Test 2, Part 1 - Problem 4.

Thursday, November 10, 2022 11:36 PM

4. Reduced / Simplified strain-displacement relation:

Main eq:

$$e = DN^e u^e = \frac{1}{2A}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}\begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \end{bmatrix}$$

Area:

$$A = \frac{1}{2}bh = \frac{1}{2}x_2 y_3$$

$$2A = x_2 y_3$$

1st row:

$$\frac{1}{2A}\left[(y_2 - y_3)u_{x_1} + (y_3 - y_1)u_{x_2} + (y_1 - y_2)u_{x_3}\right]$$

$$\frac{1}{2A}\left[(0 - y_3)u_{x_1} + (y_3 - 0)u_{x_2} + (0 - 0)u_{x_3}\right]$$

$$\frac{1}{2A}\left[-y_3 u_{x_1} + y_3 u_{x_2}\right]$$

$$\frac{1}{x_2 y_3}\left[-y_3 u_{x_1} + y_3 u_{x_2}\right]$$

$$-\frac{y_3 u_{x_1}}{x_2 y_3} + \frac{y_3 u_{x_2}}{x_2 y_3} \rightarrow \frac{u_{x_2} - u_{x_1}}{x_2 y_3}$$

2nd Row:

2nd Row:

$$\frac{1}{2A}\left[(x_3-x_2)u_{y1}+(x_1-x_3)u_{y2}+(x_2-x_1)u_{y3}\right]$$

$$\frac{1}{2A}\left[(x_3-x_2)u_{y1}+(0-x_3)u_{y2}+(x_2-0)u_{y3}\right]$$

$$\frac{1}{x_2y_3}\left[x_3u_{y1}-x_2u_{y2}+x_3u_{y2}+x_2u_{y3}\right]$$

$$\frac{x_3u_{y1}}{x_2y_3}-\frac{x_2u_{y2}}{x_2y_3}+\frac{x_3u_{y2}}{x_2y_3}+\frac{x_2u_{y3}}{x_2y_3}$$

$$\frac{x_3(u_{y1}+u_{y2})}{x_2y_3}+\frac{u_{y3}-u_{y1}}{y_3}$$

3rd row:

$$\frac{1}{x_2y_3}\left[(x_3-x_2)u_{x1}+(y_2-y_3)u_{y1}+(x_1-x_3)u_{x2}+(y_3-y_1)u_{y2}+(x_2-x_1)u_{x3}+(y_1-y_2)u_{y3}\right]$$

$$\frac{x_3(u_{x1}+u_{y2})}{x_2y_3}+\frac{u_{x3}-u_{x1}}{y_3}+\frac{u_{y2}-u_{y1}}{x_2}$$