CO1: Static Electric Fields

Coordinate Systems

Points or vectors can be represented in curvilinear coordinate systems, either orthogonal or nonorthogonal. The three common coordinate systems are Cartesian (rectangular), circular cylindrical, and spherical. A point P can be represented as (x, y, z) in Cartesian coordinates.

Charge Distributions

Electric fields can be produced by various charge configurations including point, line, surface, and volume charges.

Coulomb's Law

The force F between two point charges Q_1 and Q_2 is along the line joining them, directly proportional to the product Q_1Q_2, and inversely proportional to the square of the distance R between them.

$$F = \frac{Q_1Q_2}{4\pi\epsilon_0 R^2} \tag{1}$$

If point charges Q_1 and Q_2 are located at points with position vectors $\mathbf{r}1$ and $\mathbf{r}2$, the force $\mathbf{F}12$ on Q_2 due to Q_1 is given by:

$$\mathbf{F}12 = \frac{Q_1Q_2(\mathbf{r}_2 - \mathbf{r}_1)}{4\pi\epsilon_0|\mathbf{r}_2 - \mathbf{r}_1|^3} \tag{2}$$

Electric Field Intensity

The electric field intensity $\mathbf{E}$ at a point is the force per unit charge that would be exerted on a positive test charge placed at that point.

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 R^2}\mathbf{a}_R \tag{3}$$

For N point charges $Q_1, Q_2, \ldots, Q_N$ located at $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N$, the electric field intensity at point $\mathbf{r}$ is:

$$\mathbf{E} = \sum \frac{Q_i(\mathbf{r} - \mathbf{r}_i)}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}_i|^3} \tag{4}$$

The electric field $\mathbf{E}$ due to different charge distributions is given by:

Line charge: $\mathbf{E} = \int \frac{\rho_L dl}{4\pi\epsilon_0 R^2}\mathbf{a}_R$ Surface charge: $\mathbf{E} = \int \frac{\rho_s dS}{4\pi\epsilon_0 R^2}\mathbf{a}_R$ Volume charge: $\mathbf{E} = \int \frac{\rho_v dv}{4\pi\epsilon_0 R^2}\mathbf{a}_R$

Electric Flux Density

Related to electric field intensity by $\mathbf{D} = \epsilon\mathbf{E}$.

Gauss's Law

The total electric flux through any closed surface is equal to the total charge enclosed by that surface.

$$\oint \mathbf{D} \cdot d\mathbf{S} = Q_{enc} \tag{5}$$

Divergence

The divergence of a vector field $\mathbf{D}$ is a measure of the flux per unit volume emanating from a point.

$$\nabla \cdot \mathbf{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \tag{6}$$

Divergence Theorem

States that the total outward flux of a vector field $\mathbf{D}$ through a closed surface S is equal to the volume integral of the divergence of $\mathbf{D}$ over the volume V enclosed by S.

$$\oint \mathbf{D} \cdot d\mathbf{S} = \iiint (\nabla \cdot \mathbf{D}) dv \tag{7}$$

Potential and Potential Difference

The potential difference V_{AB} between points A and B is the work done in moving a unit positive charge from A to B.

$$V_{AB} = -\int_A^B \mathbf{E} \cdot d\mathbf{l} \tag{8}$$

Potential Gradient

Electric field intensity is related to the electric potential V as:

$$\mathbf{E} = -\nabla V \tag{9}$$

Boundary Conditions on E and D

The lines of force or flux lines (or the direction of $\mathbf{E}$) are always normal to equipotential surfaces.

Electric Current

Electric current I is the rate of flow of charge.

Current Density

Current density $\mathbf{J}$ is related to the electric field intensity $\mathbf{E}$ through the conductivity σ of the material:

$$\mathbf{J} = \sigma \mathbf{E} \tag{10}$$

Equation of Continuity

Expresses the conservation of charge.

CO2: Static Magnetic Fields

Biot-Savart's Law

$$d\mathbf{H} = \frac{I d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2} \tag{11}$$

Ampere's Circuital Law

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc} \tag{12}$$

Differential Form of Ampere's Circuital Law

$$\nabla \times \mathbf{H} = \mathbf{J} \tag{13}$$

Curl

The curl of a vector field $\mathbf{H}$ is a measure of the circulation per unit area at a point.

Stokes' Theorem

Relates the line integral of a vector field around a closed path to the surface integral of the curl of the vector field over any surface bounded by that path.

Lorentz Force Equation

The force on a moving charge in a magnetic field.

Force on a Current Element in a Magnetic Field

$$d\mathbf{F} = I d\mathbf{l} \times \mathbf{B} \tag{14}$$

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