

3.1 Let A, B, C be objects in \mathcal{C} and hence objects in \mathcal{C}^{op} . Let $f \in \text{Hom}_{\mathcal{C}^{op}}(A, B)$ and $g \in \text{Hom}_{\mathcal{C}^{op}}(B, C)$. Thus $f \in \text{Hom}_{\mathcal{C}}(B, A)$ and $g \in \text{Hom}_{\mathcal{C}}(C, B)$. Since \mathcal{C} is a category, the morphism $f \circ_{\mathcal{C}} g$ exists in \mathcal{C} . Define $\circ_{\mathcal{C}^{op}}$ as $g \circ_{\mathcal{C}^{op}} f := f \circ_{\mathcal{C}} g$. We are guaranteed that the composition of morphisms exists since \mathcal{C} is a category. We gain associativity for the same reason. The composition of identity in \mathcal{C}^{op} becomes composition of identity on the other side in \mathcal{C} , which is still an identity.

3.2 $\text{End}_{\text{Set}}(A)$ is defined as $\text{Hom}_{\text{Set}}(A, A)$. This is the set of all set-functions $A \rightarrow A$; in other words, the set A^A . Thus $|\text{End}_{\text{Set}}(A)| = |A^A| = |A|^{|A|}$ (the last equality from a previous exercise.)

3.3 Let $f : a \rightarrow b$ be some morphism. Want to show that $f \circ 1_a =_{(1)} f =_{(2)} 1_b \circ f$.

(1). $1_a = (a, a) \implies a \sim a, f = (a, b) \implies a \sim b$. The transitive property allows us to conclude that $a \sim b$, picking that as the composition, so $1_a \circ f = f$.

(2). $f = (a, b) \implies a \sim b, 1_b = (b, b) \implies b \sim b$. The transitive property gives us that $a \sim b$, which corresponds to f . So $f \circ 1_b = f$.

3.5 Example 3.3 describes a category of a set of objections with a reflexive, transitive relation between them. \hat{S} has a set of objects, the objects being subsets of S . The reflexive, transitive relation is \subseteq . In all aspects it takes the form of the abstract category described in Example 3.3.

3.7 Objects in our coslice category are arrows in \mathcal{C}^A from some fixed object A to an arbitrary object Z :

$$\begin{array}{c} A \\ \downarrow f \\ Z \end{array}$$

A morphism $f_1 \rightarrow f_2$ is a commutative diagram of the following form:

$$\begin{array}{ccc} A & & \\ \downarrow f_1 & \searrow f_2 & \\ Z_1 & \xrightarrow{\tau} & Z_2 \end{array}$$

We obtain composition of morphisms by observing that since the following diagram commutes:

$$\begin{array}{ccccc} & & A & & \\ & \swarrow f_1 & \downarrow f_2 & \searrow f_3 & \\ Z_1 & \xrightarrow{\tau} & Z_2 & \xrightarrow{\sigma} & Z_3 \end{array},$$

this diagram must also commute:

$$\begin{array}{ccc} & A & \\ \swarrow f_1 & & \searrow f_3 \\ Z_1 & \xrightarrow{\sigma\tau} & Z_3 \end{array}.$$