

2.1 Let S be a finite set with n elements. Since S is finite, we can list out its elements s_1, s_2, \dots, s_n . We can specify a function $f : S \rightarrow S$ by specifying one element of S for each element of S .

First we choose $f(s_1)$. There are n possible choices, these being any of s_1 through s_n . Then, in choosing the next element, there are $n - 1$ choices, these being all the elements of S except the one chosen for $f(s_1)$. We may not choose the same element as $f(s_1)$, since then the function would no longer be injective. Similarly, we have $n - 2$ choices for $f(s_3)$, $n - 3$ choices for $f(s_4)$ and so on. Finally, at $f(s_n)$ there is only 1 choice. Ultimately, there are $n!$ injections from $S \rightarrow S$. These are also surjective, since we pick every element of S to output to. So there are $n!$ bijections from $S \rightarrow S$.

2.2

(\Rightarrow) Let $f : A \rightarrow B$ have a right inverse, with $A \neq \emptyset$. Want to show that f is surjective.