)}$$



→ PSD of a PAM signal stretches from $-\infty$ to ∞.