Notes on FMCW and Meteorological Radar

Stuart Ballantyne

January 8, 2022

Contents

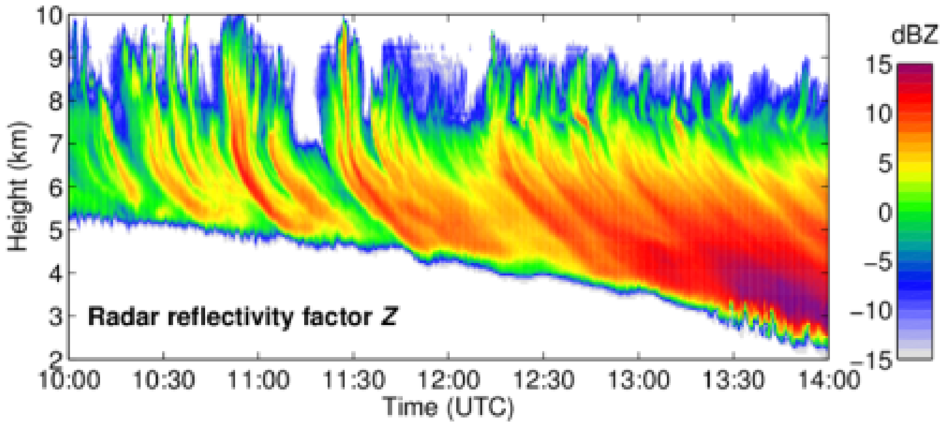

Figure 1: Example cloud radar output.[1]

1 Radar range equation

1.1 Point target

Consider an isotropically emitting antenna with transmit power P_t. The power density Q at a distance r from the antenna is given by

$$Q = \frac{P_t}{4\pi r^2}.$$

To account for the directivity and efficiency of the antenna one must include the (transmit) gain G_t. Hence,

$$Q = \frac{G_t P_t}{4\pi r^2}.$$

The power reflected by a target with *radar cross-section* σ (units of cross-sectional area) is then

$$P_{refl} = Q\sigma = \frac{G_t P_t \sigma}{4\pi r^2}.$$

Assuming the power is reflected isotropically, then

$$Q_r = \frac{P_{refl}}{4\pi r^2} = \frac{G_t P_t \sigma}{(4\pi r^2)^2}.$$

The receiver antenna has an effective area A given by

$$A = \frac{G_r \lambda^2}{4\pi},$$

where G_r is the receiver antenna gain. The received power is simply the product $P_r = AQ_r$, hence we finally arrive at

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi)^3 r^4}\sigma, \tag{1}$$

which is known as the **radar range equation**.

1.2 Meteorological targets

For a distributed meteorological target, one writes

$$\sigma = \eta V, \tag{2}$$

where η is the volume reflectivity (cross-sectional area per unit volume) and V is the volume sampled by the radar.

[Derive volume term]

In the Rayleigh scattering regime, one can write

$$\eta = \sum_i \sigma_i = \frac{\pi^5}{\lambda^4}|K|^2 \sum_i D_i^6 = \frac{\pi^5}{\lambda^4}|K|^2 Z \tag{3}$$

Thus, the **meteorological radar range equation** is

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi)^3 r^4} \cdot \frac{1}{2\ln 2} \cdot \frac{\pi r^2 \theta \phi c \tau}{8} \cdot \eta = \frac{P_t G_t G_r \lambda^2 \theta \phi c \tau}{1024(\ln 2)\pi^2 r^2}\eta = \frac{P_t G_t G_r \theta \phi c \tau \pi^3 |K|^2}{1024(\ln 2)\lambda^2 r^2} Z, \tag{4}$$

where we have inserted a factor $1/(2\ln 2)$ to account for the Gaussian antenna gain profile.[5]

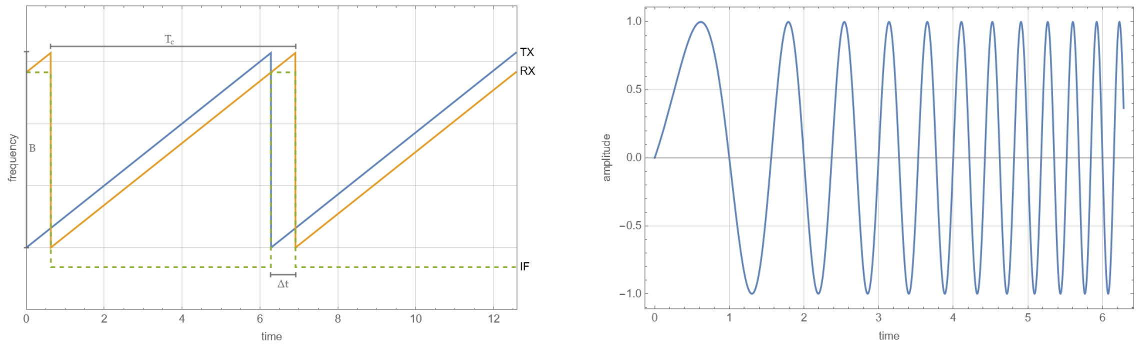

Figure 2: Left: transmitted (TX) and received (RX) sawtooth chirps and their difference frequency (IF). Right: illustration of TX chirp waveform (own work).

Wavelength (mm)	3.19
Range res. (m)	10
Transmit power (dBm)	25
Gain (dBi)	52
Beamwidth (°)	0.43
Chirp rep. freq. (GHz)	6510
Noise figure (dB)	5.3

Table 1: Parameters for the St. Andrews 94-GHz cloud profiling radar.[2]

2 Frequency-modulated continuous wave

3 Radar reflectivity

References

[1] *The Clouds Group: Research*, University of Reading, `http://www.met.reading.ac.uk/clouds/research.html` (visited on 01/07/2022).

[2] D. A. Robertson and R. I. Hunter, "A solid state 94 GHz FMCW Doppler radar demonstrator for cloud profiling", in Proc. SPIE, Radar Sensor Technology XXI, Vol. 10188 (2017), pp. 345–351.

[3] J. F. Shaeffer, "Target reflectivity", in *Principles of modern radar*, Vol. 1, edited by M. A. Richards, J. A. Scheer, and W. A. Holm, 3rd ed. (2010) Chap. 6.

[4] R. J. Keeler and R. J. Serafin, "Meterological radar", in *Radar handbook*, edited by M. Skolnik, 3rd ed. (2008) Chap. 19.

[5] J. R. Probert-Jones, "The radar equation in meteorology", Quarterly Journal of the Royal Meteorological Society **88**, 485–495 (1962).