Report: Simulation of a Vibrating System

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1 Equations satisfied by the system using Newton's second law of motion.

1.1

$$m_1\ddot{x}_1 = -k_1x_1 + k_2(x_2 - x_1)$$

1.2

$$m_2\ddot{x}_2 = -k_2(x_2 - x_1) + k_3(x_3 - x_2)$$

1.3

$$m_3\ddot{x}_3 = -k_3(x_3 - x_2) - k_4x_3$$

2 System of equations expressed as an eigen problem.

$$Av = \lambda v$$

$$A = \begin{bmatrix} \frac{k_1+k_2}{m_1} & \frac{-k_1}{m_1} & 0\\ \frac{-k_2}{m_2} & \frac{k_2+k_3}{m_2} & \frac{-k_2}{m_2}\\ 0 & \frac{-k_3}{m_3} & \frac{k_3}{m_3} \end{bmatrix} \text{ where } v \text{ is an eigenvector and } \lambda \text{ is a corresponding scalar eigenvalue}.$$

3 Eigenvector matrix and eigenvalue matrix.

$$v = \begin{bmatrix} -0.2798 & 0.6620 & 0.5239 \\ 0.8229 & 0.0446 & 0.4409 \\ -0.4944 & 0.7482 & 0.7288 \end{bmatrix} \lambda = \begin{bmatrix} 22.2041 & 0 & 0 \\ 0 & 7.8367 & 0 \\ 0 & 0 & 3.2925 \end{bmatrix}$$

4 Frequencies and amplitudes.

$$\omega = \begin{bmatrix} 4.7121 & 2.7994 & 1.8145 \end{bmatrix}$$