4dt905_sb224sc_A3

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1 Assignment 3

1.1 Simple and Multiple Linear Regression pt2

Author: Samuel Fredric Berg

Student ID: sb224sc

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1.2 Conceptual

$$Y = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \hat{\beta}_3 X_3 + \hat{\beta}_4 X_1 X_2 + \hat{\beta}_5 X_1 X_3$$

$$\hat{\beta}_0 = 50(Intercept)$$

$$\hat{\beta}_1 = 20(GPA)$$

$$\hat{\beta}_2 = 0.07(IQ)$$

$$\hat{\beta}_3 = 35(Level)$$

$$\hat{\beta}_4 = 0.01(GPA \cdot IQ)$$

$$\hat{\beta}_5 = -10(GPA \cdot Level)$$

$$X_3 = 1 \text{ for College, } 0 \text{ for High School}$$

1.

$$Y_c = 50 + 20X_1 + 0.07X_2 + 35 + 0.01X_1X_2 - 10X_1$$

$$Y_h = 50 + 20X_1 + 0.07X_2 + 0.01X_1X_2$$

$$Y_c - Y_h = 35 - 10X_1$$

$$35 - 10X_1 = 0 \implies X_1 = 3.5$$

Thue, when GPA > 3.5 High School graduates earn more than College graduates.

Answer: $\boxed{iii}$

2.

$$X_1 = 4.0$$

$$X_2 = 110$$

$$X_3 = 1$$

$$Y = 50 + 20(4.0) + 0.07(110) + 35 + 0.01(4.0)(110) - 10(4.0)$$

$$Y = 137.1$$

Answer: $\boxed{\$137,100}$

3. False. The magnitude of a coefficient does not indicate statistical importance. To determine statistical importance we need to look at the p-values associated with that coefficient, not just its absolute value. In the presented case, the units of predictor X_2 (IQ) are generally > 100. A small coeffeicient for the $X_2 \cdot X_1$ term might still result in a large contribution to the model and be highly statistically significant.

Answer: $\boxed{False}$

1.3 Practical

1.3.1 Imports

[1]:
```
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
import statsmodels.api as sm
```

```python
import numpy as np
```

1.3.2 Assignment 2

1.3.3 Load data

[2]:
```python
df = pd.read_csv("../data/Boston.csv", index_col=0)
```

1.3.4 Number of predictors and names

[3]:
```python
df_names = df.columns.tolist()
print(f"Number of columns: {len(df_names)}")
print(f"Column names: {df_names}")
```

```
Number of columns: 14
Column names: ['crim', 'zn', 'indus', 'chas', 'nox', 'rm', 'age', 'dis', 'rad',
'tax', 'ptratio', 'black', 'lstat', 'medv']
```

1.3.5 Statistical summary of predictors

[4]:
```python
df.describe()
```

[4]:

	crim	zn	indus	chas	nox	rm
count	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000
mean	3.613524	11.363636	11.136779	0.069170	0.554695	6.284634
std	8.601545	23.322453	6.860353	0.253994	0.115878	0.702617
min	0.006320	0.000000	0.460000	0.000000	0.385000	3.561000
25%	0.082045	0.000000	5.190000	0.000000	0.449000	5.885500
50%	0.256510	0.000000	9.690000	0.000000	0.538000	6.208500
75%	3.677083	12.500000	18.100000	0.000000	0.624000	6.623500
max	88.976200	100.000000	27.740000	1.000000	0.871000	8.780000

	age	dis	rad	tax	ptratio	black
count	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000
mean	68.574901	3.795043	9.549407	408.237154	18.455534	356.674032
std	28.148861	2.105710	8.707259	168.537116	2.164946	91.294864
min	2.900000	1.129600	1.000000	187.000000	12.600000	0.320000
25%	45.025000	2.100175	4.000000	279.000000	17.400000	375.377500
50%	77.500000	3.207450	5.000000	330.000000	19.050000	391.440000
75%	94.075000	5.188425	24.000000	666.000000	20.200000	396.225000
max	100.000000	12.126500	24.000000	711.000000	22.000000	396.900000

	lstat	medv
count	506.000000	506.000000
mean	12.653063	22.532806
std	7.141062	9.197104
min	1.730000	5.000000
25%	6.950000	17.025000

```
50%      11.360000   21.200000
75%      16.955000   25.000000
max      37.970000   50.000000
```

1.3.6 Number of datapoints

[5]:
```
print(f"Number of datapoints: {len(df)}")
```

```
Number of datapoints: 506
```

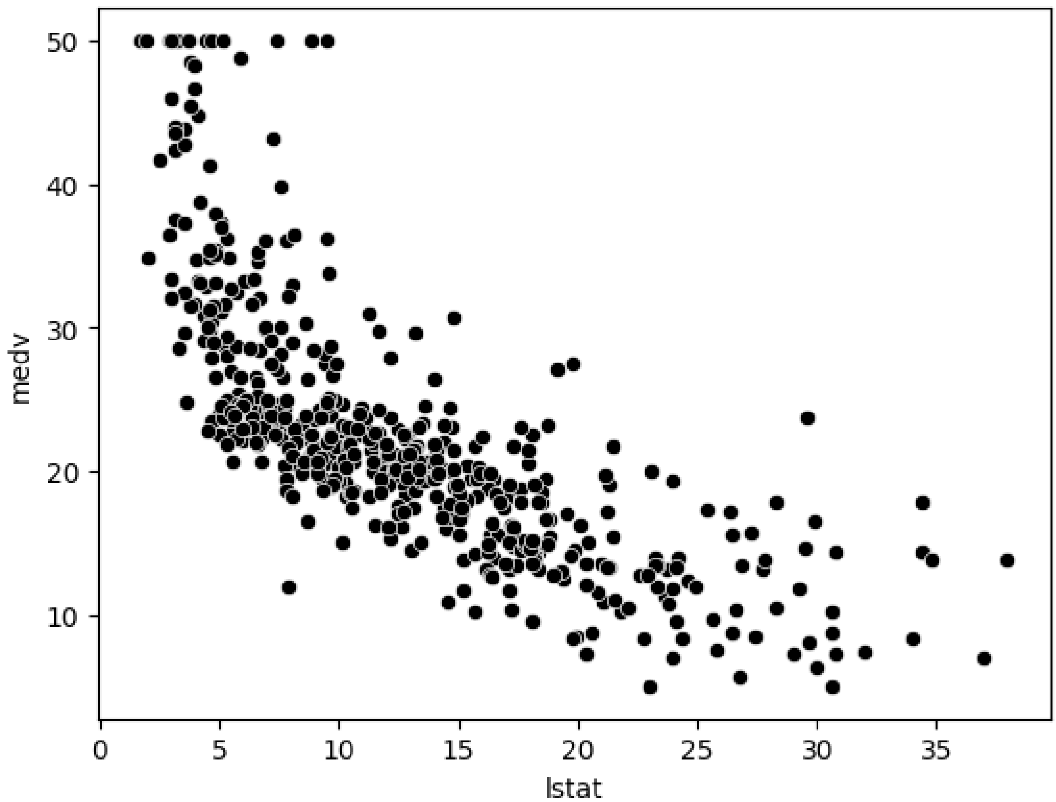
1.3.7 Display data in table format

[6]:
```
print(df.head(5))
```

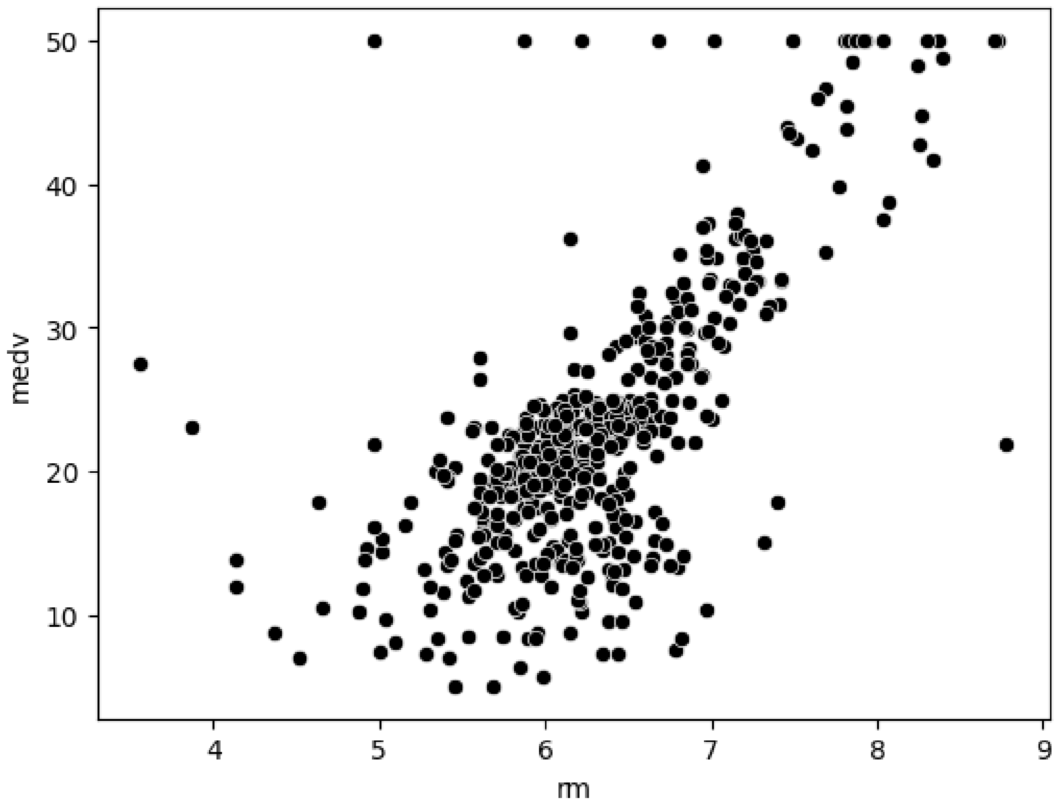
```
      crim    zn  indus  chas    nox     rm   age     dis  rad  tax  ptratio  \
1  0.00632  18.0   2.31     0  0.538  6.575  65.2  4.0900    1  296     15.3
2  0.02731   0.0   7.07     0  0.469  6.421  78.9  4.9671    2  242     17.8
3  0.02729   0.0   7.07     0  0.469  7.185  61.1  4.9671    2  242     17.8
4  0.03237   0.0   2.18     0  0.458  6.998  45.8  6.0622    3  222     18.7
5  0.06905   0.0   2.18     0  0.458  7.147  54.2  6.0622    3  222     18.7

    black  lstat  medv
1  396.90   4.98  24.0
2  396.90   9.14  21.6
3  392.83   4.03  34.7
4  394.63   2.94  33.4
5  396.90   5.33  36.2
```

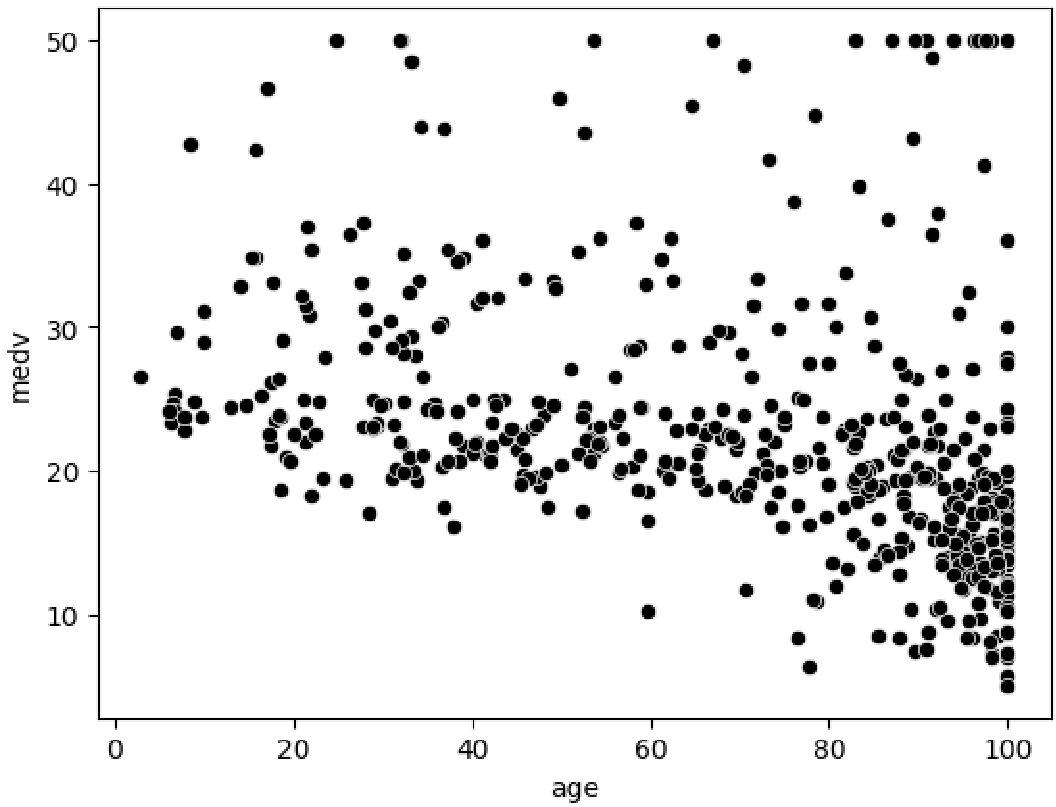
1.3.8 Plot lstat, rm and age against medv

[7]:
```
sns.scatterplot(x="lstat", y="medv", data=df, color="black")
plt.show()

sns.scatterplot(x="rm", y="medv", data=df, color="black")
plt.show()

sns.scatterplot(x="age", y="medv", data=df, color="black")
plt.show()
```



50
40
30
20
10
medv
0
5
10
15
20
25
30
35
lstat


50
40
30
20
10
medv
4
5
6
7
8
9
rm



1.3.9 Linear regression

[8]:
```
regression1 = sm.OLS(df["medv"], sm.add_constant(df["lstat"])).fit()
print(regression1.summary())

regression2 = sm.OLS(df["medv"], sm.add_constant(df["rm"])).fit()
print(regression2.summary())

regression3 = sm.OLS(df["medv"], sm.add_constant(df["age"])).fit()
print(regression3.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.544
Model:                            OLS   Adj. R-squared:                  0.543
Method:                 Least Squares   F-statistic:                     601.6
Date:                Sun, 25 Jan 2026   Prob (F-statistic):           5.08e-88
Time:                        16:43:09   Log-Likelihood:                -1641.5
No. Observations:                 506   AIC:                             3287.
Df Residuals:                     504   BIC:                             3295.
Df Model:                           1
```

```
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         34.5538      0.563     61.415      0.000      33.448      35.659
lstat         -0.9500      0.039    -24.528      0.000      -1.026      -0.874
==============================================================================
Omnibus:                      137.043   Durbin-Watson:                   0.892
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              291.373
Skew:                           1.453   Prob(JB):                     5.36e-64
Kurtosis:                       5.319   Cond. No.                         29.7
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
                            OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.484
Model:                            OLS   Adj. R-squared:                  0.483
Method:                 Least Squares   F-statistic:                     471.8
Date:                Sun, 25 Jan 2026   Prob (F-statistic):           2.49e-74
Time:                        16:43:09   Log-Likelihood:                -1673.1
No. Observations:                 506   AIC:                             3350.
Df Residuals:                     504   BIC:                             3359.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const        -34.6706      2.650    -13.084      0.000     -39.877     -29.465
rm             9.1021      0.419     21.722      0.000       8.279       9.925
==============================================================================
Omnibus:                      102.585   Durbin-Watson:                   0.684
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              612.449
Skew:                           0.726   Prob(JB):                    1.02e-133
Kurtosis:                       8.190   Cond. No.                         58.4
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
                            OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.142
Model:                            OLS   Adj. R-squared:                  0.140
Method:                 Least Squares   F-statistic:                     83.48
Date:                Sun, 25 Jan 2026   Prob (F-statistic):           1.57e-18
```

```
Time:                        16:43:09   Log-Likelihood:                -1801.5
No. Observations:                 506   AIC:                             3607.
Df Residuals:                     504   BIC:                             3615.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         30.9787      0.999     31.006      0.000      29.016      32.942
age           -0.1232      0.013     -9.137      0.000      -0.150      -0.097
==============================================================================
Omnibus:                      170.034   Durbin-Watson:                   0.613
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              456.983
Skew:                           1.671   Prob(JB):                    5.85e-100
Kurtosis:                       6.240   Cond. No.                         195.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

1.3.10 Interpretation of regressions

lstat For `lstat` gives R-squared value of (0.544), which means that approximately 54.4% of the variance can be explained the model. The prod F-statistic (5.08e-88) indicates that the model is statistically significant due to the value < 0.05. The negative coefficient (-0.9500) indicates that as `lstat` increases, `medv` tends to decrease, suggesting an inverse relationship between these two variables.

rm The same interpretation can be made for `rm`, which has an R-squared value of (0.484),prod F-statistic value of (2.49e-74) and a coefficient of (9.1021). Here the positive coefficient indicates that as `rm` increases, `medv` also tends to increase, suggesting a direct relationship between these two variables.

age Same interpretation can be made for `age`, which has an R-squared value of (0.142), prod F-statistic value of (1.57e-18) and a coefficient of (-0.1232). Here the negative coefficient indicates that as `age` increases, `medv` tends to decrease, suggesting an inverse relationship between these two variables.

[9]:
```python
print(regression1.conf_int())
print(regression2.conf_int())
print(regression3.conf_int())
```

```
               0          1
const  33.448457  35.659225
lstat  -1.026148  -0.873951
               0          1
const -39.876641 -29.464601
```

```
rm       8.278855   9.925363
                0          1
const  29.015752  32.941604
age    -0.149647  -0.096679
```

Inicates the lower and upper bounds of the 95% confidence interval. First row (y intercept) and second row (slope of predictor). Smaller intervals in the slope indicates that the model is more precise.

The second one indicates highly positive correlation but it has a greater interval. Meanwhile the first and third one indicates negative correlation with smaller intervals.

1.3.11 Use model

[10]:
```python
use_lstat = pd.DataFrame({"lstat": [5, 10, 15]})
use_lstat = sm.add_constant(use_lstat)
predictor1 = regression1.get_prediction(use_lstat).summary_frame(alpha=0.05)
print(predictor1[["mean", "obs_ci_lower", "obs_ci_upper"]])

use_rm = pd.DataFrame({"rm": [5, 6.5, 8]})
use_rm = sm.add_constant(use_rm)
perdictor2 = regression2.get_prediction(use_rm).summary_frame(alpha=0.05)
print(perdictor2[["mean", "obs_ci_lower", "obs_ci_upper"]])

use_age = pd.DataFrame({"age": [25, 50, 75]})
use_age = sm.add_constant(use_age)
perdictor3 = regression3.get_prediction(use_age).summary_frame(alpha=0.05)
print(perdictor3[["mean", "obs_ci_lower", "obs_ci_upper"]])
```

```
        mean  obs_ci_lower  obs_ci_upper
0  29.803594     17.565675     42.041513
1  25.053347     12.827626     37.279068
2  20.303101      8.077742     32.528459
        mean  obs_ci_lower  obs_ci_upper
0  10.839924     -2.214474     23.894322
1  24.493088     11.480391     37.505784
2  38.146251     25.058353     51.234149
        mean  obs_ci_lower  obs_ci_upper
0  27.899610     11.090368     44.708852
1  24.820542      8.043748     41.597335
2  21.741474      4.971031     38.511917
```

Interpretation of results

lstat Inserted values for `lstat` where 5, 10, 15 this means that with 95% confidence the predicted `medv` values will respectivly be between (17.56, 42.04), (12.82, 37.27) and (8.07, 32.52) approximatly.

rm & age Same interpretation can be made for `rm` and `age` where inserted values are 5, 6.5, 8 for `rm` and 25, 50, 75 for `age`.

[11]:
```python
regression = sm.OLS(df["medv"], sm.add_constant(df[["lstat", "rm", "age"]])).
    ↪fit()
print(regression.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.639
Model:                            OLS   Adj. R-squared:                  0.637
Method:                 Least Squares   F-statistic:                     296.2
Date:                Sun, 25 Jan 2026   Prob (F-statistic):          1.20e-110
Time:                        16:43:09   Log-Likelihood:                -1582.4
No. Observations:                 506   AIC:                             3173.
Df Residuals:                     502   BIC:                             3190.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -1.1753      3.182     -0.369      0.712      -7.427       5.076
lstat         -0.6685      0.054    -12.298      0.000      -0.775      -0.562
rm             5.0191      0.454     11.048      0.000       4.127       5.912
age            0.0091      0.011      0.811      0.418      -0.013       0.031
==============================================================================
Omnibus:                      138.819   Durbin-Watson:                   0.851
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              415.436
Skew:                           1.296   Prob(JB):                     6.15e-91
Kurtosis:                       6.603   Cond. No.                         985.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

Here the R-squared value (0.639) indicates that approximately 63.9% of the variance in `medv` can be explained by the model. The prod F-statistic (1.20e-110) indicates that the model is statistically significant due to the value < 0.05.

[12]:
```python
regression = sm.OLS(df["medv"], sm.add_constant(df.drop(columns=["medv"]))).
    ↪fit()
print(regression.summary())
```

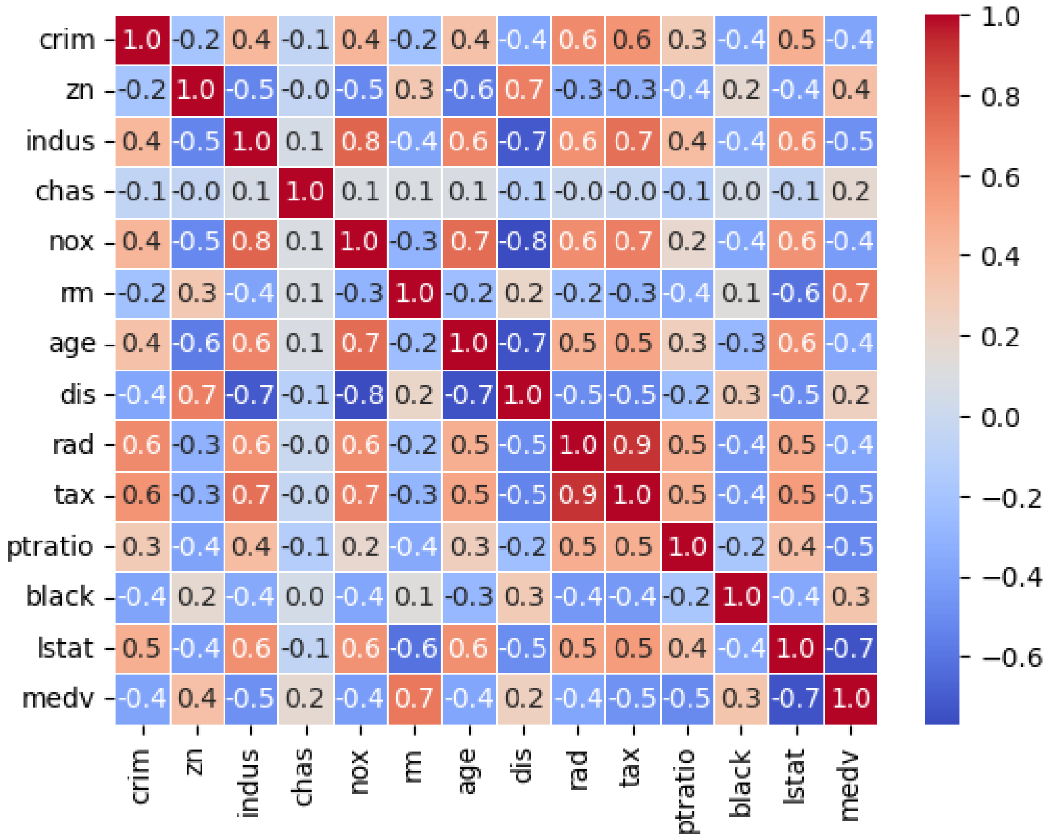
```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.741
Model:                            OLS   Adj. R-squared:                  0.734
Method:                 Least Squares   F-statistic:                     108.1
Date:                Sun, 25 Jan 2026   Prob (F-statistic):          6.72e-135
Time:                        16:43:09   Log-Likelihood:                -1498.8
No. Observations:                 506   AIC:                             3026.
```

```
Df Residuals:                     492   BIC:                             3085.
Df Model:                          13
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         36.4595      5.103      7.144      0.000      26.432      46.487
crim          -0.1080      0.033     -3.287      0.001      -0.173      -0.043
zn             0.0464      0.014      3.382      0.001       0.019       0.073
indus          0.0206      0.061      0.334      0.738      -0.100       0.141
chas           2.6867      0.862      3.118      0.002       0.994       4.380
nox          -17.7666      3.820     -4.651      0.000     -25.272     -10.262
rm             3.8099      0.418      9.116      0.000       2.989       4.631
age            0.0007      0.013      0.052      0.958      -0.025       0.027
dis           -1.4756      0.199     -7.398      0.000      -1.867      -1.084
rad            0.3060      0.066      4.613      0.000       0.176       0.436
tax           -0.0123      0.004     -3.280      0.001      -0.020      -0.005
ptratio       -0.9527      0.131     -7.283      0.000      -1.210      -0.696
black          0.0093      0.003      3.467      0.001       0.004       0.015
lstat         -0.5248      0.051    -10.347      0.000      -0.624      -0.425
==============================================================================
Omnibus:                      178.041   Durbin-Watson:                   1.078
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              783.126
Skew:                           1.521   Prob(JB):                    8.84e-171
Kurtosis:                       8.281   Cond. No.                     1.51e+04
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 1.51e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Here the R-squared value (0.741) indicates that approximately 74.1% of the variance in `medv` can be explained by the model. The prod F-statistic (6.72e-135) indicates that the model is statistically significant due to the value < 0.05.

Correlation matrix

[13]:
```python
sns.heatmap(df.corr(), annot=True, cmap="coolwarm", fmt=".1f", linewidths=0.5)
plt.show()
```



This matrix shows the correlation coefficients between each pair of variables in the dataset. A correlation coefficient close to 1 indicates a strong positive correlation, while a coefficient close to -1 indicates a strong negative correlation. A coefficient around 0 suggests no correlation between the variables.

Example interpretations:

`crim` and `zn` have a correlation coefficient of -0.2, indicating a weak negative correlation. This suggests that as the value of `crim` increases, the value of `zn` tends to decrease slightly.

1.3.12 Use multiple linear regression model

[14]:
```
selected_predictor_values = pd.DataFrame(
    pd.MultiIndex.from_product(
        [[5, 10, 15], [5, 6.5, 8]], names=["lstat", "rm"]
    ).to_frame(index=False)
)
print(selected_predictor_values)

regression = sm.OLS(df["medv"], sm.add_constant(df[["lstat", "rm"]])).fit()
selected_predictor_values = sm.add_constant(selected_predictor_values)
```

```
predictions = regression.get_prediction(selected_predictor_values)
pred_summary = predictions.summary_frame(alpha=0.05)

print(pred_summary[["mean", "obs_ci_lower", "obs_ci_upper"]])
```

```
   lstat   rm
0      5  5.0
1      5  6.5
2      5  8.0
3     10  5.0
4     10  6.5
5     10  8.0
6     15  5.0
7     15  6.5
8     15  8.0
        mean  obs_ci_lower  obs_ci_upper
0  20.903875      9.889729     31.918021
1  28.546057     17.635923     39.456192
2  36.188239     25.225479     47.150999
3  17.692084      6.722152     28.662016
4  25.334266     14.437027     36.231505
5  32.976448     21.995024     43.957872
6  14.480292      3.537875     25.422709
7  22.122474     11.221204     33.023745
8  29.764656     18.747835     40.781477
```

Interpretation of results Row one indicates that for value of `lstat` (5) and `rm` (5.0) will with 95% confidence result in a `medv` value between (9.88, 31.91), meanwhile row nine indicates that for value of `lstat` (15) and `rm` (8.0) will with 95% confidence result in a `medv` value between (18.74, 40.78). Same goes for all the subsequent rows with different values for `lstat` and `rm` which also provides a new boundary for all combinations of them.

1.3.13 Assignment 3

[15]:
```
X = df[["lstat", "rm", "nox", "dis", "ptratio"]]
Y = df["medv"]
X = sm.add_constant(X)
model1 = sm.OLS(Y, X).fit()

print(model1.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.708
Model:                            OLS   Adj. R-squared:                  0.705
Method:                 Least Squares   F-statistic:                     242.6
Date:                Sun, 25 Jan 2026   Prob (F-statistic):          3.67e-131
Time:                        16:43:09   Log-Likelihood:                -1528.7
```

```
No. Observations:                 506   AIC:                             3069.
Df Residuals:                     500   BIC:                             3095.
Df Model:                           5
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         37.4992      4.613      8.129      0.000      28.436      46.562
lstat         -0.5811      0.048    -12.122      0.000      -0.675      -0.487
rm             4.1633      0.412     10.104      0.000       3.354       4.973
nox          -17.9966      3.261     -5.519      0.000     -24.403     -11.590
dis           -1.1847      0.168     -7.034      0.000      -1.516      -0.854
ptratio       -1.0458      0.114     -9.212      0.000      -1.269      -0.823
==============================================================================
Omnibus:                      187.456   Durbin-Watson:                   0.971
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              885.498
Skew:                           1.584   Prob(JB):                    5.21e-193
Kurtosis:                       8.654   Cond. No.                         545.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

[16]:
```python
df["lstat_rm"] = df["lstat"] * df["rm"]
X = df[["lstat", "rm", "lstat_rm", "nox", "dis", "ptratio"]]
X = sm.add_constant(X)
model2 = sm.OLS(Y, X).fit()

print(model2.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.778
Model:                            OLS   Adj. R-squared:                  0.775
Method:                 Least Squares   F-statistic:                     290.8
Date:                Sun, 25 Jan 2026   Prob (F-statistic):          2.48e-159
Time:                        16:43:09   Log-Likelihood:                -1459.9
No. Observations:                 506   AIC:                             2934.
Df Residuals:                     499   BIC:                             2963.
Df Model:                           6
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          3.1518      4.880      0.646      0.519      -6.435      12.739
lstat          1.8115      0.196      9.237      0.000       1.426       2.197
rm             8.3344      0.491     16.971      0.000       7.370       9.299
```

```
lstat_rm      -0.4185      0.034    -12.488      0.000      -0.484      -0.353
nox          -12.3651      2.885     -4.286      0.000     -18.033      -6.697
dis           -1.0184      0.148     -6.893      0.000      -1.309      -0.728
ptratio       -0.7152      0.103     -6.967      0.000      -0.917      -0.514
==============================================================================
Omnibus:                      246.928   Durbin-Watson:                   1.079
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             2792.613
Skew:                           1.836   Prob(JB):                         0.00
Kurtosis:                      13.908   Cond. No.                     2.36e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 2.36e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Interpretation of results By utilizing `lstat`, `rm`, `nox`, `dis` and `ptratio` columns, the model achives an R-squared value of (0.705), but by just adding the interaction between `lstat` and `rm` the R-squared value increases to (0.775). This indicates that the interaction between `lstat` and `rm` contributes a better prediction of `medv` then just using the individual predictors alone.

Adding non-linear term

[17]:
```python
df["lstat_rm_squared"] = df["lstat_rm"] ** 2
X = df[["lstat", "rm", "lstat_rm", "lstat_rm_squared", "nox", "dis", "ptratio"]]
X = sm.add_constant(X)
model3 = sm.OLS(Y, X).fit()

print(model3.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.781
Model:                            OLS   Adj. R-squared:                  0.778
Method:                 Least Squares   F-statistic:                     253.9
Date:                Sun, 25 Jan 2026   Prob (F-statistic):         8.05e-160
Time:                        16:43:09   Log-Likelihood:                -1455.8
No. Observations:                 506   AIC:                             2928.
Df Residuals:                     498   BIC:                             2961.
Df Model:                           7
Covariance Type:            nonrobust
================================================================================
====
                       coef    std err          t      P>|t|      [0.025
0.975]
--------------------------------------------------------------------------------
----
const               10.5522      5.499      1.919      0.056      -0.253
```

```
21.357
lstat                  1.5468      0.216      7.167      0.000       1.123
1.971
rm                     7.6004      0.552     13.777      0.000       6.516
8.684
lstat_rm              -0.4468      0.035    -12.864      0.000      -0.515
-0.379
lstat_rm_squared       0.0004      0.000      2.845      0.005       0.000
0.001
nox                  -12.2898      2.865     -4.290      0.000     -17.918
-6.662
dis                   -1.0641      0.148     -7.209      0.000      -1.354
-0.774
ptratio               -0.7112      0.102     -6.977      0.000      -0.912
-0.511
==============================================================================
Omnibus:                      217.415   Durbin-Watson:                   1.059
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             2007.945
Skew:                           1.622   Prob(JB):                         0.00
Kurtosis:                      12.204   Cond. No.                     3.02e+05
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 3.02e+05. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Preform ANOVA

[18]:
```python
ANOVA_results = sm.stats.anova_lm(model2, model3)
print(ANOVA_results)
```

```
   df_resid          ssr  df_diff     ss_diff         F    Pr(>F)
0     499.0  9500.381881      0.0         NaN       NaN       NaN
1     498.0  9348.435955      1.0  151.945925  8.094303  0.004623
```

Conclusion from ANOVA ANOVA test indicates that the third model (with interaction and non-linear term) is significantly better then the second model (with interaction) due to the p-value being less than 0.05 (0.004) and it having a better R-squared value.

Add polynomial

[19]:
```python
for exp in range(2, 6):
    df[f"lstat_poly_{exp}"] = df["lstat"] ** exp

X = df[
    [
        "lstat",
```

```python
        "rm",
        "lstat_rm",
        "lstat_poly_2",
        "lstat_poly_3",
        "lstat_poly_4",
        "lstat_poly_5",
        "nox",
        "dis",
        "ptratio",
    ]
]
X = sm.add_constant(X)
model4 = sm.OLS(Y, X).fit()

print(model4.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.792
Model:                            OLS   Adj. R-squared:                  0.787
Method:                 Least Squares   F-statistic:                     188.0
Date:                Sun, 25 Jan 2026   Prob (F-statistic):         1.80e-161
Time:                        16:43:09   Log-Likelihood:                -1443.5
No. Observations:                 506   AIC:                             2909.
Df Residuals:                     495   BIC:                             2956.
Df Model:                          10
Covariance Type:            nonrobust
================================================================================
                   coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
const           33.9246      7.663      4.427      0.000      18.869      48.981
lstat           -5.6422      1.426     -3.957      0.000      -8.444      -2.841
rm               6.5291      0.711      9.183      0.000       5.132       7.926
lstat_rm        -0.3055      0.052     -5.878      0.000      -0.408      -0.203
lstat_poly_2     0.8633      0.187      4.622      0.000       0.496       1.230
lstat_poly_3    -0.0495      0.012     -4.153      0.000      -0.073      -0.026
lstat_poly_4     0.0013      0.000      3.795      0.000       0.001       0.002
lstat_poly_5 -1.279e-05   3.64e-06     -3.514      0.000   -1.99e-05   -5.64e-06
nox            -13.7513      2.823     -4.871      0.000     -19.298      -8.204
dis             -1.0326      0.145     -7.127      0.000      -1.317      -0.748
ptratio         -0.7407      0.101     -7.324      0.000      -0.939      -0.542
==============================================================================
Omnibus:                      232.049   Durbin-Watson:                   1.116
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             2275.267
Skew:                           1.742   Prob(JB):                         0.00
Kurtosis:                      12.787   Cond. No.                     3.38e+08
==============================================================================
```

```
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 3.38e+08. This might indicate that there are
strong multicollinearity or other numerical problems.
```

[20]:

```python
ANOVA_results = sm.stats.anova_lm(model2, model4)
print(ANOVA_results)
```

```
   df_resid          ssr  df_diff     ss_diff         F    Pr(>F)
0     499.0  9500.381881      0.0         NaN       NaN       NaN
1     495.0  8903.772453      4.0  596.609428  8.292038  0.000002
```

Conclusion from ANOVA The p-value decreased significantly (0.000002) indicating that the fourth model (with interaction and polynomial term) is statistically significantly better then the third model (with interaction and non-linear term), however with high degree polynomials there is a risk of overfitting the model to the training data, which can lead to poor generalization to new data.

[21]:

```python
df["log_rm"] = np.log(df["rm"])

X = df[
    [
        "lstat",
        "lstat_poly_2",
        "lstat_poly_3",
        "lstat_poly_4",
        "lstat_poly_5",
        "rm",
        "log_rm",
        "nox",
        "dis",
        "ptratio",
    ]
]
X = sm.add_constant(X)
model5 = sm.OLS(Y, X).fit()

print(model5.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.804
Model:                            OLS   Adj. R-squared:                  0.800
Method:                 Least Squares   F-statistic:                     202.6
Date:                Sun, 25 Jan 2026   Prob (F-statistic):          7.10e-168
Time:                        16:43:09   Log-Likelihood:                -1428.4
No. Observations:                 506   AIC:                             2879.
Df Residuals:                     495   BIC:                             2925.
```

```
Df Model:                          10
Covariance Type:            nonrobust
================================================================================
                   coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
const          172.9866     13.954     12.397      0.000     145.571     200.402
lstat           -8.5527      1.227     -6.969      0.000     -10.964      -6.141
lstat_poly_2     1.0064      0.178      5.654      0.000       0.657       1.356
lstat_poly_3    -0.0582      0.011     -5.087      0.000      -0.081      -0.036
lstat_poly_4     0.0015      0.000      4.672      0.000       0.001       0.002
lstat_poly_5 -1.521e-05   3.52e-06     -4.323      0.000   -2.21e-05    -8.3e-06
rm              25.1967      2.732      9.224      0.000      19.830      30.564
log_rm        -137.4038     16.761     -8.198      0.000    -170.336    -104.472
nox            -16.6408      2.734     -6.087      0.000     -22.012     -11.270
dis             -0.9709      0.141     -6.885      0.000      -1.248      -0.694
ptratio         -0.7843      0.097     -8.116      0.000      -0.974      -0.594
==============================================================================
Omnibus:                      221.958   Durbin-Watson:                   1.064
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             2718.500
Skew:                           1.567   Prob(JB):                         0.00
Kurtosis:                      13.914   Cond. No.                     9.63e+08
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 9.63e+08. This might indicate that there are
strong multicollinearity or other numerical problems.
```

[22]:
```python
ANOVA_results = sm.stats.anova_lm(model2, model5)
print(ANOVA_results)
```

```
   df_resid          ssr  df_diff      ss_diff          F        Pr(>F)
0     499.0  9500.381881      0.0          NaN        NaN           NaN
1     495.0  8386.756361      4.0   1113.62552  16.431997  1.190966e-12
```

Conclusion from ANOVA Increase in R-squared value and the very low p-value indicates that the fifth model (with interaction, higher degree polynomial term and logarithmic term) is statistically significantly better then the fourth model (with interaction and polynomial term).

Load data 2

[23]:
```python
df2 = pd.read_csv("../data/Carseats.csv", index_col=0)
print(df2.describe(), "\n")
print(df2["ShelveLoc"].value_counts(), "\n")
print(df2["Urban"].value_counts(), "\n")
print(df2["US"].value_counts())
```

```
            Sales    CompPrice      Income  Advertising  Population  \
count  400.000000  400.000000  400.000000   400.000000  400.000000
mean     7.496325  124.975000   68.657500     6.635000  264.840000
std      2.824115   15.334512   27.986037     6.650364  147.376436
min      0.000000   77.000000   21.000000     0.000000   10.000000
25%      5.390000  115.000000   42.750000     0.000000  139.000000
50%      7.490000  125.000000   69.000000     5.000000  272.000000
75%      9.320000  135.000000   91.000000    12.000000  398.500000
max     16.270000  175.000000  120.000000    29.000000  509.000000

            Price         Age   Education
count  400.000000  400.000000  400.000000
mean   115.795000   53.322500   13.900000
std     23.676664   16.200297    2.620528
min     24.000000   25.000000   10.000000
25%    100.000000   39.750000   12.000000
50%    117.000000   54.500000   14.000000
75%    131.000000   66.000000   16.000000
max    191.000000   80.000000   18.000000

ShelveLoc
Medium    219
Bad        96
Good       85
Name: count, dtype: int64

Urban
Yes    282
No     118
Name: count, dtype: int64

US
Yes    258
No     142
Name: count, dtype: int64
```

[24]:

```
X = pd.get_dummies(df2, columns=["ShelveLoc", "Urban", "US"])
for column in X.select_dtypes("bool"):
    X[column] = X[column].astype(int)

X = X.drop(columns=["Sales"])
X = sm.add_constant(X)
Y = df2["Sales"]
model = sm.OLS(Y, X).fit()

print(model.summary())
```

```
                            OLS Regression Results
```

```
==============================================================================
Dep. Variable:                  Sales   R-squared:                       0.873
Model:                            OLS   Adj. R-squared:                  0.870
Method:                 Least Squares   F-statistic:                     243.4
Date:                Sun, 25 Jan 2026   Prob (F-statistic):         1.60e-166
Time:                        16:43:09   Log-Likelihood:                -568.99
No. Observations:                 400   AIC:                             1162.
Df Residuals:                     388   BIC:                             1210.
Df Model:                          11
Covariance Type:            nonrobust
================================================================================
====
                       coef    std err          t      P>|t|      [0.025
0.975]
--------------------------------------------------------------------------------
----
const                3.3853      0.253     13.370      0.000       2.887
3.883
CompPrice            0.0928      0.004     22.378      0.000       0.085
0.101
Income               0.0158      0.002      8.565      0.000       0.012
0.019
Advertising          0.1231      0.011     11.066      0.000       0.101
0.145
Population           0.0002      0.000      0.561      0.575      -0.001
0.001
Price               -0.0954      0.003    -35.700      0.000      -0.101
-0.090
Age                 -0.0460      0.003    -14.472      0.000      -0.052
-0.040
Education           -0.0211      0.020     -1.070      0.285      -0.060
0.018
ShelveLoc_Bad       -1.1405      0.118     -9.629      0.000      -1.373
-0.908
ShelveLoc_Good       3.7096      0.121     30.652      0.000       3.472
3.948
ShelveLoc_Medium     0.8162      0.107      7.605      0.000       0.605
1.027
Urban_No             1.6312      0.138     11.789      0.000       1.359
1.903
Urban_Yes            1.7541      0.139     12.629      0.000       1.481
2.027
US_No                1.7847      0.146     12.243      0.000       1.498
2.071
US_Yes               1.6006      0.148     10.783      0.000       1.309
1.892
==============================================================================
Omnibus:                        0.811   Durbin-Watson:                   2.013
```

```
Prob(Omnibus):                  0.667   Jarque-Bera (JB):                0.765
Skew:                           0.107   Prob(JB):                        0.682
Kurtosis:                       2.994   Cond. No.                     3.32e+18
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The smallest eigenvalue is 4.43e-30. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.
```

Conclusion from model summary The model achieves an R-squared value (0.873) which indicates that $\approx 87.3\%$ of the variance can be explained by the model. The prob F-statistic (1.60e-166) indicates that the model is statistically significant due to the value < 0.05.

[25]:
```python
X = df2.drop(columns=["Sales", "Population", "Education", "Age", "Urban", "US"])
X = pd.get_dummies(X, columns=["ShelveLoc"])

for column in X.select_dtypes("bool"):
    X[column] = X[column].astype(int)

X["Income:Advertising"] = df2["Income"] * df2["Advertising"]
X["Price:Age"] = df2["Price"] * df2["Age"]
Y = df2["Sales"]
X = sm.add_constant(X)
model = sm.OLS(Y, X).fit()

print(model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  Sales   R-squared:                       0.870
Model:                            OLS   Adj. R-squared:                  0.868
Method:                 Least Squares   F-statistic:                     328.2
Date:                Sun, 25 Jan 2026   Prob (F-statistic):          2.90e-168
Time:                        16:43:09   Log-Likelihood:                -573.74
No. Observations:                 400   AIC:                             1165.
Df Residuals:                     391   BIC:                             1201.
Df Model:                           8
Covariance Type:            nonrobust
================================================================================
======
                           coef    std err          t      P>|t|      [0.025
0.975]
--------------------------------------------------------------------------------
------
const                    4.1957      0.352     11.903      0.000       3.503
4.889
```

```
CompPrice               0.0934      0.004     22.492      0.000       0.085
0.102
Income                  0.0098      0.003      3.756      0.000       0.005
0.015
Advertising             0.0534      0.021      2.544      0.011       0.012
0.095
Price                  -0.0759      0.003    -25.591      0.000      -0.082
-0.070
ShelveLoc_Bad          -0.8966      0.143     -6.280      0.000      -1.177
-0.616
ShelveLoc_Good          3.9982      0.149     26.769      0.000       3.705
4.292
ShelveLoc_Medium        1.0942      0.133      8.221      0.000       0.833
1.356
Income:Advertising      0.0009      0.000      3.124      0.002       0.000
0.001
Price:Age              -0.0004   2.69e-05    -13.713      0.000      -0.000
-0.000
==============================================================================
Omnibus:                        1.537   Durbin-Watson:                   1.988
Prob(Omnibus):                  0.464   Jarque-Bera (JB):                1.326
Skew:                           0.129   Prob(JB):                        0.515
Kurtosis:                       3.116   Cond. No.                     2.80e+19
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The smallest eigenvalue is 2.18e-29. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.
```

Conclusion form model summary Achives an R-squared value (0.870) which is worse than the previous model and prob F-statistic (2.90e-168) which indicates that the model is more statistically significant due to the value < 0.05.

Beat the teacher

[26]:
```
X = df2.drop(columns=["Sales"])
X = pd.get_dummies(X, columns=["ShelveLoc", "US", "Urban"])

for column in X.select_dtypes("bool"):
    X[column] = X[column].astype(int)

X["Income:Advertising"] = df2["Income"] * df2["Advertising"]
Y = df2["Sales"]
X = sm.add_constant(X)
model = sm.OLS(Y, X).fit()
```

```python
print(model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  Sales   R-squared:                       0.876
Model:                            OLS   Adj. R-squared:                  0.872
Method:                 Least Squares   F-statistic:                     227.6
Date:                Sun, 25 Jan 2026   Prob (F-statistic):          5.48e-167
Time:                        16:43:09   Log-Likelihood:                -565.00
No. Observations:                 400   AIC:                             1156.
Df Residuals:                     387   BIC:                             1208.
Df Model:                          12
Covariance Type:            nonrobust
================================================================================
======
                       coef    std err          t      P>|t|      [0.025
0.975]
--------------------------------------------------------------------------------
------
const                3.5106      0.255     13.767      0.000       3.009
4.012
CompPrice            0.0931      0.004     22.630      0.000       0.085
0.101
Income               0.0107      0.003      4.125      0.000       0.006
0.016
Advertising          0.0684      0.022      3.043      0.003       0.024
0.113
Population           0.0002      0.000      0.456      0.649      -0.001
0.001
Price               -0.0952      0.003    -35.962      0.000      -0.100
-0.090
Age                 -0.0454      0.003    -14.367      0.000      -0.052
-0.039
Education           -0.0220      0.020     -1.125      0.261      -0.060
0.016
ShelveLoc_Bad       -1.1051      0.118     -9.356      0.000      -1.337
-0.873
ShelveLoc_Good       3.7570      0.121     31.005      0.000       3.519
3.995
ShelveLoc_Medium     0.8586      0.107      7.988      0.000       0.647
1.070
US_No                1.8361      0.146     12.604      0.000       1.550
2.123
US_Yes               1.6744      0.150     11.199      0.000       1.380
1.968
Urban_No             1.6884      0.139     12.173      0.000       1.416
1.961
Urban_Yes            1.8222      0.140     13.031      0.000       1.547
```

```
2.097
Income:Advertising     0.0008      0.000      2.791      0.006       0.000
0.001
==============================================================================
Omnibus:                        1.390   Durbin-Watson:                   2.036
Prob(Omnibus):                  0.499   Jarque-Bera (JB):                1.229
Skew:                           0.131   Prob(JB):                        0.541
Kurtosis:                       3.070   Cond. No.                     1.15e+19
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The smallest eigenvalue is 1.73e-30. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.
```

Conclusion from Beat the teacher model I was able to to get slightly better R-squared value (0.876) with a prob F-statistic (5.48e-167) which is in between the two previous models significance. I did this by not dropping any predictors and adding interaction term between `Income` and `Advertising`.