AE 6042 Computational Fluid Dynamics
Assignment # 2

Note: Please show your work.

1) A one-sided finite difference expression for $\left(\frac{\partial u}{\partial x}\right)_i$ can be shown to be

$$\left(\frac{\partial u}{\partial x}\right)_i \approx \frac{3u_i - 4u_{i-1} + u_{i-2}}{2\Delta x}$$

a) Write the leading term in the truncation error. Assume that the nodes are equally spaced.

b) What is the order of accuracy of this difference representation?

2) Develop a finite difference $\frac{\partial^2 u}{\partial x^2}$ for a <u>non-uniformly spaced</u> mesh. Note: You do not need to show the truncation error terms but your approximation must be at least 1st order accurate.

3) Consider the following equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}$$

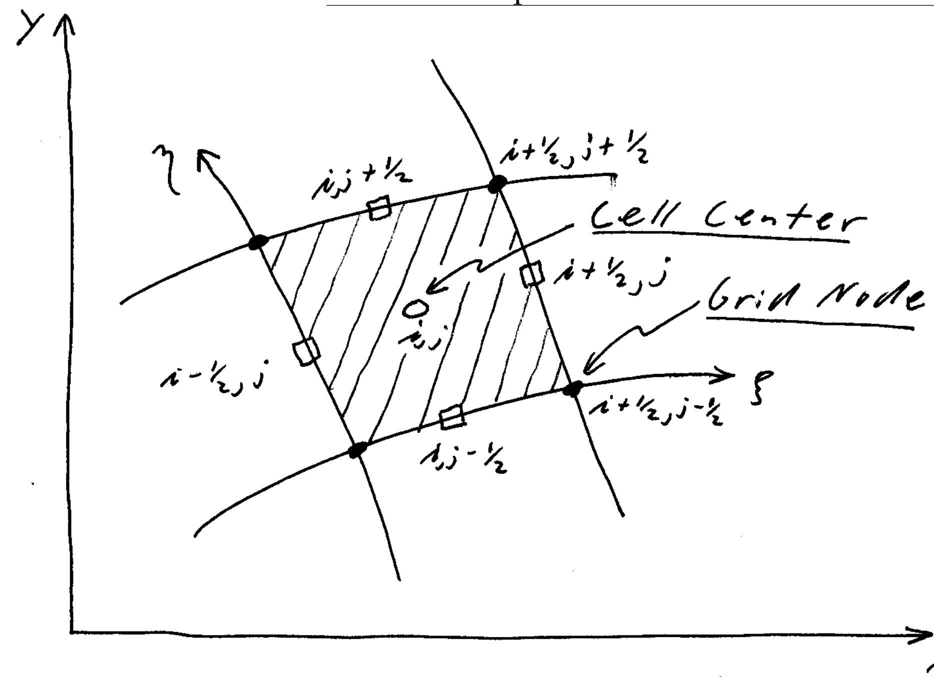
a) Is this equation in conservation law form? Explain your answer.

b) What is the classification of this PDE? (i.e. elliptic, parabolic or hyperbolic?) Please show your work.

4) Classify the following system of PDE's:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0$$

5) In class we showed that on a uniform Cartesian grid, finite difference and finite volume discretizations can be equivalent. In Problems 5 and 6 we determine whether or not this is true on a <u>non-uniform, curvilinear grid</u>. A cell-centered formulation is used <u>with the computational stencil shown below:</u>



Consider the following explicit finite volume scheme:

$$U_{i,j}^{n+1} = U_{i,j}^{n} - \frac{\Delta t}{V_{i,j}}\left[\left(\overleftrightarrow{F}\cdot\vec{n}\Delta S\right)_{i+1/2,j} + \left(\overleftrightarrow{F}\cdot\vec{n}\Delta S\right)_{i,j+1/2} + \left(\overleftrightarrow{F}\cdot\vec{n}\Delta S\right)_{i-1/2,j} + \left(\overleftrightarrow{F}\cdot\vec{n}\Delta S\right)_{i,j-1/2}\right]$$

a) Write an expression for $\left(\overleftrightarrow{F}\cdot\vec{n}\Delta S\right)_{i+1/2,j}$ as a function of $F_{i+1/2,j}$, $G_{i+1/2,j}$, and as a function of x and y at the grid nodes. F and G are fluxes in the x and y directions, respectively and you should choose $\Delta\xi=\Delta\eta=1$

b) Write similar expressions for the other 3 faces.

6) Utilizing the computational stencil sketched above, now consider the following explicit finite difference scheme:

$$U_{i,j}^{n+1} = U_{i,j}^{n} - \Delta t\left[(y_\eta F - x_\eta G)_{i+1/2,j} - (y_\eta F - x_\eta G)_{i-1/2,j} + (x_\xi G - y_\xi F)_{i,j+1/2} - (x_\xi G - y_\xi F)_{i,j-1/2}\right]$$

a) If the inverse metrics are computed using the following expressions:

$$x_{\eta_{i+1/2,j}} = (x_{i+1/2,j+3/2} - x_{i+1/2,j-3/2})/3$$
$$y_{\eta_{i+1/2,j}} = (y_{i+1/2,j+3/2} - y_{i+1/2,j-3/2})/3$$
$$x_{\xi_{i,j+1/2}} = (x_{i+3/2,j+1/2} - x_{i-3/2,j+1/2})/3$$
$$y_{\xi_{i,j+1/2}} = (y_{i+3/2,j+1/2} - y_{i-3/2,j+1/2})/3$$
$$Note\ \Delta\xi = \Delta\eta = 1$$

determine whether or not the resulting finite difference scheme is exactly equivalent to the finite volume scheme you developed in parts a and b of Problem 5.

b) If the inverse metrics are computed using the following expressions:

$$x_{\eta_{i+1/2,j}} = x_{i+1/2,j+1/2} - x_{i+1/2,j-1/2}$$
$$y_{\eta_{i+1/2,j}} = y_{i+1/2,j+1/2} - y_{i+1/2,j-1/2}$$
$$x_{\xi_{i,j+1/2}} = x_{i+1/2,j+1/2} - x_{i-1/2,j+1/2}$$
$$y_{\xi_{i,j+1/2}} = y_{i+1/2,j+1/2} - y_{i-1/2,j+1/2}$$
$$Note\ \Delta\xi = \Delta\eta = 1$$

determine whether or not the resulting finite difference scheme is exactly equivalent to the finite volume scheme you developed in parts a and b of Problem 5.

7) If the initial condition is a uniform freestream flow throughout the computational domain (i.e. $\rho = \rho_\infty,\ \rho u = (\rho u)_\infty, \ldots, U = U_\infty,\ F = F_\infty,\ G = G_\infty\ everywhere$) <u>and</u> there are no effects from the boundaries then numerical solvers should maintain uniform freestream flow. A scheme which has this property is said to provide "perfect freestream capture."

Let's consider whether or not the finite difference scheme given in Problem 6 has this property on a non-uniform, curvilinear grid.

a) With the inverse grid metrics discretized as given in part a of Problem 6, <u>show</u> whether or not the scheme provides perfect freestream capture.

b) With the inverse grid metrics discretized as given in part b of Problem 6, <u>show</u> whether or not the scheme provides perfect freestream capture.