The Semantic Web Logic Exercises 2

This exercises are about quantifiers: we start with some practice about translating simple English sentences into predicate calculus, and then go on to more elaborate ones. The point of these exercises is to give you practice in translating between ordinary language and logical symbolism, a key part of ontology building. Some of the English sentences that I give you may be ambiguous, or difficult to interpret: if so, you should try to say what the ambiguity is, and which meaning you have chosen.

1. Translate into predicate calculus:

   (a) Some dogs are hairy

   (b) All poodles are dogs

   (c) Some hairy things are dogs

   (d) Not every hairy thing is a dog

   (e) There is no dog which does not bark

   (f) Every student has read a Java textbook.

   (g) John lost his laptop in the library or the cafeteria.

   (h) A student who has a computer will work at home.

2. Translate the following formulas into English. For example the formula $\exists x.(car(x) \lor owns(john, x))$ translates into John owns a car or there exists a car which John owns.

   (a) $\exists x(bankaccount(x) \land openedat(john, x, barclays))$

   (b) $\exists x(lecturer(x) \land \forall y.(student(y) \rightarrow knows(y, x)))$

   (c) $\forall x \exists y(advisor(x) \land student(y) \land advises(x, y))$

   (d) $\forall x(student(x) \land (clever(x) \lor worksHard(x))) \rightarrow passExam(x)$

   (e) $\exists x(student(x) \land clever(x) \land \neg passExam(x))$

3. Say that an argument is sound or *logically valid* if there is no possible model (world) in which the premises are true and the conclusion false. Are the following arguments logically valid, or not? Why?

   (a)   All As are B
       All Bs are C
       ‾‾‾‾‾‾‾‾‾‾‾
       All As are C

   (b)   Some As are B
       All Bs are C
       ‾‾‾‾‾‾‾‾‾‾‾
       All As are C

   (c)   Some As are B
       All Bs are C
       ‾‾‾‾‾‾‾‾‾‾‾
       Some As are C

   (d)   All As are B
       All Cs are B
       ‾‾‾‾‾‾‾‾‾‾‾
       All As are C

4. Say that something is *best* if it is better than anything else. Suppose that you have a predicate `better`(x, y) which says that x is better than y.

   (a) Translate 'a is best' into predicate calculus

(b) What follows from 'a is best' and 'b is best'?

Say that y is x's favourite if x prefers y to anything else. Suppose you have a predicate prefers(x, y, z) which says that x prefers y to z.

(c) Translate into predicate calculus 'y is x's favourite'.

(d) Translate into predicate calculus 'Everyone is someone's favourite'. Is this sentence ambiguous?

5. Translate into predicate calculus 'Every farmer who owns a donkey beats it'. (Not that this is not the same sentence as 'Every farmer who owns a donkey beats a donkey'.)

6. (In discussion with your group)
We say that a sentence is *contradictory* if there is no logically possible model in which it is true. Say whether each of these sentences is contradictory, and, if they are not, describe a model in which it is true. You can also argue with a partner as to whether these are contradictory or not, with one person taking the stance that there is a model in which it can be true and the other arguing there is not. Note for some cases you may have to make assertions about your model that are not physically possible!

(a) There is a square house all of whose walls face south

(b) Epimenides, the Cretan, spoke the truth when he said that everything that Cretans say is false

(c) Mt. Everest is lower than Mt. Cook

(d) Mt. Everest is lower than Mt. Cook, Mt. Cook is lower than Mt. Whistler, and Mt. Whistler is lower than Mt. Everest

(e) There is a mountain which is higher than every mountain.

(f) $2 + 2 \neq 4$

(g) Tim Berners-Lee believes that $2 + 2 \neq 4$

(h) Tim Berners-Lee knows that $2 + 2 \neq 4$

(i) It is raining but Tim Berners-Lee doesn't believe it

(j) It is raining but I don't believe it

7. (In discussion with your group)
We say that a sentence is *tautologous* if it is true in every logically possible model, and we say that a sentence is *contingent* if it is neither contradictory nor tautologous, but possible in at least one model. For each of these sentences, say whether it is tautologous, contradictory, or contingent, and translate it into predicate calculus.

(a) Somebody lives in London

(b) Nobody lives in London

(c) At least two people live in London

(d) At least 10,000 people live in London

(e) Some concert pianists are French

(f) Every black dog is a dog

(g) Every fake Picasso is a Picasso

(h) Every suspected criminal is suspected

(i) Every suspected criminal is a criminal

(j) All ravens are black

(k) If two people are brothers then they are siblings

Arguments like those in Question 2 are called *syllogisms*, and their logical properties have been known for over 2000 years: see Aristotle, *First Analytics* I, ch. 4ff.

Questions (with a few alterations and additions) from Benson Mates, *Elementary Logic* (Oxford 1972) and Raymond Bradley, Norman Schwarz *Possible Worlds* (Blackwell 1979).
FURTHER PRACTICE: Russell, S. and Norvig, P. (2009). *Artificial Intelligence- A Modern Approach (Third Edition)*. Prentice Hall. Chapters 7 and 8.