M-ary PAM:-

For each symbol transmission within the time interval of T_b to carry more bits, there must be more than two signaling symbols to choose from. By increasing the number of symbols to M in the signal set, we ensure that the information transmitted by each symbol will also increase with M.

$$I_m = \log_2 M \text{ bits}$$

In general, the information I_m transmitted by M-ary symbol. Hence, to transmit n bits, we need only $M = 2^n$ pulses or M-ary signaling. This style of M-ary signaling is known as PAM, because the data is conveyed by the varying pulse amplitude

Example:-

→ Determine the PSD of the quaternary (4-ary) baseband signaling, when the message bits 1 and 0 are equally likely.

Sol:-

The 4-ary line code has four distinct symbols corresponding to the four different combinations of two message bits. One such mapping is

$$a_k = \begin{cases} -3 & \text{message bits } 00 \\ -1 & \text{message bits } 01 \\ +1 & \text{message bits } 10 \\ +3 & \text{message bits } 11 \end{cases}$$

$$R_0 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k^2$$

with in the summation, 1/4 of the a_k will be ± 1 and ± 3. Thus,

$$R_0 = \lim_{N\to\infty} \frac{1}{N}\left[\frac{N}{4}(-3)^2 + \frac{N}{4}(-1)^2 + \frac{N}{4}(1)^2 + \frac{N}{4}(3)^2\right] = 5$$

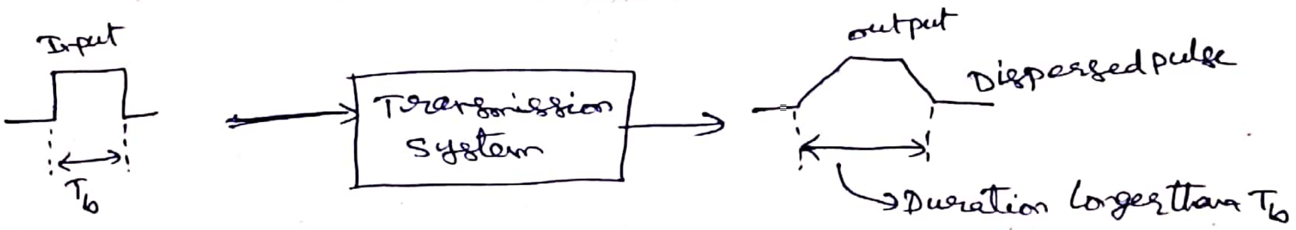
$$= \lim_{N\to\infty} \frac{1}{N}\left[\frac{9N}{4} + \frac{N}{4} + \frac{N}{4} + \frac{9N}{4}\right] = \frac{1}{N}\cdot\frac{20N}{4} = 5 \qquad \therefore S_x(f) = \frac{5}{T_s}$$

pulse shaping:-

Intersymbol Interference (ISI):-

In a Communication system, when the data is being transmitted in the form of pulses (i.e. bits), the output produced at the receiver due to other bits or symbols interferes with the output produced by the desired bit. This is known as ISI.

The ISI arises due to the imperfections in the overall frequency response of the system. when a short pulse of duration T_b seconds is transmitted through a bandlimited system, then the frequency components contained in the input pulse are differentially attenuated and more importantly differentially delayed by the system.



Due to this, the pulse appearing at the output of the system will be dispersed over an interval which is Longer than T_b seconds.

Effects:-

i. In the absence of ISI and noise, the transmitted bit can be decoded correctly at the receiver.

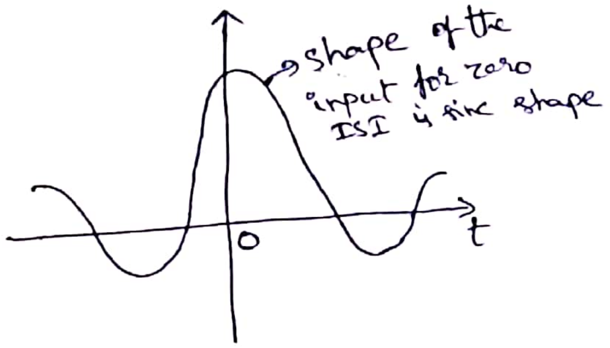
ii. The presence of ISI will introduce errors in the decision at the receiver output.

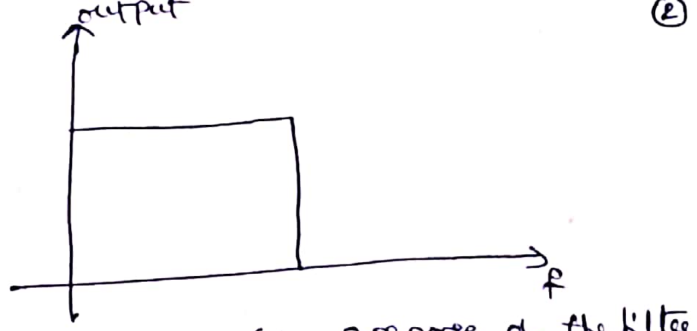
(iii). Hence, the receiver can make an error in deciding whether it has received a logic 1 or a logic 0.

Remedy to Reduce ISI:-

1. It has been proved that the function which produces a zero ISI is a sinc function.

ii. The sinc pulse transmitted to have a zero ISI, which is known as Nyquist pulse shaping.





frequency response of the filter

Nyquist criterion for Distortionless Baseband binary transmission:-

If ISI is absence, then

$$y(t_i) = \mu a_i$$

This expression shows that under those conditions, the i^{th} transmitted bit can be decoded correctly. Inorder to minimize the effects of ISI, we have to design the transmitting and receiving filters properly. To determine the transfer functions of the transmitting and receiving filters, to reconstruct the transmitted data sequence $\{b_k\}$. This is achieved by first extracting and then decoding the corresponding sequence of weights from the output $y(t)$.

$$y(t) = \mu \sum_{k=-\infty}^{\infty} a_k\, p(t - kT_b)$$

output $y(t)$ is dependent on a_k, the received pulse $p(t)$ and the scaling factor μ.

The decoding should be such that the contribution of the weighted pulse.

$$p(iT_b - kT_b) = \begin{cases} 1 & \text{for } i = k \\ 0 & \text{for } i \neq k \end{cases}$$ where $p(0) = 1$ due to normalizing

To transform this above condition into frequency domain.

$$P_\delta(f) = F\left[p(nT_b)\right] = R_b \sum_{n=-\infty}^{\infty} P(f - nR_b) \qquad \left[\because R_b = 1/T_b\right]$$

$P_\delta(f) \rightarrow$ Fourier transform of an infinite periodic sequence of unity impulses whose strengths are weighted by the respective sample values of $p(t)$. Hence, it can be written as

$$P_\delta(f) = \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} [p(mT_b)\,\delta(t-mT_b)]\, e^{-j2\pi f t}\, dt$$

where $\sum_{m=-\infty}^{\infty} p(mT_b)\,\delta(t-mT_b)$ represents the sequence of unit impulses weighted by the respective samples.

Let $(m=i-k)$, if $i=k$ then $m=0$ and if $i\neq k$ then $m\neq 0$.

then

$$P_\delta(f) = \int_{-\infty}^{-\infty} p(0)\,\delta(t)\, e^{-j2\pi f t}\, dt$$

Using the shifting property of delta function.

$$P_\delta(f) = p(0)$$

But, $p(0)=1$ due to normalization, hence $P_\delta(f)=1$

$$1 = R_b \sum_{n=-\infty}^{\infty} P(f-nR_b)$$

therefore

$$\sum_{n=-\infty}^{\infty} P(f-nR_b) = 1/R_b = T_b$$

This expression is called as the nyquist criterion for distortionless baseband transmission in the absence of noise.

Raised Cosine Spectrum:-

The difficulties are identified from the Nyquist Criterion are

i. It is necessary that the amplitude characteristics of $P(f)$ should be flat from $-B_0$ to B_0 and zero outside this band. But, abrupt transition at $\pm B_0$ is not physically realizable.

ii. Due to discontinuity of $P(f)$ at $\pm B_0$, there is practically no margin of error in sampling times at the receiver end.

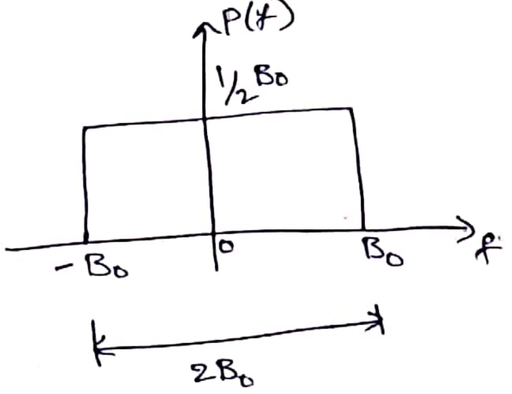
To reduce/minimize these two difficulties, to put an condition on the overall frequency response $P(f)$ to satisfy as

$$\sum_{n=-\infty}^{\infty} P(f-nR_b) = T_b = \frac{1}{R_b}$$

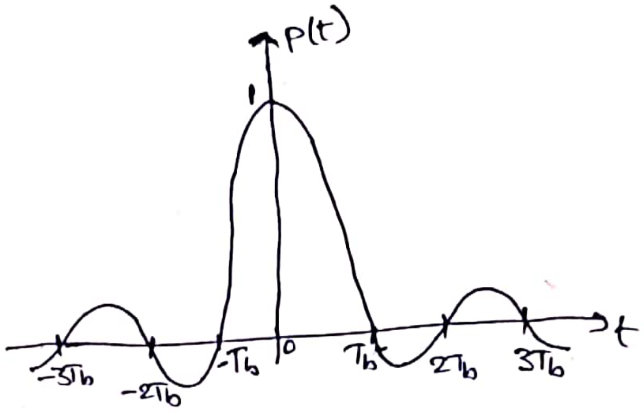
Expanding the summation sign

$$\cdots P(f+R_b)+P(f)+P(f-R_b)+P(f-2R_b)\cdots = T_b$$

$$B_0 = \frac{R_b}{2} \Rightarrow R_b = 2B_0$$



Graphical representation of P(f)



Time domain representation

$$\cdots P(f+2B_0)+P(f)+P(f-2B_0)+P(f-4B_0)+\cdots = \frac{1}{2B_0}$$

To retain only the three terms on LHS which correspond to $n=-1, n=0, n=1$ and restrict the frequency band of interest to $(-B_0, B_0)$ to get

$$P(f+2B_0)+P(f)+P(f-2B_0)=\frac{1}{2B_0} \quad \text{and given } -B_0 \le f \le B_0$$

It is possible to devise several bandlimited functions which will satisfy above equation, one of them is called as the raised cosine spectrum. this spectrum consists of a flat portion and a roll off portion. the raised cosine spectrum is expressed as

$$P(f) = \begin{cases} \frac{1}{2B_0} & 0 \le |f| \le f_1 \\ \frac{1}{4B_0}\left\{1-\sin\left[\frac{\pi(|f|-B_0)}{2B_0-2f_1}\right]\right\} & f_1 \le |f| \le 2B_0-f_1 \\ 0 & |f| \ge 2B_0-f_1 \end{cases}$$

the relation between frequency parameter f_1 and the bandwidth B_0 are related to

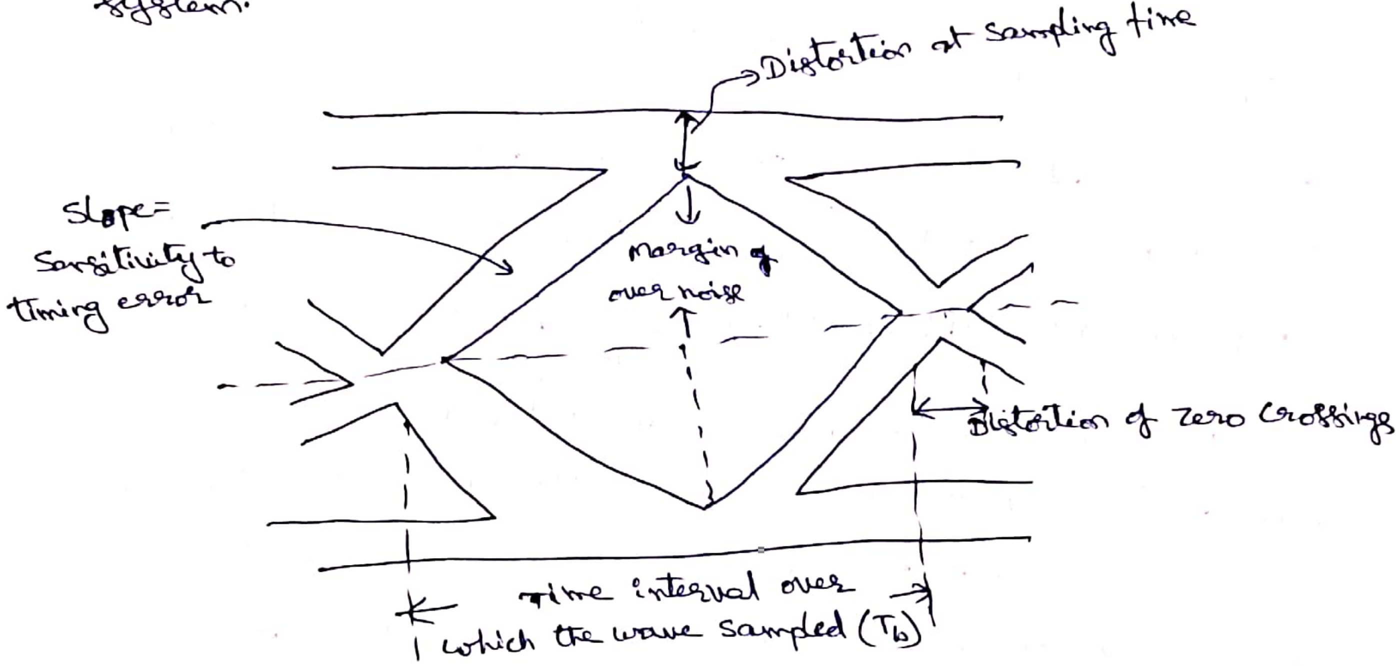
$$\alpha = 1 - f_1/f_m$$

where α is called as the roll of factor. It indicates the excess bandwidth over the ideal solution B_0. the transmission bandwidth B_T is defined as

$$B_T = 2B_0 - f_1 = B_0(1+\alpha).$$

Eye pattern:-

Eye pattern is a pattern displayed on the screen of a Cathode Ray oscilloscope (CRO). The shape of this pattern resembles with the shape of human eye. The eye pattern is obtained on the C.R.O. by applying the received signal to vertical deflection plates (Y-plates) of the C.R.O and a sawtooth wave at the transmission symbol rate ($1/T_b$) to the horizontal deflection plates (X-plates). The interior region of the eye pattern is called as the eye opening. The eye pattern provides a great deal of information about the performance of the system.



i. The width of the eye opening defines the time interval over which the received wave can be sampled, without an error due to ISI, the best time for sampling is when the eye is open widest.

ii. The sensitivity of the system to the timing error is determined by the rate of closure of the eye as the sampling rate is varied.

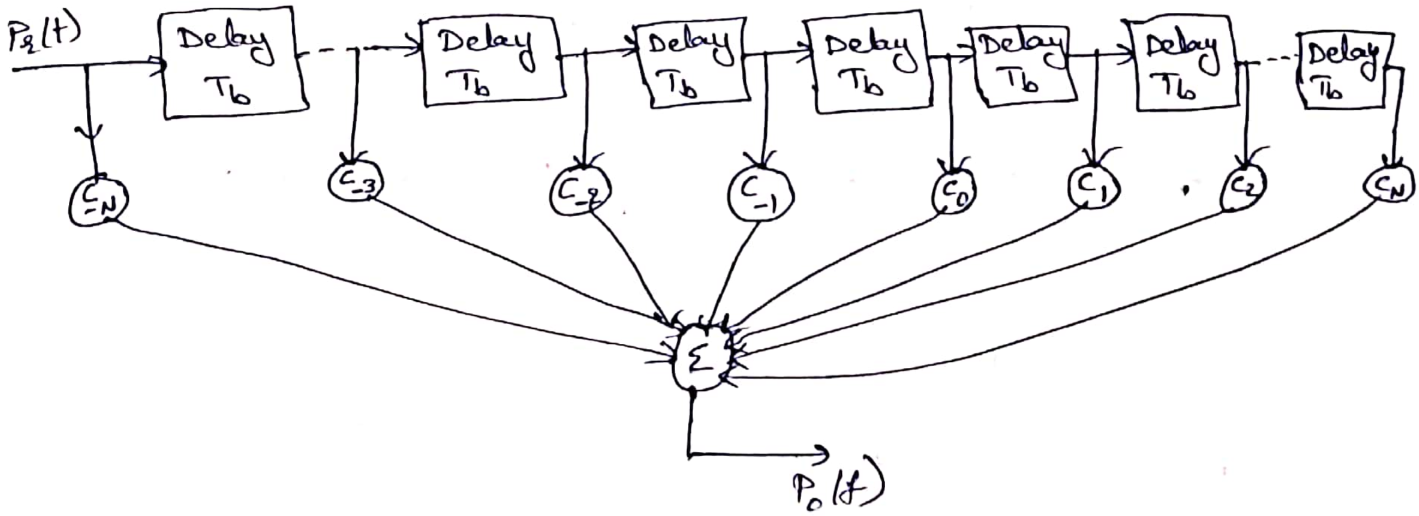
iii. The height of eye opening at a specified sampling time defines the margin over noise

Equalizers:-

A data modulated baseband pulse train is often attenuated and distorted by the transmission medium. The attenuation can be compensated by the preamplifier, whereas the distortion can be compensated by an equalizer. Channel distortion is in the form of dispersion, which is caused by an attenuation of certain critical frequency components of the baseband data pulse train. Theoretically, an equalizer should have a frequency characteristic that is the inverse of that of the distortive channel medium. Unfortunately, the equalizer could also enhance the received channel noise by boosting its components at these critical frequencies. This undesirable phenomenon is known as noise enhancement or noise amplification.

(i) Zero-Forcing equalizer design:-

It is really not necessary to eliminate or minimize ISI with neighboring pulses for all t. All that is needed is to eliminate or minimize interference among neighbouring pulses at their respective sampling instants only. The design goal is to force the equalizer output pulse to have zero ISI values at the sampling instants. The goal of is for the equalizer output pulses to satisfy the Nyquist's first criterion.



The time delay between successive taps is chosen to be T_b, the same interval for each data symbol in baseband modulation.

The output $P_o(t)$ is the sum of pulses of the form $C_K P_R(t-KT_b)$.

$$P_o(t) = \sum_{n=0}^{2N} C_{n-N} P_R(t-nT_b)$$

The samples of $P_o(t)$ at $t=KT_b$ are

$$P_o(KT_b) = \sum_{n=0}^{2N} C_{n-N} P_R(KT_b - nT_b) \qquad K=0,\pm1,\pm2,\pm3 \cdots$$

By using a more convenient notation $P_R[K]$ to denote $P_R(KT_b)$ and $P_o[K]$ to denote $P_o(KT_b)$ can be expressed as

$$P_o[K] = \sum_{n=0}^{2N} C_{n-N} P_R[K-n] \qquad K=0,\pm1,\pm2,\cdots$$

Considering the delay in the transversal filter, we can rewrite Nyquist's first criterion to require that samples $P_o[K]=0$ for $K \neq N$ and $P_o[N]=1$.

$$P_o[K] = \begin{cases} 1 & K=N \\ 0 & K=0,\cdots N-1, N+1,\cdots 2N \end{cases}$$

$$\underbrace{\begin{bmatrix} P_o[0] \\ \vdots \\ P_o[N] \\ \vdots \\ P_o[2N] \end{bmatrix}}_{P_o} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} P_R[0] & P_R[-1] & \cdots P_R[-2N+1] & P_R[-2N] \\ P_R[1] & P_R[0] & P_R[-2N+2] & P_R[-2N+1] \\ \vdots & \vdots & \vdots & \vdots \\ P_R[2N-1] & P_R[2N-2] & P_R[0] & P_R[-1] \\ P_R[2N] & P_R[2N-1] & P_R[1] & P_R[0] \end{bmatrix}}_{P_R} \underbrace{\begin{bmatrix} C_{-N} \\ C_{-N+1} \\ \vdots \\ C_{-1} \\ C_0 \\ C_1 \\ \vdots \\ C_{N-1} \\ C_N \end{bmatrix}}_{C}$$

In this compact expression, the $(2N+1) \times (2N+1)$ matrix P_R has identical entries along all the diagonal lines. Such a matrix is known as the Toeplitz matrix and is commonly encountered in describing a convolutive relationship.

ii. Mean Square Error (MSE) equalizer design:-

Another design approach aimed at minimizing the mean square error (MSE) between equalizer output response $P_o[K]$ and the desired zero ISI response. This is known as minimum MSE (MMSE) method for designing transversal filter equalizers. The MMSE design does not try to force the pulse samples to zero at 2N points. Instead, we minimize the squared errors averaged over a set of output samples. This method involves more simultaneous equations. Thus we must find the equalizer tap values to minimize the average (mean) square error over a larger window of length 2K+1, that is, we aim to minimize the MSE.

$$MSE \triangleq \frac{1}{2K+1} \sum_{k=N-K}^{N+K} \left(P_o[k] - \delta[k-N] \right)^2$$

where we use a function known as the Kronecker delta

$$\delta[k] = \begin{cases} 1 & k=0 \\ 0 & k \neq 0 \end{cases}$$

The equalizer output sample values are

$$P_o[K+N] = \sum_{n=0}^{2N} c_{n-N}\, P_r[K+N-n] \quad K=0,\pm1,\pm2 \cdots \pm K$$

The solution to this minimization problem can be better represented in matrix form as

$$c = P_r^{+} P_o$$

where P_r^{+} represents the ~~moore~~ moore-Penrose pseudo-inverse of the non-square matrix P_r of size $(2K+1)\times(2N+1)$

$$P_r = \begin{bmatrix} P_r(N-K) & P_r(N-K-1) & \cdots\; P_r[-N-K+1] & P_r[-N-K] \\ P_r[N-K+1] & P_r[N-K] & P_r[-N-K+2] & P_r[-N-K+1] \\ \vdots & \vdots & \vdots & \\ P_r[N+K-1] & P_r[N+K-2] & P_r[-N+K] & P_r[-N+K-1] \\ P_r[N+K] & P_r[N+K-1] & P_r[-N+K+1] & P_r[-N+K] \end{bmatrix}$$

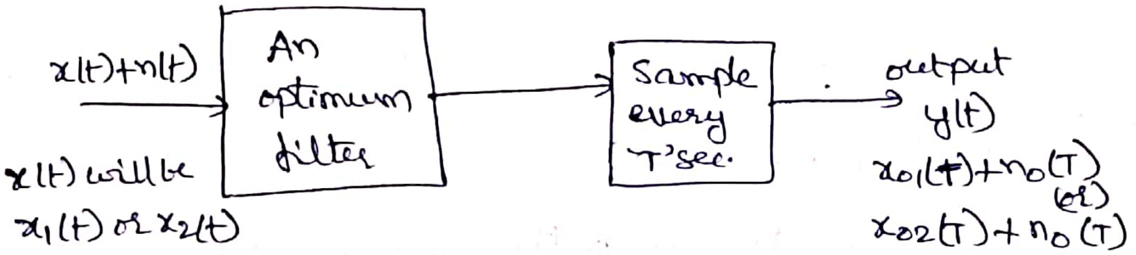
optimum filter/receiver:-

The integrator or dump filter is an optimum filter/receiver, which is used to minimize the probability of error (P_e). A generalized filter to ~~[illegible]~~ receive binary coded signals, it is known as optimum filter.

Let the received signal be a binary waveform and assumed that it is a polar NRZ signal.

For binary 1, $x_1(t) = +A$ for one bit period T

binary 0, $x_2(t) = -A$ for one bit period T



Input to the receiver is $x(t)+n(t)$

output from the receiver $x_{01}(T)+n_0(T)$ or $x_{02}(T)+n_0(T)$

Therefore, the decision boundary will be in the middle of x_{01} and x_{02}

$$= \frac{x_{01}(T)+x_{02}(T)}{2}$$

The probability density function ~~[illegible]~~ (PDF) for $n_0(t)$ is

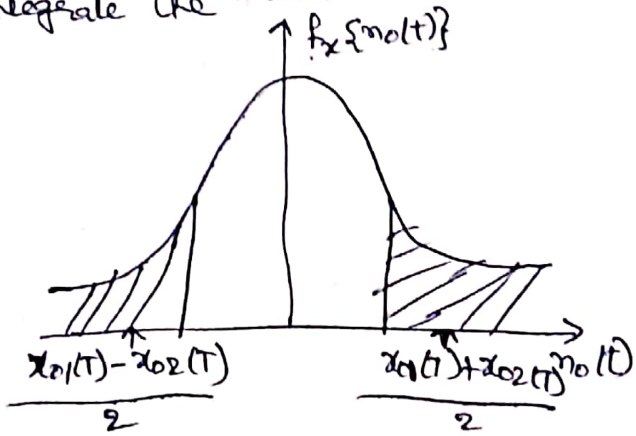
$$f_x\{n_0(t)\} = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-[n_0(t)]^2/2\sigma^2}$$

$n_0(t) \rightarrow$ random function, $\sigma \rightarrow$ Standard deviation

Hence, to evaluate the P_e, it must integrate the area under the PDF curve from $n_0(T) \geq \frac{x_{01}(T)-x_{02}(T)}{2}$

Then

$$P_e = P\left[n_0(T) \geq \frac{x_{01}(T)-x_{02}(T)}{2}\right]$$



$$P_e = \int_{\frac{x_{01}(T)-x_{02}(T)}{2}}^{\infty} f_X[n_0(t)]\, d[n_0(t)]$$

$$P_e = \int_{\frac{x_{01}(T)-x_{02}(T)}{2}}^{\infty} \frac{1}{\sigma\sqrt{2\pi}}\, e^{-[n_0(t)]^2/2\sigma^2}$$

Let $\frac{[n_0(t)]^2}{2\sigma^2} = y^2 \Rightarrow n_0(t) = \sigma\sqrt{2}\, y$ and $d[n_0(t)] = \sqrt{2}\,\sigma\, dy$

$n_0(t) = \frac{x_{01}(T)-x_{02}(T)}{2} \Rightarrow y = \frac{x_{01}(T)-x_{02}(T)}{2\sqrt{2}\,\sigma}$

Substitute above values in P_e. Then

$$P_e = \int_{\frac{x_{01}(T)-x_{02}(T)}{2\sqrt{2}\sigma}}^{\infty} \frac{1}{\sigma\sqrt{2\pi}}\, e^{-y^2}\cdot \sigma\sqrt{2}\, dy$$

$$= \frac{1}{\sqrt{\pi}} \int_{\frac{x_{01}(T)-x_{02}(T)}{2\sqrt{2}\sigma}}^{\infty} e^{-y^2}\, dy$$

Let us rearrange the equation

$$P_e = \frac{1}{2}\left[\frac{2}{\sqrt{\pi}} \int_{\frac{x_{01}(T)-x_{02}(T)}{2\sqrt{2}\sigma}} e^{-y^2}\, dy\right]$$

Let $\frac{2}{\sqrt{\pi}} \int_{\mu}^{\infty} e^{-y^2} dy = \mathrm{erfc}(\mu)$

then

$$\boxed{P_e = \frac{1}{2}\,\mathrm{erfc}\left(\frac{x_{01}(T)-x_{02}(T)}{2\sqrt{2}\,\sigma}\right)}$$

Matched Filter receiver/ detection :-

If a filter produces an output in such a way that it maximizes the ratio of output peak power to mean noise power in its frequency response, then that filter is called matched filter.

To evaluate the probability of error for matched filter, let us again start with optimum filter and shall consider the special case of white Gaussian Noise.

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left[\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\,\sigma}\right]$$

where

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2 = \int_{-\infty}^{\infty} \frac{|X(f)|^2}{S_{ni}(f)}\, df$$

$S_{ni}(f) \rightarrow$ white Gaussian noise with PSD of $\frac{N_0}{2}$, then

$$\left(\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right)^2_{max} = \int_{-\infty}^{\infty} \frac{|X(f)|^2}{\frac{N_0}{2}}\, df = \frac{2}{N_0}\int_{-\infty}^{\infty} |X(f)|^2\, df$$

Also, parseval's theorem states that

$$\int_{-\infty}^{\infty} |X(f)|^2\, df = \int_{-\infty}^{\infty} x^2(t)\, dt = \int_{0}^{T} x^2(t)\, dt$$

In the last integral we have taken limits from 0 to T because $x(t)$ exist from 0 to T only. We know that $x(t) = x_1(t) - x_2(t)$. Then

$$\int_{-\infty}^{\infty} |X(f)|^2\, df = \int_{0}^{T} [x_1(t) - x_2(t)]^2\, dt$$

$$= \int_{0}^{T} x_1^2(t)\, dt + \int_{0}^{T} x_2^2(t)\, dt - 2\int_{0}^{T} x_1(t)\, x_2(t)\, dt$$

where $\int_{0}^{T} x_1^2(t)\, dt = \int_{0}^{T} x_2^2(t)\, dt = E$ i.e. energy of the signal

$$\int_{0}^{T} x_1(t)\, x_2(t)\, dt = E_1 = E_2 = E_{12} = -E$$

Then $$\int_{-\infty}^{\infty} |X(f)|^2\, df = [E + E - 2(-E)] = 4E$$

Substituting this values:

$$\left[\frac{x_{01}(T)-x_{02}(T)}{\sigma}\right]^2_{max} = \frac{2}{N_0}\cdot 4E = \frac{8E}{N_0}$$

Therefore

$$\left(\frac{x_{01}(T)-x_{02}(T)}{\sigma}\right) = 2\sqrt{2}\cdot\sqrt{\frac{E}{N_0}}$$

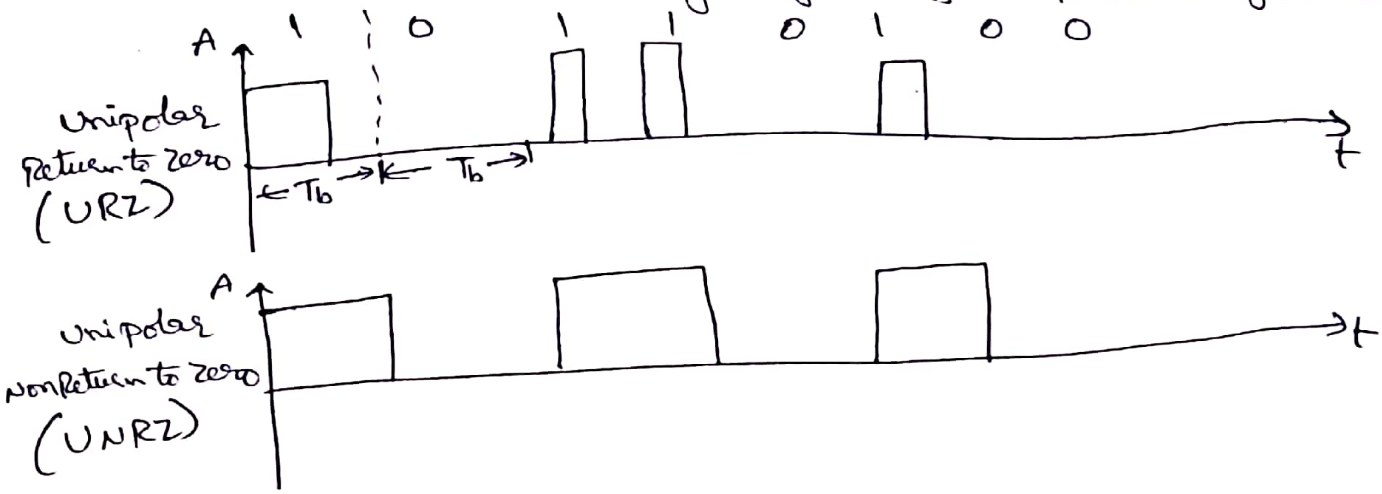
we can rearrange this equation as per the probability of error standard equation. Then

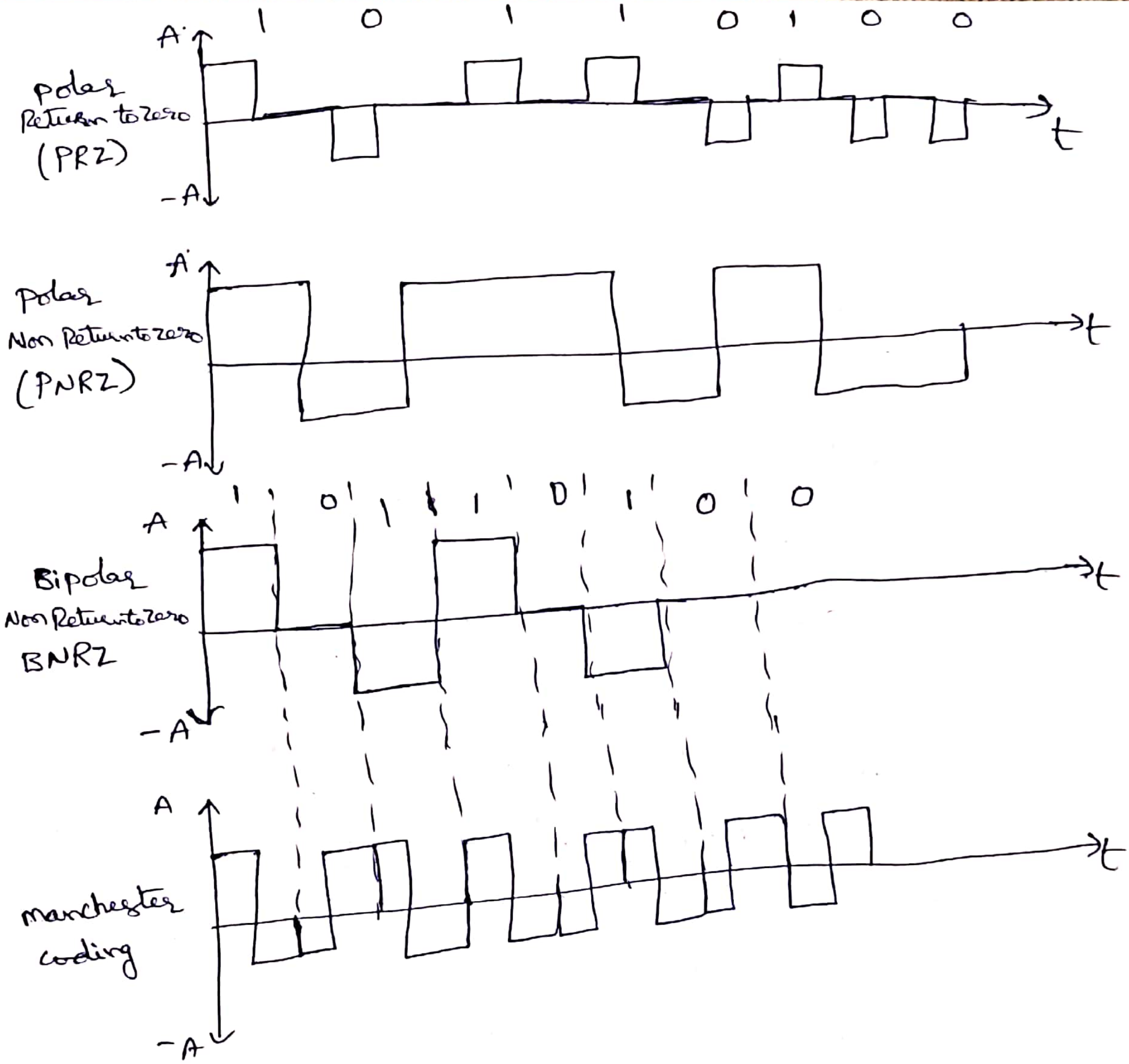
$$\boxed{P_e = \frac{1}{2}\,erfc\sqrt{\frac{E}{N_0}}}$$

Line Coding:-

The digital data can be transmitted by various Transmission or line codes such as on-off, polar, bipolar and so on. This is called line coding. Each type of line code has its advantages & disadvantages

1. Transmission Bandwidth – must be as small as possible
2. power efficiency – should be as small as possible
3. Error detection & correction capability:- must be possible to detect & correct
4. Adequate timing content:- must be possible to extract timing or clock information from the signal.
5. Transparency:- must be possible to transmit a digital signal correctly regardless the pattern of 1's and 0's.




1 0 1 1 0 1 0 0
A
polar Return to zero (PRZ)
-A
t
Polar Non Return to zero (PNRZ)
Bipolar Non Return to Zero BNRZ
manchester coding