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Missing Scores

1)

To start out, there were no ties, Alan had the top aggregate, and the marks are in alphabetical order. The total for each subject is $5 + 4 + 3 + 2 + 1 = 15$. With that we can calculate the total aggregate for all 5 subjects which will be $15 * 5 = 75$.

We will now assign points for Alan, since we know Alan scored 24 aggregate points, we can calculate that he scored 5 for four of his subjects and a 4 for one of the other which adds up to 24. From the conditions stated, Ellen topped Mathematics and score 3 for Science. This means Alan is not able to have a score of 5 for Mathematics but is able to for Science. Therefore, Alan scored 4 for Mathematics.

	English	History	Mathematics	French	Science	Total
Alan	5	5	4	5	5	24
Barbara						
Charles						
David						
Ellen			5		3	
Total	15	15	15	15	15	75

One of the condition states that the aggregate marks are in alphabetical order, this means $\text{Ellen} < \text{David} < \text{Charles} < \text{Barbara} < \text{Alan}$.

We will now compute the minimum scores that each student might get. As we know Ellen has the lowest aggregate and that 1 mark are still available, we can assign three of those to Ellen. This will make Ellen's minimum score of 11.

Since David scored higher aggregate than Ellen, David's minimum score will be 12. Charles scored a higher aggregate than David, Charles's minimum score will be 13. Lastly Barbara scored a higher aggregate than Charles, so her minimum score will have to be 14.

Computing the current aggregate that we have $75 - 24(\text{Alan}) - 14(\text{Barbara}) - 13(\text{Charles}) - 12(\text{David}) - 11(\text{Ellen})$ which gives us a remainder of 1. If this 1 aggregate was given to either Charles, David, or Ellen, there will be ties and therefore will not satisfy one of the given conditions while if we give it to Alan, Alan will have scored 25 aggregate which also does not satisfy the other condition. Therefore, Barbara will now have a minimum score of 15.

	English	History	Mathematics	French	Science	Total
Alan	5	5	4	5	5	24
Barbara						15
Charles						13
David						12
Ellen	1	1	5	1	3	11

Total	15	15	15	15	15	75
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We will now compute the marks each student receives for each subject. There are four 4 marks, four 3 marks, five 2 marks, and two 1 marks left. We know that Charles scored the same marks for four of his subjects. Let's say that Charles scored 4 for English, History, French, and Science, and scored 1 for Mathematics, that will leave Charles with an aggregate of 17. This is not allowed as Charles will have a higher aggregate than Barbara. If Charles scored 2 for the above subjects and 1 for Mathematics, he would have an aggregate of 9 which is lesser than Ellen, which is also not right. Since there are not enough 1's for Charles to have 4 of the same marks, we can rule out that Charles will receive more than one 1 mark. Therefore, all that is left for Charles will be four marks of 3. Since $3 * 4 = 12$, the other subject will be of 1 mark. Charles is not able to score a mark of 3 for Science as Ellen has already scored 3 for Science, therefore making Charles scoring 1 for Science.

	English	History	Mathematics	French	Science	Total
Alan	5	5	4	5	5	24
Barbara						15
Charles	3	3	3	3	1	13
David						12
Ellen	1	1	5	1	3	11
Total	15	15	15	15	15	75

David is not able to score higher than Charles, therefore David will score mostly 2's in his subjects. If David scored all 2's, he would have 10 aggregate marks which is lesser than Ellen, which means he will score a 4 mark for one of his subjects. If David scored 4 for more than one subject, he would have a higher aggregate than Charles. Since Alan has already scored 4 for Mathematics, David is not allowed to score 4 for Mathematics.

	English	History	Mathematics	French	Science	Total
Alan	5	5	4	5	5	24
Barbara						15
Charles	3	3	3	3	1	13
David	2	2	2	2	4	12
Ellen	1	1	5	1	3	11
Total	15	15	15	15	15	75

With the updated table above, we can see that Barbara will have scored 4 for English, History, and French while scoring 1 for Mathematics, and 2 for Science. This gives Barbara $4 + 4 + 4 + 2 + 1 = 15$ total aggregate which is higher than Charles, and lesser than Alan.

	English	History	Mathematics	French	Science	Total
Alan	5	5	4	5	5	24
Barbara	4	4	1	4	2	15
Charles	3	3	3	3	1	13
David	2	2	2	2	4	12
Ellen	1	1	5	1	3	11
Total	15	15	15	15	15	75

Therefore, Barbara scored 1 for Mathematics and 3 students scored the same marks in at least 4 subjects; they are Alan, Charles, and David.

2)

Team	Played	Won	Lost	Drawn	Goals for	Goals against	Points
A						1	4
B	1						
C					5	0	6
D						4	
E	4			2		2	2

From the information given in the table above, we can tell that team E has played against all the other teams and only received 2 points, which indicates that they have lost 2 matches, and drew the other 2, leading to 0 wins. Since Team E has 2 goals against, Team E lost both matches with a 0-1 game where the other team scored 1 goal against team E.

Team	Played	Won	Lost	Drawn	Goals for	Goals against	Points
A						1	4
B	1						
C					5	0	6
D						4	
E	4	0	2	2	0	2	2

Looking at Team C, they have already received 6 points. This means that they can either have played 3 games and won all their games to receive 6 points, or they have played 4 games that would lead to winning 2 games and drawing the other 2. We know that Team C is not able to have played 4 games as Team B has only played 1 game overall which is a game with Team E since team E has played all games. Therefore, Team C has played 3 games and won all 3 of their games.

Team	Played	Won	Lost	Drawn	Goals for	Goals against	Points
A						1	4
B	1						
C	3	3	0	0	5	0	6
D						4	
E	4	0	2	2	0	2	2

Looking at Team A, they have been awarded with 4 points meaning they could have played 2 games, won both games or 3 games, won 2 and lost 1. It is not possible for Team A to have played 4 games as Team B has only played with Team E, which means Team A's maximum number of games is 3. Since Team C has played 3 games and won all, this means that Team A has played Team C and lost, contributing to their 1 "goals against". This also means that Team A has played 3 games, winning 2 of them and losing 1 to Team C. Since we know that there are 7 goals that have been scored so far and Team A won 2 games, Team A must score a goal each of the other 2 games to have won.

Team	Played	Won	Lost	Drawn	Goals for	Goals against	Points
A	3	2	1	0	2	1	4
B	1						
C	3	3	0	0	5	0	6
D						4	
E	4	0	2	2	0	2	2

Now we will fill in for Team B. As we know Team B has only played 1 game, which is with Team E, it is not possible for Team B to have won against Team E. This is because Team E has only lost 2 games, which is against Team A and Team C. Therefore, Team B drew against Team E with 0 goals each.

Team	Played	Won	Lost	Drawn	Goals for	Goals against	Points
A	3	2	1	0	2	1	4
B	1	0	0	1	0	0	1
C	3	3	0	0	5	0	6
D						4	
E	4	0	2	2	0	2	2

With the updated table above, we can see that Team A and Team C must have played against Team D to satisfy their overall of 3 games each. Since we know that Team E has lost to Team A, and Team C and drew with Team B leaving with 1 more draw left, this makes Team D the last team to draw against team E with a 0-0 game. Team A has scored 2 goals, 1 for each of their winning games, which means Team A scored 1 goal against Team E, and 1 goal against Team D since Team A lost to Team C. This leaves Team D's "goals against" $4 - 1 = 3$. This leaves Team C to have scored 3 goals against Team D. Just to be sure, since Team A scored a goal against team E, Team C can only score 1 goal also. Team C also scored a goal against Team A, leaving Team C with 3 "goals for" remaining which satisfies the 3 "goals against" that team D has remain.

Therefore, Team C has played against Team D and scored 3 goals against them.