2310040036

D. Lahari

CO2 ASSIGNMENT

1. In a conducting medium the magnetic field is given as $\bar{H} = xy^2z\hat{x} + 5(x+1)y^2z^2\hat{j} + (x+10)y^2z^2\hat{z}$. Find the conduction current density at point (4,1,-1) meter. Also, find the current enclosed by square loop $y=2, 0<x<3, 0<z<1$.

A. Given,

$$\bar{H} = xy^2z\hat{x} + 5(x+1)y^2z^2\hat{j} + (x+10)y^2z^2\hat{z}.$$

⇒ conduction current density:

$$\Delta \times H = J$$

$$J = \Delta \times H$$

$$J = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy^2z & 5(x+1)y^2z^2 & (x+10)y^2z^2 \end{vmatrix}$$

$$J = \hat{i}\left(\frac{\partial}{\partial y}(x+10)y^2z^2 - \frac{\partial}{\partial z}5(x+1)y^2z^2\right) -$$

$$\hat{j}\left(\frac{\partial}{\partial x}(x+10)y^2z^2 - \frac{\partial}{\partial z}xy^2z\right) +$$

$$\hat{z}\left(\frac{\partial}{\partial x}5(x+1)y^2z^2 - \frac{\partial}{\partial y}xy^2z\right).$$

$J = \hat{i}\,((x+10)2yz^2 - 5(x+1)y^2(2z)) - \hat{j}\,(y^2z^2 - xy^2) + \hat{z}\,(5y^2z^2 - x(2y)z)$

at $(4,1,-1)$

$J = \hat{i}\,((4+10)2(1)(-1)^2 - 5(4+1)1^2 2(-1)) - \hat{j}\,(1^2(-1)^2 - 4(1)^2) + \hat{z}\,(5(1)^2(-1)^2 - 4(2(1)(-1)))$

$J = \hat{i}\,(14\times 2\times 1 - 5\times 5\times(-2)) - \hat{j}\,(1-4) + \hat{z}\,(5+8)$

$J = \hat{i}\,(28+50) - \hat{j}\,(-3) + \hat{z}\,(13)$

$J = 78\hat{i} + 3\hat{j} + 13\hat{z}$

⇒ current enclosed:

From Ampere's circuit law:

$$\oint_c H \cdot dl = I_{enclosed}$$

∵ loop is at $y = 2$

$$I_{enclosed} = \int_s (\nabla \times H) \cdot ds$$

$$I_{enclosed} = \int_0^3 \int_0^1 J_y \, dz\, dx$$

$$I_{enclosed} = \int_0^3 \int_0^1 y^2z^2 - xy^2 \, dz\, dx$$

at $y = 2$

$$I_{enclosed} = \int_0^3 \int_0^1 4z^2 - 4x \, dz \, dx$$

$$I_{enclosed} = \int_0^3 4\left(\frac{z^3}{3}\right) - 4xz \Big|_0^1 dx$$

$$= \int_0^3 x - 4xz \, dx$$

$$= \int_0^3 \frac{4}{3} - 4x \, dx$$

$$= \frac{4}{3}x - 4\frac{x^2}{2} \Big|_0^3$$

$$= \frac{4}{\cancel{3}} \cancel{3} - \cancel{4}^{2} \cdot \frac{9}{\cancel{2}}$$

$$= 4 - 18$$

$$= 14$$

2. Find $\iint F\,ds$ using divergence theorem where $F = 6xyz\,i + 24x\,j + 2yz\,k$ and S is the surface of the cube bounded by $x=0, x=1, y=0, y=1$ and $z=0$ and $z=1$.

As. Given

$$F = 6xyz\,i + 24x\,j + 2yz\,k$$

By using Divergence theorem

$$\iint_S F\,ds = \iiint_V (\nabla\cdot F)\,dV$$

$$\nabla\cdot F = \frac{\partial}{\partial x}(6xyz) + \frac{\partial}{\partial y}(24xj) + \frac{\partial}{\partial z}(2yz)$$

$$= 6yz + 0 + 2y$$

$$\iint F\,ds = \int_0^1\int_0^1\int_0^1 6yz + 2y\,dx\,dy\,dz$$

$$= \int_0^1\int_0^1 6yz + 2y\,dy\,dz$$

$$= \int_0^1 6z\frac{y^2}{2} + \cancel{2}\cdot\frac{y^2}{\cancel{2}}\Big|_0^1\,dz$$

$$= \int_0^1 6z\cdot\frac{1}{2} + 1\,dz$$

$$= \int_0^1 3z + 1\,dz$$

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$$= \frac{3z^2}{2} + z \Big|_0^1$$

$$= \frac{3}{2} + 1 = \frac{5}{2}$$

3. Find the electric field intensity at a distance of 40 cm from a charge of 4 μC in vaccum.

A. The electric field in intensity at a distance r from a point charge Q in vaccum is given by coulomb's law.

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$Q = 4\,\mu C = 4\times10^{-3}\,C$

$r = 40\,cm = 0.40\,m$

$\varepsilon_0 = 8.854\times10^{-12}\,F/m$

$\frac{1}{4\pi\varepsilon_0} = 9\times10^{9}\,Nm^2/C^2$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$E = 9\times10^{9}, \frac{4\times10^{-3}}{(0.40)^2}$$

$$E = \frac{36 \times 10^6}{0.16}$$

$$E = 225 \times 10^6$$

$$E = 2.25 \times 10^8 \ V/m$$

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