Assignment - 4 : Q5

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Assignment-4
Question-5

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5) We want to maximise $f^t C f$

given constraints - $f^t f = 1$

$e^t e = 1$

$f^t e =$

where e is the eigen vector i.e. $Ce = \lambda_1 e$

The equivalent condition using lagrange multiplier is maximizing

$$J(f) = f^t C f - \lambda(f^t f - 1) - u(f^t e)$$

$$\frac{\partial J(f)}{\partial f} = 2f^t C - 2\lambda f^t - u e^t = 0$$

$$\Rightarrow \quad 2f^t C = 2\lambda f^t + u e^t \qquad -(2)$$

Post Multiply with e

$$\rightarrow \quad 2f^t Ce = 2\lambda f^t e + u e^t e$$

$$Ce = \lambda_1 e \qquad \text{by } e^n (1)$$

$$\Rightarrow \quad 2f^t \lambda_1 e = 2\lambda \cdot 0 + u \cdot 1$$

$$\Rightarrow \quad 2\lambda_1 f^t e = 2\lambda \cdot 0 + u \cdot 1$$

$$\rightarrow \quad 2\lambda_1 \times 0 = 0 + u$$

$$\Rightarrow \quad \boxed{0 = u} \qquad -(3)$$

hence, using e^n (2) and (3)

$2f^t C = 2\lambda f^t$

$\Rightarrow$ taking transpose

$C^t f = \lambda f$

$\Rightarrow$ $\boxed{Cf = \lambda f}$

hence f is an eigen vector

hence $f^t C f = f^t \lambda f = \underline{\lambda}$

$f^t C f$ will be maximised when λ is maximised

→ As given already that e is already an eigen vector and ~~that~~, $f \perp e$, max value λ can take is the second highest eigen value

$\Rightarrow$ f is eigen vector with second highest eigen value.