Report: Simulation of a Vibrating System

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1 Equations satisfied by the system using Newton's second law of motion.

The equations are the result of the object and its mass experiencing forces from two horizontal sides. Spring values affect the distance travelled from both sides.

1.1

$m_1\ddot{x}_1 = -k_1x_1 + k_2(x_2 - x_1)$

1.2

$m_2\ddot{x}_2 = -k_2(x_2 - x_1) + k_3(x_3 - x_2)$

1.3

$m_3\ddot{x}_3 = -k_3(x_3 - x_2) - k_4x_3$

2 System of equations expressed as an eigen problem.

k and m are matrices representing spring values and mass respectively. Thus,

$\left[\frac{k}{m}\right]\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}\ddot{x}_1\\\ddot{x}_2\\\ddot{x}_3\end{bmatrix}$.

Without damping, we choose a purely oscillatory solution.

$x = ve^{i\omega t} \longrightarrow \ddot{x} = -\omega^2 ve^{i\omega t} = -\omega^2 x \longrightarrow Ax = -\omega^2 x$ where A is the spring and mass matrix. This becomes an eigen problem where $\lambda = -\omega^2$ so $Av = \lambda v$.

$A = \begin{bmatrix}\frac{-k_1-k_2}{m_1} & \frac{k_2}{m_1} & 0\\ \frac{k_2}{m_2} & \frac{-k_2-k_3}{m_2} & \frac{k_3}{m_2}\\ 0 & \frac{k_3}{m_3} & \frac{-k_3-k_4}{m_3}\end{bmatrix}$ where v is an eigenvector and λ is a corresponding scalar eigenvalue.

3 Eigenvector matrix and eigenvalue matrix.

$$v = \begin{bmatrix} 0.1163 & 0.8321 & 0.3803 \\ -0.8495 & -0.2305 & 0.6768 \\ 0.5146 & -0.5045 & 0.6303 \end{bmatrix} \lambda = \begin{bmatrix} -25.7567 & 0 & 0 \\ 0 & -8.1925 & 0 \\ 0 & 0 & -3.0508 \end{bmatrix}$$

4 Frequencies and amplitudes.

Frequency is the value $\omega = \sqrt{-\lambda}$. It is the number of occurrences in one period. Amplitude is an eigenvector, X of the form $AX = \lambda X$ and is the measure of change over a period. $\omega = \begin{bmatrix} 4.7121 \\ 2.7994 \\ 1.8145 \end{bmatrix}$