Modeling and Simulation of Open-loop Handling Dynamics of Tractor Trailer Combination

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Abstract

The tractor trailer system is a combination of different bodies joined via joints, thus has different stability parameters compared to the normal passenger cars. Different instabilities that occur in these types of vehicles and the various control strategies and algorithms that have been implemented to minimize or prevent them have been discussed in this paper. A tractor trailer model was built from equations of motion that are developed using concept of Lagrange's equations. Simulations were performed on this model and tractor trajectory, yaw and fifth wheel (hitch point) angle responses were studied to step, impulse and sine wave steering inputs. The results indicate the different instabilities induced into the system due to the different steering inputs.

Introduction

A system of articulated vehicles also known as tractor-trailer system, consists of single or multiple trailing or trailer units, towed by a driver unit(tractor). These two bodies are hinged together with the tractor unit providing linear and lateral inputs to the trailer, manipulating its velocity and relative position. This input is not only used to maneuver the trailer but also to stabilize it. The tractor may have more than two axles or four wheels of which all axles may not be powered. This system covers combinations of car trailer/semi-trailer, truck-trailer/semi-trailer and tractor-trailers.

Tractor trailer combinations have become one of the most widely used freight vehicles and have been playing a crucial role in road transportation throughout the world. Thus, it makes it important for us to study the aspects of tractor trailer combinations related to safety of the system. Statistics show that tractor trailer combinations have been part of many fatal accidents over the years. According to the Large Truck and Bus Crash Facts provided by Federal Motor Carrier Safety Administration, a total of 64,100 crashes have been caused by tractor trailer combinations in 2013, of which 2,342 have been fatal [1].

In order to study the stability of tractor trailer combinations, it becomes necessary for us to understand the difference between a single vehicle unit and such combination vehicles (articulated vehicles) such as tractor trailer systems. The following points are some of the unique characteristics of articulated vehicles [2]:

1. In contrast to a single vehicle unit, articulated vehicles have a relatively small stability region and poor maneuverability primarily due to the length of such vehicle combinations.
2. The controllability and stability with respect to yaw and roll motion at high speeds are inferior.
3. There is a phase lag between the tractor movement and trailer movement. In addition to this, there is a larger lag between the driver's input and trailer response, which requires good driving skills to manage.
4. Rearward amplification refers to the ratio of peak lateral acceleration of the trailer at its C.G to the that of the tractor during lane changing/obstacle avoidance maneuvers. A higher value of rearward amplification means that there is a greater tendency for rollover.

The paper has been organized in the following way. The first section consists of an overview of various instabilities in a tractor-trailer system and their causes. In the second section, a brief literature review of the various control approaches adopted previously, and the corresponding control systems developed to cope up with these instabilities have been discussed. The third section consists of the basic modeling framework with equations describing lateral dynamics of the tractor-trailer system to identify the instability occurring in the system for different steering inputs. The final section consists of the conclusion and plan for future work.

Literature Review

There are various configurations possible for articulated vehicles depending on the type of hitch equipment, type of trailer and its dimensions and loading conditions. Various types of hitch equipment used and the classification of articulated vehicles depending on the type of leading vehicle and trailer have been extensively discussed and documented [3].These variable configurations significantly affect the stability of the tractor-trailer system; thus rendering the control system to be quite robust.

Any situation when there is a loss of control of the vehicle or the travel path of the vehicle is undesired, then that situation can be considered as an instability. Instabilities occurring in articulated vehicle with one articulation such as Tractor-semitrailer have been extensively studied by [4],[5] and can be divided into two types [6]:

1. Divergent instability:Occurs when the vertical force of the trailer at its hitch point exceeds a threshold value. An example of this is jackknifing, during which the hitch angle increases without any oscillations. Tractor jackknifing is commonly observed when there is relatively large sideslip angle in the rear axle of the tractor and the articulation angle is excessively high, which are caused due to the inertial

effect of the trailer, excessive steering during obstacle avoidance maneuvers or poor braking [7].
2. Unstable oscillatory motion: This instability refers to the oscillation of the trailer at high speeds, and is primarily caused due to low system damping [2]. An example of this phenomenon is trailer swing/snaking, which is caused due to loss of lateral force between trailer tire and the road. Trailer swing/snaking is commonly observed on low adhesion surfaces.

Divergent instability typically depends only on physical parameters of the towing vehicle and the vertical load on the hitch, whereas trailer snaking is dependent on parameters of both the tractor as well as the trailer. It is dependent on the mass of the trailer, location of C.G. of the trailer, moment of inertia of the trailer, cornering stiffness of the tractor and trailer tires, distance from the tractor's rear point to hitch point, wheelbase of the tractor and so on [8]. Because of the complex configuration of a tractor-trailer system, it is subject more than one of the aforementioned instabilities. Apart from this there is yet another common type of instability known as rollover instability, which is caused to excessive loading of the trailer as well as due to high location of C.G of the tractor – trailer system.

Several researchers have worked on controlling such instabilities. To control jackknifing researchers have explored methods such as active steering control, differential braking [9] as well as active braking control [7]. For example, researchers from National Taiwan University of Science and Technology developed a method to prevent jackknife for articulated vehicles using model reference adaptive control (MRAC) through differential braking [9]. Though several researchers have previously worked on jackknifing prevention through differential braking, there have been problems due to variations in level of loading of the trailer. Thus, in this paper [9] an adaptive control algorithm was employed. Their primary objective was to optimize the fifth wheel angle (hitch angle) to a desired value for various loading conditions. A 3 DOF model of a tractor semi – trailer system based on Lagrange equations of motion has been adopted to design the controller and perform simulation studies. A desired value of the fifth wheel angle was predicted using the steering wheel angle (δ) according to the method described in [10]. Basically, the idea is to pick any point on the tractor and treat it as the tracking point for the trailer and compute the appropriate fifth wheel angle. This value is compared with the actual fifth wheel angle, which is obtained as an output from the vehicle model, and the MRAC controller is used to apply differential braking to reduce the error between desired and actual states. A scaled model was constructed to validate the results obtained from computer simulations. It was concluded that MRAC controller could improve the fifth wheel angle response despite variations in loading conditions, thus making it a viable alternative for preventing jackknifing. Yet another approach was followed by researchers from Concordia University, where jackknifing was prevented through active braking control [7] by controlling/tracking the desired yaw rates of tractor and trailer. The results were evaluated using a co-simulation environment involving MATLAB/SIMULINK and TruckSim under two different maneuvers on a slippery road. A single tractor 4 DOF linear model is used as a reference, an active braking control scheme is developed and used in conjunction with a PID based yaw moment controller. The control scheme is used to reduce the error between the actual and desired yaw moments. It must be noted that in this paper [7], the PID parameters have been optimized and tuned for a particular vehicle and load configuration.

Yet another example of an instability is trailer snaking, during which the trailer unit oscillates behind the tractor unit due to uneven roads surface, wind gusts or driver inputs. To prevent this problem various passive systems have been used in the past such as the four-bar mechanism, damping systems, each having its own limitations which have been counteracted using active systems such as Direct Yaw Moment (DYM) controller. There are two types of DYM units to stabilize the lateral instabilities, one based on side-slip angles and another based on yaw rate control, the latter being more stable. Also, active braking systems have been developed, in which braking is applied on individual trailer wheels to develop different moments and eliminate undesired yaw movements. This system was effective during the lane change maneuvers for the tractor-trailer systems[11] [12]. The other approach is by use of steerable axles on the trailers, the self-steer system, which allows free angular movement of the trailer axles after a threshold value of moment at wheels is reached. Command steer in this system, the side-slip angle between the tractor and rear axle wheels is zero. In a pivotal bogie system, the fixed axles are replaced by ball race-based axles, allowing them to move in relation to the trailer body. Steerable steering systems are effective in reducing trailer swings at low and high speeds. The bogie system instills more sway at high speeds but reduces the sway during low speed cornering [3]. In [3], it is discussed how different controllers related to steering performance affect the swaying motion of the trailers. Path sensing systems have been developed that identify the lateral deviation of trailer C.G. from the centerline of the lane. It was observed that the linear control systems gave steering inputs that were smoother and with less flutter, indicating that the implementation of linear feedback control systems with forward feed systems to prevent or maintain trailer sway within desired limits.

To prevent rollovers passive systems have been deployed which are only useful for steady-state circular motion and are not effective when it comes to transient maneuvers[13],[14]. Two different approaches that were undertaken to combat the issue of roll stability, one relating to rollover index and the other relating to the state of the vehicle system. Since, the loading conditions keep changing, the location of C.G keeps varying, this in turn affects the roll dynamics (roll angle and roll time). Rollover index which indicates the likelihood of rolling is calculated. This value is then used as a trigger to initiate the differential braking (larger amount of brake force on the outer wheels compared to the inner) to prevent rolling. The sprung mass is calculated using the suspension deflection while the C.G. approximation is done using lateral acceleration measurement and speed. All these parameters are constantly measured by the control unit to calculate rollover index and trigger the control algorithm as and when the rollover index value crosses the threshold [13]. Another method to prevent rollover is through implementing control strategy for active steering of semi-trailer. In this method, path following control is implemented and the fifth wheel of the tractor is the point to followed by the trailer end. This strategy is modified to incorporate the roll stability by passing the parameters such as lateral acceleration, wheel rates, roll rates, vehicle parameters to the LQR (Linear Quadratic Regulator) which uses the Kalman filter for estimation of the states and generate control inputs for feedback. These inputs were used to calculate the lateral load transfer ratio (LTR=1 vehicle rollover) for every wheel as it is closely related to the rollover. When an increase in this value was observed the trailer is steered towards side with higher LTR while maintaining limited deviation from its path and avoiding the rollover [14].

While it is important to maintain stability of a tractor – trailer system, one of the major challenges in this field is designing a control strategy to balance between the stability requirements and handling performance, as they often have conflicting objectives due to the limited number of control channels available. In this paper, we aim to study the lateral dynamics of the tractor trailer system using a 4 – DOF single track model, and use this model to observe instabilities introduced due to different steering inputs such as step, impulse and sine wave.

Vehicle Data

After reviewing the common issues causing instabilities in tractor-trailer combined system, a study is conducted to focus on simulating the yaw-plane dynamics of the tractor-trailer combination. To perform open-loop simulations, a 3-axle tractor-trailer system is selected. The figure below shows a schematic of 3-axle tractor trailer combination with its dimensional notations.

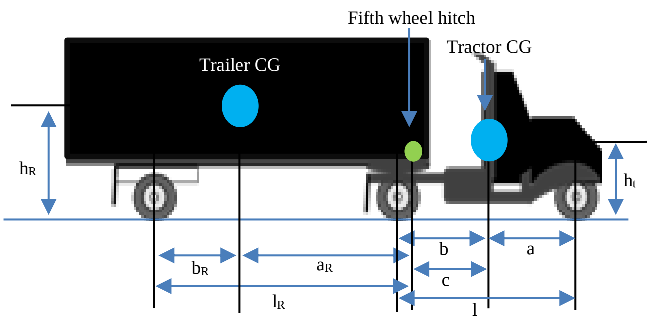

Figure 1. Tractor-semitrailer dimensions

Table 1. Vehicle dimensions

Distance of front axle of tractor from tractor CG (a)	1175 mm
Distance of rear axle of tractor from tractor CG (b)	2310 mm
Distance of fifth wheel hitch from tractor CG (c)	1860 mm
Wheelbase of tractor (l)	3485 mm
Distance of fifth wheel hitch from trailer CG (a_R)	5090 mm
Distance of trailer axle from trailer CG (b_R)	2305 mm
Distance of trailer axle from fifth wheel hitch (l_R)	7395 mm
Height of tractor CG from ground plane (h_t)	1100 mm
Height of trailer CG from ground plane (h_R)	1650 mm

Modeling Framework

The schematic of tractor-trailer system is shown in Figure 2. It is the simplest model that helps us understand both lateral as well as yaw instabilities in the system. The modelling process starts with fixing the reference frame. The equations for motion of the body are then formulated with respect to these reference frames. In this model, we have selected three reference frames for studying the motion of the system in X-Y plane. First is the O-X-Y coordinate system which is the inertial reference frame, second, the G-x-y coordinate system which is fixed in the body of tractor and third is the G_R-x_R-y_R coordinate system which is fixed in the body of the trailer.

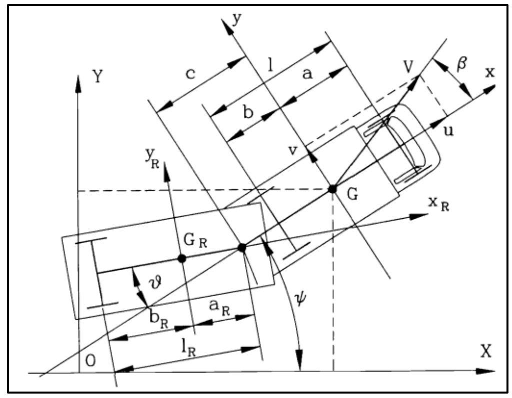

Figure 2. Schematic of tractor-trailer combination showing reference frames and generalized coordinates. [4]

$$T = \frac{1}{2}m_T V_G^2 + \frac{1}{2}m_R V_{GR}^2 + \frac{1}{2}J_T\dot{\psi}^2 + \frac{1}{2}J_R\left(\dot{\psi} - \dot{\theta}\right)^2 \quad (1)$$

In this model, a 3-axle tractor-trailer system is selected to perform simulations and understand the dynamics of this complex multi-body system. Tires of each axle are assumed to be single tires that have stiffness equal to the sum of stiffnesses of all the tires on that axle. δ is the steering wheel angle of the front tractor wheels. Only front axle of the tractor can be steered all the other axles are non-steerable. The steering angle, δ, hitch angle, ϴ, and the body side-slip angle, β, all the angles are assumed to be small and hence the system is linearized. The mass of tractor is denoted by m_T and that of the trailer is denoted by m_R and the corresponding moments of inertias are J_T and J_R respectively. In the figure, we can see that, the tractor's yaw movement is given by ψ and that of the trailer relative to tractor is given by ϴ. The distance of centre of mass of the tractor from front and rear axle of the tractor is denoted by a and b. c is the distance of fifth wheel hitch point from the centre of mass of the tractor. For the trailer system, a_R is the distance of the hitch point from the trailer centre of mass and b_R is the distance of trailer axle from the trailer centre of mass. The velocity of centre of mass of tractor is v_G and that of the trailer is v_{GR}. Modelling and simulation of the tractor-trailer system is done under the above conditions and assumptions.

A single rigid body has six degrees of freedom which requires six equations to describe the motion of the body. Since, the tractor-trailer system has more than one rigid body to deal with, the description of motions of these bodies become tedious using the Newton's laws of motion. Hence, for studying multi-body dynamics, Lagrange's analytical method is used to describe the motion of the bodies. First, the total kinetic energy of the system is calculated as shown below.

The equations of motion obtained in the form of Lagrange's equation are[4], [15]:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = Q_i \quad (2)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{X}}\right) - \frac{\partial T}{\partial X} = Q_X \quad (3)$$

where the generalized coordinates q_i are X, Y, ψ and ϴ and Q_i are the generalised forces corresponding to the generalized coordinates. Since, we have four generalised coordinates, we can define the equations of motion of the tractor-trailer system in four equations as given below.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Y}}\right) - \frac{\partial T}{\partial Y} = Q_Y \quad (4)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\psi}}\right) - \frac{\partial T}{\partial \psi} = Q_\psi \quad (5)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} = Q_\theta \quad (6)$$

For us, since, the study of the project is focused on lateral dynamics of the tractor-trailer combination, we are neglecting the equation of motion obtained by using X as a generalized coordinate.

On solving these equations, we obtain a set of equations that describe the motion of the tractor-trailer system. But there is a scope for further simplification in these equations. Since, the values of ϴ and δ are well within the range for which the linearization of trigonometric function is possible, we further reduce the Lagrange's equations to its simplest version. Thus, the equations of motion obtained after linearization are given below:

$$m(\dot{v} - V_R) - ma_R(c + a_R)\dot{r} + \dot{m}_R a\ddot{\theta} = Q_Y \quad (7)$$

$$J_1\dot{r} - J_2\ddot{\theta} - m_R(c + a_R)(\dot{v} + V_R) - m_R a_R \dot{V}\theta = Q_\psi \quad (8)$$

$$J_3\ddot{\theta} - J_2\dot{r} + m_R a_R(\dot{v} + V_R) + m_R a_R \theta \dot{V} = Q_\theta \quad (9)$$

Considering that the driver drives the tractor-trailer smoothly without abrupt steering inputs. In such case, the side-slip angle can be given as:

$$\alpha_i = \theta + \beta - \frac{r}{V}(c + l_i) + \frac{\dot{\theta}}{V} l_i - \delta_i \quad (10)$$

Also, for smooth driving, it can be assumed that there is a linear relation between the lateral force and the side-slip angle. Thus, assuming that there is no lateral load transfer and no external forces and moments are acting on the system, on substituting the value of α we can obtain the following equations for generalized forces:

$$Q_Y = Q_{Y,\beta}\beta + Q_{Y,r}r + Q_{Y,\dot{\theta}}\dot{\theta} + Q_{Y,\delta}\delta + F_{ye} + F_{ye,R} \quad (11)$$

$$Q_\psi = Q_{\psi,\beta}\beta + Q_{\psi,r}r + Q_{\psi,\dot{\theta}}\dot{\theta} + Q_{\psi,\delta}\delta + M_{ze} + M_{ze,R} - (c + a_R)F_{ye,R} \quad (12)$$

$$Q_\theta = Q_{\theta,\beta}\beta + Q_{\theta,r}r + Q_{\theta,\dot{\theta}}\dot{\theta} + Q_{\theta,\delta}\delta - M_{ze,R} + a_R F_{ye,R} \quad (13)$$

here,

$$Q_{Y,\beta} = Y_\beta - \sum_{\forall i_R} C_i \quad (14)$$

$$Q_{Y,r} = Y_r + \frac{1}{V}\left(\sum_{\forall i_R}(c + l_i)C_i\right) \quad (15)$$

$$Q_{Y,\dot{\theta}} = -\frac{1}{V}\left[\sum_{\forall i_R} l_i C_i\right] \quad (16)$$

$$Q_{Y,\theta} = -\sum_{\forall i_R} C_i \quad (17)$$

$$Q_{Y,\delta} = Y_\delta \quad (18)$$

$$Q_{\psi,\beta} = N_\beta + \sum_{\forall i_R}\{M_{1,i}\} \quad (19)$$

$$Q_{\psi,r} = N_r - \frac{1}{V}\left[\sum_{\forall i_R}\{(c + l_i)M_{1,i}\}\right] \quad (20)$$

$$Q_{\psi,\dot{\theta}} = \frac{1}{V}\left[\sum_{\forall i_R}\{l_i M_{1,i}\}\right] \quad (21)$$

$$Q_{\psi,\theta} = \sum_{\forall i_R}\{M_{1,i}\} \quad (22)$$

$$Q_{\psi,\delta} = N_\delta \quad (23)$$

$$Q_{\theta,\beta} = -\sum_{\forall i_R}\{M_{2,i}\} \quad (24)$$

$$Q_{\theta,r} = \frac{1}{V}\left[\sum_{\forall i_R}\{(c + l_2)M_{2,i}\}\right] \quad (25)$$

$$Q_{\theta,\dot{\theta}} = -\frac{1}{V}\left[\sum_{\forall i_R}\{l_i M_{2,i}\}\right] \quad (26)$$

$$Q_{\theta,\theta} = Q_{\theta,\beta} \quad (27)$$

$$Q_{\theta,\delta} = 0 \quad (28)$$

and,

$$M_{1,i} = (c + l_i)C_i + (M_{zi})_\alpha + h_{G_R}(f_0 + KV^2)C_i \quad (29)$$

$$M_{2,i} = l_i C_i + (M_{zi})_\alpha + h_{G_R}(f_0 + KV^2)C_i \quad (30)$$

On substituting the values of Q_Y , Q_ψ, Q_Θ in equations (7), (8) and (9), we can now write the equations of motion in the matrix form as:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\} \quad (31)$$

Elements of matrices [M], [C], [K] and {F} are given in appendix. This matrix equation is used to build a Simulink model for tractor-trailer system. Subsequently, simulations were done to study the dynamics of this articulated vehicle system.

Simulation Results

The tractor-trailer system modeled above is simulated for lateral dynamics in this paper. The tractor-trailer system is simulated for three operating scenarios. One, for a step steering input where it is assumed that the system is stimulated by giving one-degree steering input through tractor steering wheel and the response of the system is observed. Second, for a pulse steering input where again one-degree steering input is given to the system, but in this case, first, the steering wheel is given a one-degree rotation in one direction, after 0.5 seconds a rotation is given in opposite direction. And the third scenario is when the tractor is given periodic steering input in the form of sine wave.

The system was simulated at different constant speeds and its response was observed. The stimulation to the system is given at one-second timestep. In this paper, the results obtained for step steering input at a speed of 55 mph are published since, it is the average operating speed for tractor-trailers on interstate highways in the US [16].

Step Steering Input

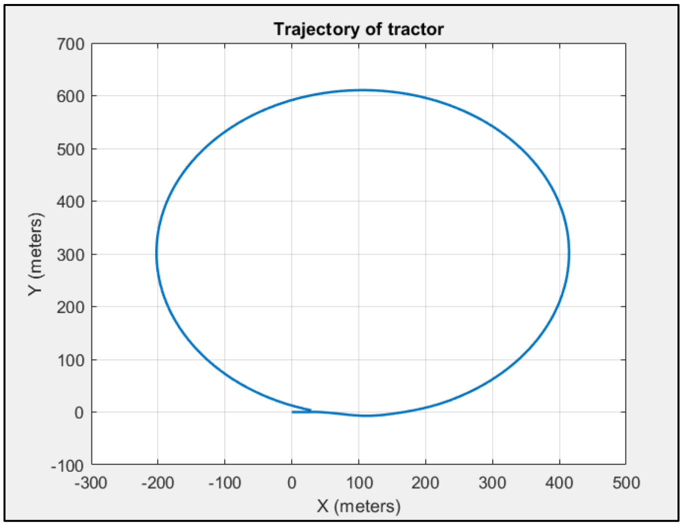

Figure 3. Trajectory of tractor in X-Y plane.

Figure (3) shows the trajectory of the tractor in X-Y plane when a step steering input is given to the system. We can see that the tractor traces an elliptical path(trajectory) in locked controls operating conditions.

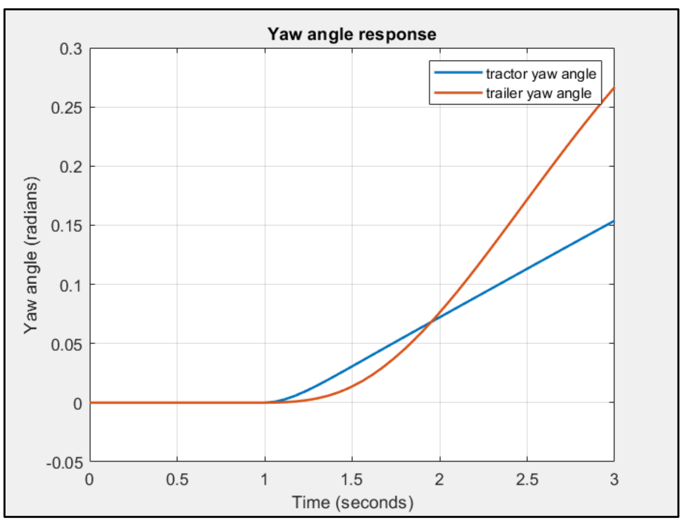

Figure 4. Yaw response for both tractor and trailer due to step steering stimulation.

This figure shows the yaw response for both tractor and trailer individually. We can observe from figure (4) that due to step steering input, the tractor which initially had straight horizontal motion has now changed its orientation. The yaw angle of the tractor continues to increase with time since a constant steering input is given to it which is usually referred to as 'locked control' model simulation. For trailer, since none of its axles are steerable, there is a delay in trailer yaw movement response to the step input given to the system. After the initial delayed response, the trailer's yaw angle becomes more than that of the tractor's yaw angle after a certain timestep as we can see from figure (4). This is due to the high moment of inertia of the trailer and its limitation of not being able to trace the trajectory of tractor exactly because of non-steerability. Thus, the trailer oscillates and takes some time for stabilization.

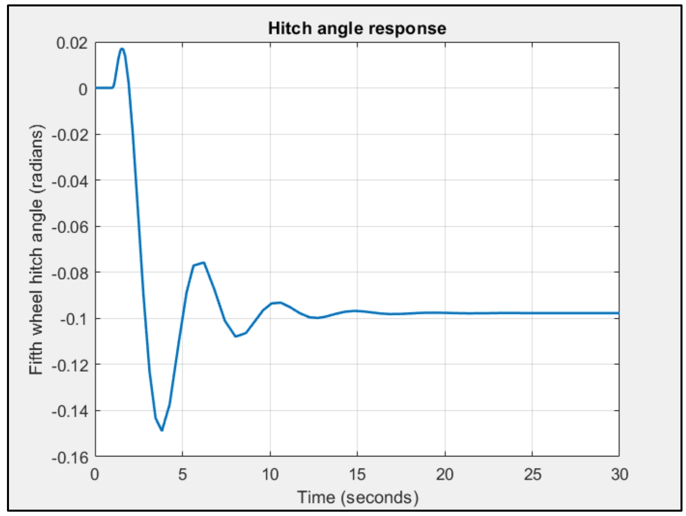

Figure 5. Hitch angle response due to step steering stimulation.

As we discussed already, there is a delay in yaw angle response of trailer to the step steering input initially and then it oscillates to later stabilize after certain timestep. Figure (5) justifies this conclusion of ours. The fifth wheel hitch first has a positive angle hence, the yaw angle of trailer assumes a value (ψ - Θ) which justifies the delay in yaw response of trailer as compared to tractor. After a timestep, the hitch angle assumes a negative value and so the yaw angle of trailer becomes (ψ + Θ) which explains why the yaw angle of trailer is greater than that of tractor, as we concluded previously. The hitch angle then oscillates for some time and later stabilizes. In figure (5), we can see the hitch angle response stabilizes at around 19 seconds. Due to the step input, the hitch angle assumes a constant angle depending on the direction of motion of the tractor.

Conclusion/Summary

In this paper, it has been attempted to provide a brief overview on the most common instabilities observed in tractor-trailer systems such as jackknifing, trailer snaking, rollover etc. Previous research efforts undertaken to control these instabilities have also been discussed through a detailed literature review. Following that, the basic equations of motion that govern the lateral motion of a tractor trailer system were derived using Lagrange equations. Using these equations, a mathematical model of the system was built, and basic simulations pertaining to lateral dynamics have been done. The trailer is unable to instantly follow the tractor when a change in the system's state is introduced and induces instability into the system before the trailer attains the same state as the tractor.

Plan for Project 2

Now that we have developed a basic model describing the system and understood some critical shortcomings by simulating, future work would be to implement control strategy to control yaw instabilities such as jackknifing and trailer snaking. Differential braking, active steering and active braking control are the commonly implemented strategies to control instabilities in such systems, our aim is to implement one of these strategies to the tractor-trailer model that we have developed in this paper.

Accountability Statement

The work was evenly distributed amongst the three students as - Literature review, Modeling and Simulation. Each of us read two common papers to get a basic understanding of the topic, following which Sanket Milind Bachuwar was responsible for doing the Literature review, Nayan Prashant Deshmukh was responsible for deriving the equations describing the dynamics and Adithya Baburaj was responsible for developing the SIMULINK model. All the three students participated in interpreting the results from the simulation.

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Definitions/Abbreviations

m_T, m_R	Mass of the tractor and trailer respectively
V_G, V_{GR}	C.G. velocity of tractor and trailer respectively
J_T, J_R	Moment of Inertia of tractor and trailer respectively
ψ	Yaw angle of tractor
θ	Hitch angle
a, b	Distance from C.G to front and rear axle of tractor respectively
c	Distance from C.G to Hitch point
l, l_R	Wheelbase of tractor and trailer respectively
a_R, b_R	Distance from C.G to front and rear axle of trailer respectively
$Q_X, Q_Y, Q_\psi, Q_\theta$	Generalized forces in each coordinate

Appendix

$$[M]=\begin{bmatrix} m & -m_R(c+a_R) & m_R a_R \\ -m_R(c+a_R) & J_1 & J_2 \\ m_R a_R & -J_2 & J_3 \end{bmatrix} \quad [K]=\begin{bmatrix} 0 & 0 & -Q_{Y,\theta} \\ 0 & 0 & -Q_{\psi,\theta} \\ 0 & 0 & -Q_{\theta,\theta} \end{bmatrix}$$

$$[C]=\begin{bmatrix} -\frac{Q_{Y,\beta}}{V} & mV - Q_{Y,r} & -Q_{Y,\dot{\theta}} \\ -\frac{Q_{\psi,\beta}}{V} & -m_R V(c+a_R) - Q_{\psi,r} & Q_{\psi,\dot{\theta}} \\ -\frac{Q_{\theta,\beta}}{V} & m_R a_R - Q_{\theta,r} & -Q_{\theta,\dot{\theta}} \end{bmatrix} \quad \{F\}=\begin{bmatrix} Q_{Y,\delta} \\ Q_{\psi,\delta} \\ 0 \end{bmatrix}[\delta] \quad \{x\}=\begin{bmatrix} y \\ \psi \\ \theta \end{bmatrix}$$