1.2 Let $\sim$ be an equivalence relation on the set S. Now let P_S be a family such that the members of P_S are sets of the form $[a] = \{b : b \in S, b \sim a\}$ where the a are members of S. We wish to show that P_S is a partition of S. We must show that the sets which are elements of P_S are nonempty, that they are disjoint, and that their union is S.

Nonempty. Each set $[a]$ contains a, since $a \sim a$ by the reflexive property of equivalence relations.

Disjoint. Let $a \in A, a \in A'$, where $A, A' \in P_S$. Let $b \in A$. A is some $[c]$, and therefore $b \sim c$. But also $a \sim c$, since $a \in A$. And $c \sim a$, and thus $b \sim a$. Now A' is some $[d]$, and $a \sim d$. But $b \sim a \sim d$, so $b \sim d$. Thus $b \in A'$. By the same logic, an arbitrary member of A' is in A. So $A' = A$. Thus the members of P_S are disjoint.

Union is S. Consider the union $S' = \bigcup_{k \in S} [k]$. Let $a \in S$. Then clearly $a \in S'$, since $[a]$ is one of the components of the union. Let $a \in S'$. Then $a \sim k$ for some k in S, so a must be in S.

1.3 Let P_S be a partition of S. Now consider the relation $\sim$ given by $a \sim b$ if and only if a and b are in the same set in the partition P_S. This is an equivalence relation.

Reflexive. Clearly, a is in the same set as a in P_S. So $a \sim a$.

Symmetric. If a and b are in the same set, then b and a are in the same set.

Transitive. If $a \in Q$ and $b \in Q$, and $b \in Q$ and $c \in Q$, then $a \in Q$ and $c \in Q$. So $a \sim b \wedge b \sim c \Rightarrow a \sim c$.

1.6

$(\sim)$

Reflexive. $a - a = 0 \in \mathbb{Z}$.

Symmetric. Let $a \sim b$. Then $b - a \in \mathbb{Z}$. Then $-(b - a) = a - b \in \mathbb{Z}$. So $b \sim a$.

Transitive. Let $a \sim b$ and $b \sim c$. Thus $(b - a) \in \mathbb{Z}$ and $(c - b) \in \mathbb{Z}$. Then $(c - b) + (b - a) = (c - a) \in \mathbb{Z}$, so $a \sim c$.

We can interpret the equivalence classes as the set of fractional parts of decimal real numbers. Said another way, as the real numbers modulo integers.

$(\approx)$

Reflexive. Let $(a, b) \in \mathbb{R}^2$. Then $b - b = a - a = 0 \in \mathbb{Z}$.

Symmetric. Let $(a, b), (c, d) \in \mathbb{R}^2$. Assume that $(a, b) \approx (c, d)$. So $c - a, d - b \in \mathbb{R}$. Thus $-(c - a) = a - c, -(d - b) = b - d \in \mathbb{Z}$. So $(c, d) \approx (a, b)$.

Transitive. Let $(a, b), (c, d), (e, f) \in \mathbb{R}^2$, with $(a, b) \approx (c, d)$ and $(c, d) \approx (e, f)$. Then $e - a = (e - c) + (c - a) \in \mathbb{Z}$, and $f - b = (f - d) + (d - b) \in \mathbb{Z}$.

We can interpret the equivalence classes as the unit square with bottom left corner at the origin on the $\mathbb{R}^2$ plane, including the bottom and left edges but not the top and right edges.