2.1 Let S be a finite set with n elements. Since S is finite, we can list out its elements $s_1, s_2, ..., s_n$. We can specify a function $f : S \to S$ by specifying one element of S for each element of S.

First we choose $f(s_1)$. There are n possible choices, these being any of s_1 through s_n. Then, in choosing the next element, there are $n-1$ choices, these being all the elements of S except the one chosen for $f(s_1)$. We may not choose the same element as $f(s_1)$, since then the function would no longer be injective. Similarly, we have $n-2$ choices for $f(s_3)$, $n-3$ choices for $f(s_4)$ and so on. Finally, at $f(s_n)$ there is only 1 choice. Ultimately, there are $n!$ injections from $S \to S$. These are also surjective, since we pick every element of S to output to. So there are $n!$ bijections from $S \to S$.

2.2

($\Rightarrow$) Let $f : A \to B$ have a right inverse, with $A \neq \emptyset$. Want to show that f is surjective. Let $g : B \to A$ be the right inverse of f, so that $f \circ g = id_B$. Now let $b \in B$ be arbitrary. We know that $g(b) \in A$. And $f(g(b)) = (f \circ g)(b) = b$. So f is surjective.

($\Leftarrow$) Let $f : A \to B$ be surjective. Want to show that f has a right inverse. That f is surjective means that $(\forall b \in B)(\exists a \in A) f(a) = b$. Thus, since A is nonempty, we know that $f^{-1}(b)$ is nonempty for all $b \in B$. Now, consider the family of subsets of A indexed by B given by $\{f^{-1}(b) : b \in B\}$. This family is necessarily disjoint, since if not, then the function would not be well defined (e.g. one input would have more than one output). Now we apply the axiom of choice to define a function $g : B \to A$ choosing one element from each subset where the input to g is the index to the family. Now, for any member b of B, $b = f(g(b)) = (f \circ g)(b) = id_B(b)$. So g is a right inverse of f.