4dt905_sb224sc_A4

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1 Assignment 4

1.1 Classification

Author: Samuel Fredric Berg

Student ID: sb224sc

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1.1.1 Conceptual

1. Linear Discriminant Analysis (LDA) and Quadratic Discriminant Analysis (QDA) are both generative models that assume the data follows a Gaussian (Normal) distribution. However, they differ significantly in their flexibility.

- Class Covariance:
  - **LDA** assumes that all classes share the same covariance matrix. This means that the decision boundaries between classes are linear. LDA is suitable when the classes are well-separated and the assumption of equal covariance holds true.
  - **QDA**, on the other hand, assumes each class to have its own covariance matrix. This results in quadratic decision boundaries, making QDA more flexible and capable of modeling more complex relationships between features. QDA is preferable when the classes have different variances or when the data is not linearly separable.
- Decision Boundaries:
  - **LDA** produces linear decision boundaries, which can be limiting if the true boundary between classes is non-linear.
  - **QDA** produces quadratic decision boundaries, allowing it to capture more complex patterns in the data.
- Sample Size:
  - **LDA** generally requires fewer parameters to estimate (due to the shared covariance matrix), making it more stable with smaller datasets.
  - **QDA** requires estimating a separate covariance matrix for each class, which can lead to overfitting if the dataset is small or if there are many features.
- Overfitting:
  - **LDA** is less prone to overfitting due to its simpler model structure.
  - **QDA** can overfit the training data, especially when the number of features is large relative to the number of samples.

2. K-Nearest Neighbors (KNN) is a non-parametric, instance-based learning algorithm used for classification and regression tasks.

   (a) **Role of Distance Metrics**: Preformance of KNN is enrirely dependent on how we define “closeness”. The choice of metric determines which neighbors are selected, which directly impacts the classification outcome. Common distance metrics include Euclidean, Manhattan and Minkowski distances.

   - Euclidean: Measures the straight-line distance between two points in a multi-dimensional space. It works best when features are continuous and equally scaled.
   - Manhattan: Measures the distance between two points by summing the absolute differences of their coordinates. It is useful when dealing with high-dimensional data or when features are discrete.
   - Minkowski: A generalization of both Euclidean and Manhattan distances, controlled by a parameter p. When $p = 2$, it is equivalent to Euclidean distance; when $p = 1$, it is equivalent to Manhattan distance.

   (b) **Curse of Dimensionality**: As the number of dimensions (features) increases, the volume of the feature space grows exponentially, leading to two major issues for KNN:

   - Sparsity: In the high-dimensional space, data points are very spread out. Even the nearest neighbors may be far away, making it difficult to find meaningful relationships.
   - Distance Convergence: As dimensions increase, the distances between points tend to converge, meaning that the difference between the nearest and farthest neighbors becomes negligible. This makes it challenging for KNN to distinguish between classes effectively and rendering the concept of “closeness” meaningless.

1.1.2 Practical

Imports

[53]:
```
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
import statsmodels.api as sm
import numpy as np
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis as LDA
from sklearn.discriminant_analysis import QuadraticDiscriminantAnalysis as QDA
from sklearn.metrics import accuracy_score
from sklearn.neighbors import KNeighborsClassifier as KNN
```

Load Data

[54]:
```
df = pd.read_csv("../data/Smarket.csv", index_col=0)
```

Number of features and names

[55]:
```
df_names = df.columns.tolist()
print(f"Number of columns: {len(df_names)}")
print(f"Column names: {df_names}")
```

```
Number of columns: 9
Column names: ['Year', 'Lag1', 'Lag2', 'Lag3', 'Lag4', 'Lag5', 'Volume',
```

```
'Today', 'Direction']
```

Statistical summary of features

[56]:
```python
print(df.describe())
print(df["Direction"].value_counts())
```

```
               Year        Lag1        Lag2        Lag3        Lag4  \
count  1250.000000  1250.000000  1250.000000  1250.000000  1250.000000
mean   2003.016000     0.003834     0.003919     0.001716     0.001636
std       1.409018     1.136299     1.136280     1.138703     1.138774
min    2001.000000    -4.922000    -4.922000    -4.922000    -4.922000
25%    2002.000000    -0.639500    -0.639500    -0.640000    -0.640000
50%    2003.000000     0.039000     0.039000     0.038500     0.038500
75%    2004.000000     0.596750     0.596750     0.596750     0.596750
max    2005.000000     5.733000     5.733000     5.733000     5.733000

             Lag5       Volume        Today
count  1250.00000  1250.000000  1250.000000
mean      0.00561     1.478305     0.003138
std       1.14755     0.360357     1.136334
min      -4.92200     0.356070    -4.922000
25%      -0.64000     1.257400    -0.639500
50%       0.03850     1.422950     0.038500
75%       0.59700     1.641675     0.596750
max       5.73300     3.152470     5.733000
Direction
Up      648
Down    602
Name: count, dtype: int64
```

Number of datapoints

[57]:
```python
print(f"Number of datapoints: {len(df)}")
```

```
Number of datapoints: 1250
```

Display data in table format

[58]:
```python
print(df.head(5))
```

```
   Year   Lag1   Lag2   Lag3   Lag4   Lag5  Volume  Today Direction
1  2001  0.381 -0.192 -2.624 -1.055  5.010  1.1913  0.959        Up
2  2001  0.959  0.381 -0.192 -2.624 -1.055  1.2965  1.032        Up
3  2001  1.032  0.959  0.381 -0.192 -2.624  1.4112 -0.623      Down
4  2001 -0.623  1.032  0.959  0.381 -0.192  1.2760  0.614        Up
5  2001  0.614 -0.623  1.032  0.959  0.381  1.2057  0.213        Up
```

Correlation matrix

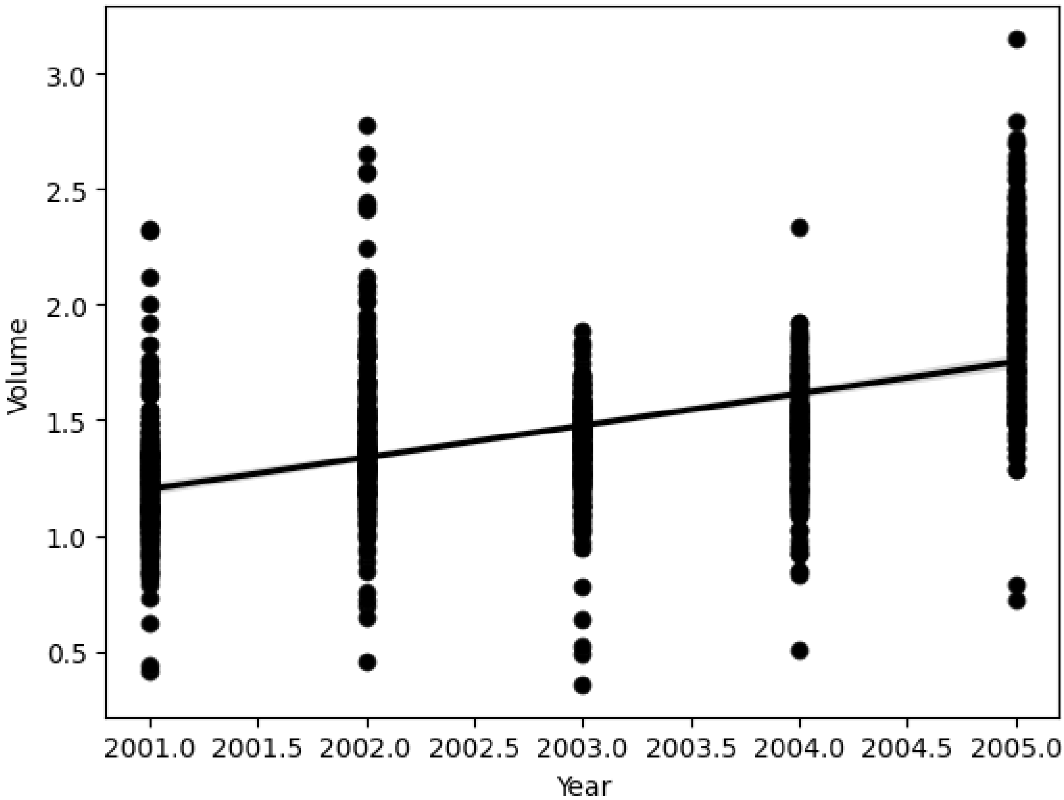
[59]:
```python
sns.heatmap(
    df.drop(columns=["Direction"]).corr(), annot=True, fmt=".2f", linewidths=0.5
```

```
)
plt.show()
```

Interpretation from matrix The matrix indicates that only `Volume` and `Year` have some correlation, meanwhile the other features seem to be minimally correlated. The value of the correlation between `Volume` and `Year` is 0.54, indicating that increasing the year, the volume tends to increase as well.

[60]:
```
sns.scatterplot(data=df, x="Year", y="Volume", color="black")
sns.regplot(data=df, x="Year", y="Volume", color="black")
```

[60]: `<Axes: xlabel='Year', ylabel='Volume'>`



Graph interpretation It reinforces the interpretation drawn from the correlation matrix, showing a positive trend between `Volume` and `Year`.

Logistics Regression

[61]:
```
X = df[["Lag1", "Lag2", "Lag3", "Lag4", "Lag5", "Volume"]]
X = sm.add_constant(X)
Y = df["Direction"].map({"Up": 1, "Down": 0})
model = sm.Logit(Y, X).fit()

print(model.summary())
```

```
Optimization terminated successfully.
         Current function value: 0.691034
         Iterations 4
                           Logit Regression Results
==============================================================================
Dep. Variable:              Direction   No. Observations:                 1250
Model:                          Logit   Df Residuals:                     1243
Method:                           MLE   Df Model:                            6
Date:                Mon, 26 Jan 2026   Pseudo R-squ.:                0.002074
Time:                        09:52:19   Log-Likelihood:                -863.79
```

```
converged:                           True   LL-Null:                       -865.59
Covariance Type:                nonrobust   LLR p-value:                    0.7319
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.1260      0.241     -0.523      0.601      -0.598       0.346
Lag1          -0.0731      0.050     -1.457      0.145      -0.171       0.025
Lag2          -0.0423      0.050     -0.845      0.398      -0.140       0.056
Lag3           0.0111      0.050      0.222      0.824      -0.087       0.109
Lag4           0.0094      0.050      0.187      0.851      -0.089       0.107
Lag5           0.0103      0.050      0.208      0.835      -0.087       0.107
Volume         0.1354      0.158      0.855      0.392      -0.175       0.446
==============================================================================
```

Interpretation of regression The p-values for all coefficients are > 0.05, indicating that none of the features are statistically significant enough for predicting the direction.

Use model

[62]:
```python
probs = model.predict()

for indecies in range(10):
    print(f"({indecies + 1})\t {probs[indecies]}")
```

```
(1)      0.5070841334630001
(2)      0.48146787817516973
(3)      0.4811388348131379
(4)      0.5152223557927659
(5)      0.5107811625911515
(6)      0.5069564604552573
(7)      0.49265087386752915
(8)      0.5092291581449722
(9)      0.5176135261687921
(10)     0.4888377794207711
```

Interpretation This shows probability of the market value going up rather than down, as defiend by `df["Direction"].map({"Up": 1, "Down": 0})`. A value > 0.5 indicates that the model predicts an increase in market value, while a value < 0.5 indicates a decrease.

Confusion Matrix

[63]:
```python
pred = []

for prob in probs:
    if prob > 0.5:
        pred.append("Up")
    else:
        pred.append("Down")
```

```
print(pd.crosstab(pred, df["Direction"], rownames=["Predicted"],␣
 ↪colnames=["Actual"]))
accuracy = np.mean(pred == df["Direction"])
print(f"Model accuracy: {accuracy}")
```

```
Actual     Down   Up
Predicted
Down        145  141
Up          457  507
Model accuracy: 0.5216
```

Interpretation From the ten previously printed values of `probs`, we assume that the the the prdiction is not confident in its classification, due the it hovering around 0.5. This is reinforced by the model accuracy of 52.16%, which is only slightly better then a coin flip (50%).

[64]:
```
X = df[["Lag1", "Lag2"]]
X = sm.add_constant(X)
model = sm.Logit(Y, X).fit()
probs = model.predict(X)

pred = []
for prob in probs:
    if prob > 0.5:
        pred.append("Up")
    else:
        pred.append("Down")

print(pd.crosstab(pred, df["Direction"], rownames=["Predicted"],␣
 ↪colnames=["Actual"]))
accuracy = np.mean(pred == df["Direction"])
print(f"Model accuracy: {accuracy}")
```

```
Optimization terminated successfully.
         Current function value: 0.691361
         Iterations 4
Actual     Down   Up
Predicted
Down        114  102
Up          488  546
Model accuracy: 0.528
```

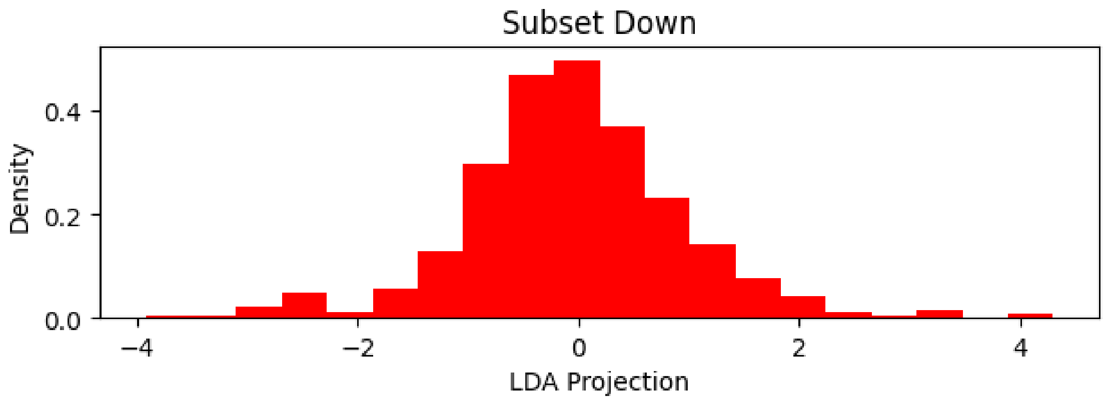
Interpretation This models accuracy is only slightly better by using only two of the predictors instead of all of them. However it is unclear if this is statistically significant without further testing.

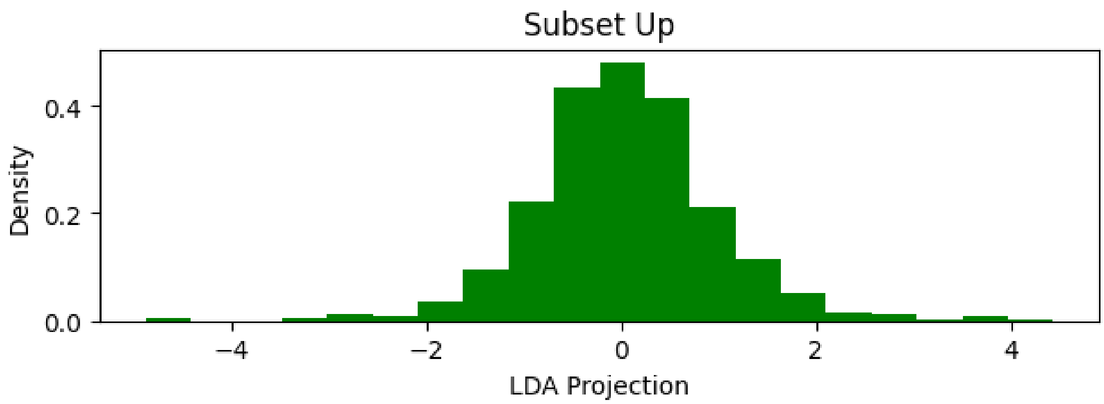
LDA

[65]:
```
X = df[["Lag1", "Lag2"]]
model = LDA()
model.fit(X, Y)
```

```python
print(f"Prior probs. of groups: {model.priors_}")
print(f"Group means: \n {model.means_}")
print(f"Coeffs. of linear discriminants: {model.coef_}")

transform = model.transform(X)
LDA_DF = pd.DataFrame({"LDA1": transform[:, 0], "Direction": Y})

figure, axes = plt.subplots(nrows=2, ncols=1)

subset_down = LDA_DF[LDA_DF["Direction"] == 0]
axes[0].hist(subset_down["LDA1"], bins=20, density=True, color="red")
axes[0].set_xlabel("LDA Projection")
axes[0].set_ylabel("Density")
axes[0].set_title("Subset Down")

subset_up = LDA_DF[LDA_DF["Direction"] == 1]
axes[1].hist(subset_up["LDA1"], bins=20, density=True, color="green")
axes[1].set_xlabel("LDA Projection")
axes[1].set_ylabel("Density")
axes[1].set_title("Subset Up")

plt.tight_layout()
plt.show()
```

```
Prior probs. of groups: [0.4816 0.5184]
Group means:
 [[ 0.05068605  0.03229734]
 [-0.03969136 -0.02244444]]
Coeffs. of linear discriminants: [[-0.07126095 -0.04433204]]
```





Interpretation

- Prior probabilities: Informs us that 51.84% of the data belongs to the "Up" class, while 48.16% belongs to the "Down" class.
- Group means: For `Lag1` the mean value for the "Down" direction ≈ 0.05 and ≈ 0.03 for the "Up" direction, while `Lag2` has a mean value of ≈ -0.04 for "Down" and ≈ 0.02 for "Up". This suggests that on average, `Lag1` and `Lag2` are slightly higher when the market goes "Up".
- Coefficients: Indicate that for an increase in `Lag1` and `Lag2`, will result in a downward trend, as both coefficients are negative.
- Graphs: Show some overlap between the two classes for both `Lag1` and `Lag2`, indicating that it might not be a good predictor of direction.

[66]:
```
LDA_class = model.predict(X)
conf_matrix = pd.crosstab(LDA_class, Y, rownames=["Predicted"],␣
↪colnames=["Actual"])
accuracy = accuracy_score(Y, LDA_class)

print(f"Confusion Matrix:\n{conf_matrix}")
print(f"Accuracy: {accuracy}")
```

```
Confusion Matrix:
Actual       0    1
```

```
Predicted
0           114  102
1           488  546
Accuracy: 0.528
```

Interpretation Same results as for the logistics regression above, which indicates that the LDA does not contribute any more than the logistics regression.

QDA

[67]:
```
model = QDA()
model.fit(X, Y)

print(f"Prior probs. of groups: {model.priors_}")
print(f"Group means: \n {model.means_}")
```

```
Prior probs. of groups: [0.4816 0.5184]
Group means:
 [[ 0.05068605  0.03229734]
 [-0.03969136 -0.02244444]]
```

Interpretation Same values as for the LDA due to same dataset and predictors being used.

Use model

[68]:
```
QDA_class = model.predict(X)
conf_matrix = pd.crosstab(QDA_class, Y, rownames=["Predicted"],␣
  ↪colnames=["Actual"])
accuracy = accuracy_score(Y, QDA_class)

print(f"Confusion Matrix:\n{conf_matrix}")
print(f"Accuracy: {accuracy}")
```

```
Confusion Matrix:
Actual       0    1
Predicted
0           109   94
1           493  554
Accuracy: 0.5304
```

Interpretation Improved accuracy of 53.04%, which is expected due to the increased utilizable parameters of QDA over LDA.

KNN Clustering

[69]:
```
train = df["Year"] < 2005

print(df[~train].shape)
```

```
(252, 9)
```

[70]:
```
train_X = df[["Lag1", "Lag2"]][train]
train_direction = df["Direction"][train]

test_X = df[["Lag1", "Lag2"]][~train]
test_direction = df["Direction"][~train]

KNN_model = KNN(n_neighbors=1)
KNN_model.fit(train_X, train_direction)
KNN_pred = KNN_model.predict(test_X)

conf_matrix = pd.crosstab(
    KNN_pred, test_direction, rownames=["Predicted"], colnames=["Actual"]
)
accuracy = accuracy_score(test_direction, KNN_pred)

print(f"Confusion Matrix:\n{conf_matrix}")
print(f"Accuracy: {accuracy}")
```

```
Confusion Matrix:
Actual     Down  Up
Predicted
Down         43  58
Up           68  83
Accuracy: 0.5

Accuracy: 0.5
```

[71]:
```
KNN_model = KNN(n_neighbors=3)
KNN_model.fit(train_X, train_direction)
KNN_pred = KNN_model.predict(test_X)

conf_matrix = pd.crosstab(
    KNN_pred, test_direction, rownames=["Predicted"], colnames=["Actual"]
)
accuracy = accuracy_score(test_direction, KNN_pred)

print(f"Confusion Matrix:\n{conf_matrix}")
print(f"Accuracy: {accuracy}")
```

```
Confusion Matrix:
Actual     Down  Up
Predicted
Down         48  55
Up           63  86
Accuracy: 0.5317460317460317
```

Interpretation The value of k seems to have a decent impact on the accuracy, with $k = 3$ yielding the highest accuracy so far of 53.17%. However, it is still unclear if this is statistically significant without further testing.