Q 6)

(i) (b) $E_L(V) \geq E_N(V)$

since $E_N(v) = \sum \min_{c_i} ||x_i - Vc_i||^2$ with atmost k nonzero

$\leq \sum ||x_i - V_k \alpha_i^k||^2$

~~since for each i, if $\alpha_i^{(k)}$ is giving~~

~~les~~ Since for any i,

if $||x_i - V_k \alpha_i^k||^2 < ||x_i - Vc_i||^2$

then $c_i \neq \arg\min ||x - Vc_i||^2$

which is a contradiction

Hence there summation ~~of~~ over all i is also less than or equal to.

(ii) ~~Algorithm.~~

Let the exact x_i ~~be~~ be

~~$V = \sum_{i=1}^{d} \alpha_i^1 V_i$~~

$x_i =$ ~~$V\alpha_i$~~ ~~V_{β_i}~~ $V\beta_i$ (since V is an orthonormal basis)

~~where~~ $\beta_{ij} =$ ~~$x_i V_{ik}$~~ $V_j^T x_i$

$\therefore \alpha_i = \arg\min_{c_i} ||x_i - Vc_i||^2$

$= \arg\min_{c_i} ||V\beta_i - Vc_i||^2$

$= \arg\min_{c_i} ||V(\beta_i - c_i)||^2$

$=$ ~~$\sum \arg\min_{c_i} \sum V_k (\beta_{ik} - c_{ik})^2$~~

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Since V is orthonormal basis.

$$\| V (B_i - C_i) \|^2 = \sum_{j=1}^{d} (B_{ij} - C_{ij})^2$$

$$\therefore \; \alpha_i = \operatorname{argmin}_{C_i} \sum (B_{ij} - C_{ij})^2$$

subject ~~for~~ to constraint atmost k non zero C_{ij}'s.

clearly this will be minimum when.

$C_{ij} = B_{ij}$ for ~~first~~ highest k $|B_{ij}|$

and $C_{ij} = 0$ for rest.

~~Algo~~

Hence the algorithm is as follows,

For ~~each~~ $j = 1$ to d

$$\underset{(1 \times d)}{B_{ij}} = V_j^T \underset{(d \times 1)}{x_i}$$

~~Sort (B_{ij})~~

~~For first k~~

for $l = 1$ to k

find ~~argmax$(|B_{ij}|)$~~ $r = \operatorname{argmax}(|B_{ij}|)$

$C_{ij} = B_{ij}$

$B_{ij} = 0$

end.

$\therefore$ Complexity of algorithm $= \underbrace{d}_{\text{for loop}} \times \underbrace{d}_{\text{dot product of } d\text{-dimensional vector}} + k$

Yes

(c) For $k=1$,

we showed in class, that to minimize the error,

$\vec{e}$ = eigen vector with maximum eigen value

$\therefore$ $E_N(w) < E_N(v)$ is not possible for $k=1$

For $k=2$, we showed ~~that~~ in question 5

~~$x_i - \alpha_{i_1} e - \alpha$~~

$\| x_i - \alpha_{i_1} v_1 - \alpha_{i_2} v_2 \|$ is minimum

when both v_1, v_2 are eigen vectors.

$\therefore$ This is also with eigen vectors as basis

Similarly, we can show for any finite k.

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