3.1 Let A, B, C be objects in C and hence objects in C^{op} . Let $f \in \operatorname{Hom}_{C^{op}}(A, B)$ and $g \in \operatorname{Hom}_{C^{op}}(B, C)$. Thus $f \in \operatorname{Hom}_{C}(B, A)$ and $g \in \operatorname{Hom}_{C}(C, B)$. Since C is a category, the morphism $f \circ_{C} g$ exists in C. Define $\circ_{C^{op}}$ as $g \circ_{C^{op}} f := f \circ_{C} g$. We are guaranteed that the composition of morphisms exists since C is a category. We gain associativity for the same reason. The composition of identity in C^{op} becomes composition of identity on the other side in C, which is still an identity.

3.2 $\operatorname{End}_{\operatorname{Set}}(A)$ is defined as $\operatorname{Hom}_{\operatorname{Set}}(A,A)$. This is the set of all set-functions $A \to A$; in other words, the set A^A . Thus $|\operatorname{End}_{\operatorname{Set}}(A)| = |A^A| = |A|^{|A|}$ (the last equality from a previous exercise.)

3.3 Let $f: a \to b$ be some morphism. Want to show that $f \circ 1_a =_{(1)} f =_{(2)} 1_b \circ f$.

(1). $1_a = (a, a) \implies a \sim a$, $f = (a, b) \implies a \sim b$. The transitive property allows us to conclude that $a \sim b$, picking that as the composition, so $1_a \circ f = f$.

(2). $f = (a, b) \implies a \sim b$, $1_b = (b, b) \implies b \sim b$. The transitive property gives us that $a \sim b$, which corresponds to f. So $f \circ 1_b = f$.

3.5 Example 3.3 describes a category of a set of objections with a reflexive, transitive relation between them. \hat{S} has a set of objects, the objects being subsets of S. The reflexive, transitive relation is \subseteq . In all aspects it takes the form of the abstract category described in Example 3.3.

3.6 Let V be a category with $\operatorname{Obj}_{V} = \mathbb{N}$, $\operatorname{Hom}_{V}(n,m)$ being the set of m by n matrices. Let $A: n \to m, B: m \to k$ be morphisms. Then define $B \circ A := BA$, the matrix product. This product is defined, since B has height the same as the width of A.

This composition is associative: consider $A: n \to m, B: m \to k, C: k \to l$. Then $C \circ (B \circ A) = C(BA) = (CB)A = (C \circ B) \circ A$.

The identity morphism 1_n is the $n \times n$ identity matrix I_n . Let $A : n \to m$. We have that $A \circ 1_n = AI_n = A = I_m A = 1_m \circ A$.

I suppose this category can be considered with the objects representing the sets of isomorphic finite-dimensional vector spaces of dimension n.

3.7 Objects in our coslice category are arrows in C^A from some fixed object A to an arbitrary object Z:



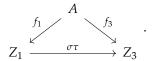
A morphism $f_1 \rightarrow f_2$ is a commutative diagram of the following form:

$$\begin{array}{c}
A \\
\downarrow f_1 & f_2 \\
Z_1 & \xrightarrow{\tau} & Z_2
\end{array}.$$

We obtain composition of morphisms by observing that since the following diagram commutes:

$$Z_1 \xrightarrow{\tau} Z_2 \xrightarrow{\sigma} Z_3$$

this diagram must also commute:



3.8 Let ISet be the category with objects being infinite sets and morphisms being set functions between them. Clearly $\operatorname{Obj}(\operatorname{ISet}) \subseteq \operatorname{Obj}(\operatorname{Set})$. For the morphisms, let A,B be two objects in ISet. Then a morphism $f \in \operatorname{Hom}_{\operatorname{Set}}(A,B)$ is a set function between A and B. Thus $f \in \operatorname{Hom}_{\operatorname{Set}}(A,B)$ as well. For the other direction, assume that $f \in \operatorname{Hom}_{\operatorname{Set}}(A,B)$. Then since f is a set function between the infinite sets A and $B, f \in \operatorname{Hom}_{\operatorname{ISet}}(A,B)$. So $\operatorname{Hom}_{\operatorname{Set}}(A,B) = \operatorname{Hom}_{\operatorname{ISet}}(A,B)$. Thus ISet is a full subcategory of Set.