3.1 Let A, B, C be objects in C and hence objects in C^{op}. Let $f \in \mathsf{Hom}_{\mathsf{C}^{op}}(A, B)$ and $g \in \mathsf{Hom}_{\mathsf{C}^{op}}(B, C)$. Thus $f \in \mathsf{Hom}_{\mathsf{C}}(B, A)$ and $g \in \mathsf{Hom}_{\mathsf{C}}(C, B)$. Since C is a category, the morphism $f \circ_{\mathsf{C}} g$ exists in C. Define $\circ_{\mathsf{C}^{op}}$ as $g \circ_{\mathsf{C}^{op}} f := f \circ_{\mathsf{C}} g$. We are guaranteed that the composition of morphisms exists since C is a category. We gain associativity for the same reason. The composition of identity in C^{op} becomes composition of identity on the other side in C, which is still an identity.

3.2 $\mathsf{End}_{\mathsf{Set}}(A)$ is defined as $\mathsf{Hom}_{\mathsf{Set}}(A, A)$. This is the set of all set-functions $A \to A$; in other words, the set A^A. Thus $|\mathsf{End}_{\mathsf{Set}}(A)| = |A^A| = |A|^{|A|}$ (the last equality from a previous exercise.)

3.3 Let $f : a \to b$ be some morphism. Want to show that $f \circ 1_a =_{(1)} f =_{(2)} 1_b \circ f$.

(1). $1_a = (a, a) \implies a \sim a$, $f = (a, b) \implies a \sim b$. The transitive property allows us to conclude that $a \sim b$, picking that as the composition, so $1_a \circ f = f$.

(2). $f = (a, b) \implies a \sim b$, $1_b = (b, b) \implies b \sim b$. The transitive property gives us that $a \sim b$, which corresponds to f. So $f \circ 1_b = f$.

3.5 Example 3.3 describes a category of a set of objections with a reflexive, transitive relation between them. $\hat{\mathsf{S}}$ has a set of objects, the objects being subsets of S. The reflexive, transitive relation is $\subseteq$. In all aspects it takes the form of the abstract category described in Example 3.3.

3.6 Let V be a category with $\mathsf{Obj}_{\mathsf{V}} = \mathbb{N}$, $\mathsf{Hom}_{\mathsf{V}}(n, m)$ being the set of m by n matrices. Let $A : n \to m, B : m \to k$ be morphisms. Then define $B \circ A := BA$, the matrix product. This product is defined, since B has height the same as the width of A.

This composition is associative: consider $A : n \to m, B : m \to k, C : k \to l$. Then $C \circ (B \circ A) = C(BA) = (CB)A = (C \circ B) \circ A$.

The identity morphism 1_n is the $n \times n$ identity matrix I_n. Let $A : n \to m$. We have that $A \circ 1_n = AI_n = A = I_m A = 1_m \circ A$.

I suppose this category can be considered with the objects representing the sets of isomorphic finite-dimensional vector spaces of dimension n.

3.7 Objects in our coslice category are arrows in C^A from some fixed object A to an arbitrary object Z:

$$\begin{tikzcd} A \arrow[d, "f"] \\ Z \end{tikzcd} .$$

A morphism $f_1 \to f_2$ is a commutative diagram of the following form:

$$\begin{tikzcd} A \arrow[d, "f_1"] \arrow[dr, "f_2"] & \\ Z_1 \arrow[r, "\tau"] & Z_2 \end{tikzcd} .$$

We obtain composition of morphisms by observing that since the following diagram commutes:

$$\begin{tikzcd} & A \arrow[dl, "f_1"'] \arrow[d, "f_2"] \arrow[dr, "f_3"] & \\ Z_1 \arrow[r, "\tau"] & Z_2 \arrow[r, "\sigma"] & Z_3 \end{tikzcd} ,$$

this diagram must also commute:

$$\begin{tikzcd} & A \arrow[dl, "f_1"'] \arrow[dr, "f_3"] & \\ Z_1 \arrow[rr, "\sigma\tau"] & & Z_3 \end{tikzcd} .$$