Control of yaw instabilities in tractor – semitrailer system using Model Reference Adaptive Control

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Abstract

This paper investigates the use of differential braking to control yaw instabilities occurring in a tractor–semitrailer system. Tractor-trailer systems are one of the most common modes of freight transportation and is subject to frequent loading and unloading. Magnitude of the differential braking force varies with the level of loading, to account for this, a Model reference adaptive control (MRAC) has been used in this paper. A reference 4 – DOF tractor trailer model, MRAC and differential braking control were developed in SIMULINK and co-simulated with TruckSim for a two-axle tractor with a one axle trailer. The simulation was run for various loading levels of the trailer from 0-9000 kg, and the variation in yaw rate of tractor and trailer, hitch rate and lateral offset of the vehicle were studied.

Introduction

Tractor-trailer systems are the most common mode of freight transportation throughout the world. Thus, making it imperative to study the common instabilities in such systems that lead to accidents. Due to its huge size, accidents caused by such system tend to be catastrophic in nature and lead to fatalities. Studies have shown that more than 64,100 crashes occurred in 2013, due to tractor trailer systems and about 5% of them have been fatal [1]. The most common instabilities observed in a tractor trailer system, and various control strategies used to tackle such instabilities have been discussed [2].

Briefly, there are three common instabilities observed in tractor trailer systems, namely, jackknifing, trailer swaying and rollover. Tractor/trailer jackknifing is due to yaw instability and is one of the primary causes of traffic accidents. During jackknifing the tractor and trailer are bent into a V-shape by an uncontrolled skidding movement [3]. Factors which influence jackknifing include: the mass of the trailer, location of C.G. of the trailer, moment of inertia of the trailer, cornering stiffness of the tractor and trailer tires, distance from the tractor's rear point to hitch point and wheelbase of the tractor [4]. During trailer swaying, the trailer oscillates behind the tractor unit due to uneven roads surface, wind gusts or driver inputs [2]. Rollover is caused due to excessive loading of the trailer as well as due to high location of C.G of the tractor – trailer system. Some of the most common control strategies to control jackknifing and trailer swaying include differential braking control [5] [6], differential braking [7] and differential steering of trailers [8].

This paper concentrates on the instabilities occurring in the yaw plane such as jackknifing and trailer swaying and doesn't take into consideration roll instability. Of all the control strategies used for controlling yaw instabilities, differential braking was found to be one of the most successful methods. Although several researchers have successfully implemented various differential braking control algorithms, it has been observed that these control strategies are very sensitive to the parameters of the tractor trailer system being considered, especially the loading levels. Since loading levels continuously keeps changing in tractor trailer systems, it becomes mandatory to develop control systems which are robust to change in loading levels of the trailer. These parametric variations lead to the need for adaptive control systems. The objective of this paper is to develop a differential braking control-based strategy that uses model reference adaptive control (MRAC) to adapt to the changes in trailer loading levels.

The paper has been structured in the following way, the first section consists of a brief introduction on adaptive control systems, MRAC and various methods employed to achieve MRAC. The second section describes in detail the adaptive control algorithm that has been used in this paper called the MIT Rule. Following which the third section explains the closed loop adaptive control system developed, which describes the reference model, plant model (TruckSim), the differential braking control strategy and MRAC controller that have been used in this paper. Finally, the fourth section provides the results of the simulation and conclusion.

Adaptive Control Systems

Adaptive control systems evolved from the need to control partially known systems. It is quite common for designers to be unaware of the parameters that govern a real process, as these characteristics are subject to change with time. As compared to a simple fixed gain PID control, adaptive controllers enable us to effectively handle parametric changes and environmental variations. Quantitatively, the aim of any control algorithm would be to determine the input $u(t)$ to keep the error $e(t) = y_m(t) - y(t)$ within prescribed values. If $y_m(t)$ is constant, then the problem is known as regulation, and if $y_m(t)$ is a function of time, the problem is known as tracking [9]. If the characteristics of the plant are already known, the solution is to design a controller to stabilize the feedback loop around the operating point or for the latter case which can minimize a cost function based on the error $e(t)$. Many analytical techniques involving quadratic performance regulators have been developed as a control solution when the differential equation of the plant is linear, and the parameters are known a priori. When the characteristics of the plant are unknown, then both regulating and tracking problems can be seen as adaptive control problems, and the suitable controllers for such systems fall into two classes: MRAC and self-tuning regulators (STR).

MRAC adjusts the controller parameters so that the output of the plant traces the output of a reference model having the same reference input. A model reference adaptive controller consists of the following components: 1. Reference model 2. Plant model 3. Controller 4. Adaptive mechanism. Reference model is used to give an ideal response to a given reference input. Often the reference model is a simplified linear model. Plant model is the actual model whose parameters are time varying and unknown. Controller is used to give the appropriate input to the plant model to control it. Adaptive mechanism is used alter the controller parameters so that the plant model can track the reference model for any change in parameters of the actual model or change in environmental conditions [10]. Methods such as MIT Rule, Lyapunov theory and augmented errorcan have been used to develop an adaptive mechanism. This paper has developed an adaptive mechanism using the MIT rule.

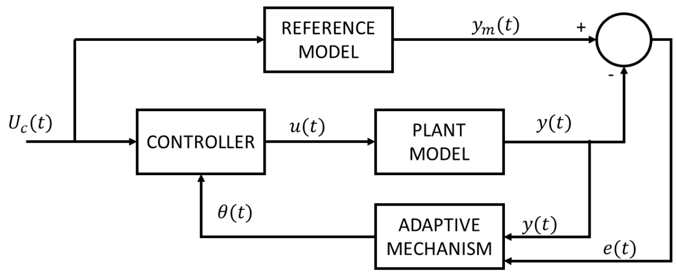

Figure 1 Model reference adaptive control

MIT Rule

MIT rule was developed in 1960s by researchers at the Massachusetts Institute of Technology and was then used to design autopilot system for aircrafts [10]. Figure 1 shows a basic layout of a model reference adaptive control using MIT Rule.

In this rule, a cost function is defined as:

$$J(\theta) = \frac{e^2}{2} \tag{1}$$

where e is the error between the output of the reference and plant model and is θ the adjustable parameter. Parameter is adjusted in such a way to minimize the cost function. Therefore, any positive change in θ results in a negative rate of change in J, thus we have

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} \tag{2}$$

where γ is a positive quantity known as the adaptation gain of the controller. Substituting equation (1) in (2) we get,

$$\frac{d\theta}{dt} = -\gamma e \left(\frac{\partial e}{\partial \theta}\right) \tag{3}$$

Where $\frac{\partial e}{\partial \theta}$ is known as the sensitivity derivative of the system and indicates how the error changes with change in θ.

Let the plant be described by the transfer function $KG(s)$, where K is an unknown parameter and $G(s)$ is a second order transfer function. Let the reference model be given by the transfer function $G_m(s) = K_0 G(s)$, where K_0 is a known parameter. We know that the error function can be written as:

$$E(s) = Y(s) - Y_m(s) \tag{4}$$

Also, we know that,

$$Y(s) = KG(s)U(s) \tag{5}$$

$$Y_m(s) = K_0 G(s) U_c(s) \tag{6}$$

Substituting (5) and (6) in (4),

$$E(s) = KG(s)U(s) - K_0 G(s) U_c(s) \tag{7}$$

Let us define a control law as:

$$u(t) = \theta * u_c \tag{8}$$

Using (7) and (8) and taking partial derivative we get,

$$\frac{\partial E(s)}{\partial \theta} = KG(s)U_c(s) = \frac{K}{K_0} Y_m(s) \tag{9}$$

(4) becomes,

$$\frac{d\theta}{dt} = -\gamma e \frac{K}{K_0} y_m = -\gamma' e y_m \tag{10}$$

where $\gamma' = \gamma \frac{K}{K_0}$.

Equation (10) when integrated with respect to time gives us the adjusting parameter. In this paper, this method has been applied to control jackknifing for a tractor trailer system for varying loading conditions of the trailer. The vehicle data used for both the reference and plant model is given below.

Vehicle Data

The vehicle data used for the model is shown below:

Table 1 Vehicle dimensions

Distance of front axle of tractor from tractor CG (a)	1175 mm
Distance of rear axle of tractor from tractor CG (b)	2310 mm
Distance of fifth wheel hitch from tractor CG (c)	1860 mm
Wheelbase of tractor (l)	3485 mm
Distance of fifth wheel hitch from trailer CG (a_R)	5090 mm
Distance of trailer axle from trailer CG (b_R)	2305 mm
Distance of trailer axle from fifth wheel hitch (l_R)	7395 mm
Height of tractor CG from ground plane (h_t)	1100 mm
Height of trailer CG from ground plane (h_R)	1650 mm

Linear tractor-trailer model

A 4-DOF system for tractor-trailer combination is modeled to study the yaw plane dynamics. The schematic diagram of the system is shown in Figure 2. In this model, a 3-axle tractor-trailer system is selected to perform simulations and understand the dynamics of this complex multi-body system. Tires of each axle are assumed to be single tires that have stiffness equal to the sum of stiffnesses of all the tires on that axle. Only front axle of the tractor can be steered all the other axles are non-steerable. The notations used for all dimensional and performance parameters are tabulated in the appendix. A single rigid body has six degrees of freedom which requires six equations to describe the motion of the body. Since, the tractor-trailer system has more than one rigid body to deal with, the description of motions of these bodies become tedious using the Newton's laws of motion. Hence, for studying multi-body dynamics, Lagrange's analytical method is used to describe the motion of the bodies. Equations of motion are obtained in terms of generalized coordinates q_j which are X, Y, ψ, θ and Q_j are the generalized forces corresponding to the generalized coordinates. Since, the study of the project is focused on lateral dynamics of the tractor-trailer combination, equation of motion obtained by X coordinate is neglected. Thus, three equations of motion for tractor-trailer combination along three generalised coordinates – Y, ψ and θ are as follows:

$$m(\dot{v}-V_R)-ma_R(c+a_R)\dot{r}+m_R a\ddot{\theta}=Q_Y \qquad (11)$$

$$J_1\dot{r}-J_2\ddot{\theta}-m_R(c+a_R)(\dot{v}+V_R)-m_R a_R\dot{V}\theta=Q_\psi \qquad (12)$$

$$\mathrm{J}_3\ddot{\theta}-\mathrm{J}_2\dot{\mathrm{r}}+\mathrm{m_R a_R}(\dot{\mathrm{v}}+\mathrm{V_R})+\mathrm{m_R a_R}\theta\dot{\mathrm{V}}=\mathrm{Q}_\theta \qquad (13)$$

These equations are used for modeling of tractor-trailer system.

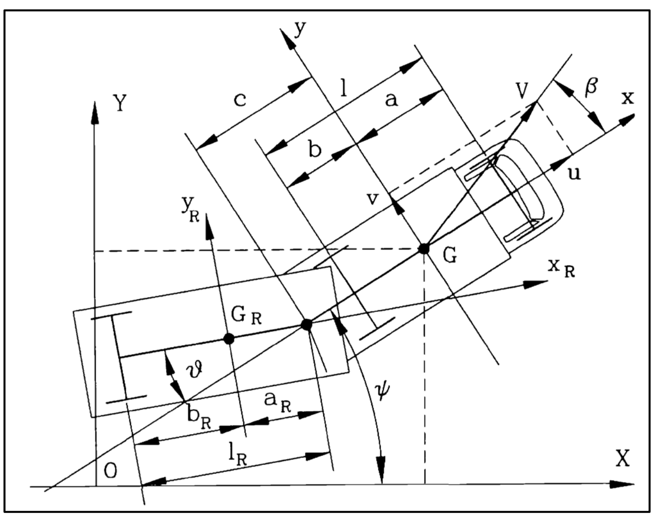

Figure 2 Schematic of tractor-trailer combination showing reference frames and generalized coordinates. [11]

Closed-loop Adaptive Control System Design

As discussed in the earlier sections, an adaptive controller is used to implement differential braking for yaw rate control of the tractor-trailer system. The plant model in this setup is a two-axle tractor with a single-axle trailer. A reference model is used to generate desired model response which is then compared with the response of plant model to obtain errors. These errors are fed into the MRAC controller which uses an adaptive mechanism to compute the adjusting parameter (**θ**). The adjusting parameter in-turn dictates the differential force needed to regulate the yaw rate of the vehicle close to the desired yaw rate.

Plant Model

TruckSim Version 8.1 was used to simulate the vehicle model. A two-axle tractor with a single axle trailer system was chosen and the dimensions were changed to match the dimensions in Table 1. The model takes wheel cylinder brake pressures as input and road wheel steer angle, longitudinal velocity, yaw rate of tractor and yaw rate of trailer are obtained as outputs from the vehicle model.

Reference Model

Vehicle differential control systems are designed so as to track the desired responses that are obtained from the reference model. The reference model used in this stability control system, gives tractor yaw rate and hitch articulation angle as outputs. Actual road wheel steer angle and longitudinal velocity of the tractor are fed to the reference model as inputs. The model computes steady state desired yaw rate for tractor and desired hitch articulation angle for a given value of road wheel angle and longitudinal velocity at any given instant. These desired values are then compared to the actual values to generate errors that in-turn actuates the control model. The steady state yaw rates and hitch articulation angles are obtained from the steady state equations of motion.

$$-Q_{Y,\beta}\beta+[mV-Q_{Y,r}]r-Q_{Y,\theta}\theta=Q_{Y,\delta}\delta \qquad (14)$$

$$-Q_{\psi,\beta}\beta-[m_R(c+a_R)V+Q_{\psi,r}]r-Q_{\psi,\theta}\theta=Q_{\psi,\delta}\delta \qquad (15)$$

$$-Q_{\theta,\beta}\beta+[m_R a_R V-Q_{\theta,r}]r-Q_{\theta,\theta}\theta=Q_{\theta,\delta}\delta \qquad (16)$$

From these equations, we can find yaw rate in terms of road wheel steer angle using Cramer's rule. Therefore, using this simple computation the reference model generates steady state yaw rates for any input of road wheel steer angle and longitudinal velocity.

The hitch articulation angle is generated in a similar manner as yaw rate using the steady state equations. The expression obtained is then differentiated to obtain the hitch articulation rate. The desired yaw rate for trailer is then obtained as the difference between the tractor yaw rate and hitch articulation rate.

Model Reference Adaptive Controller (MRAC)

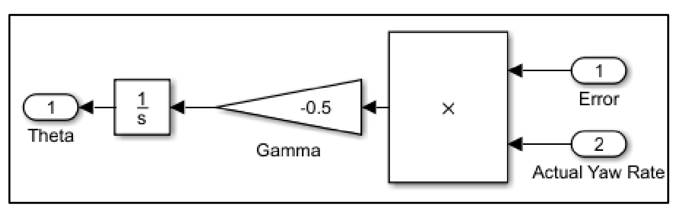

Figure 3 Simulink model of MRAC

Figure 4 shows the Simulink block diagram of the MRAC.As discussed in the previous sections of the paper, this MRAC controller is designed based on the MIT rule. The MRAC controller gets actual trailer yaw rate and error between actual and desired trailer yaw rate as input. Gamma is the control parameter or controller gain referred to as 'Adaptation gain' which needs to be tuned to obtain acceptable performance of the vehicle.

Differential Braking Strategy

The control strategy is implemented using a high level and low-level control. The MRAC and PD controllers are used to calculate the required corrective yaw moments. The application of corrective yaw moments is achieved through brake pressure distribution on the wheels of the tractor and trailer (differential braking). The steering angle from TruckSim is used as input to the reference model to generate the desired yaw rate for the tractor and fifth angle wheel response. Then the derivative of the fifth wheel angle is used to calculate the desired trailer yaw rate. The actual yaw rate for tractor and trailer from Trucksim is used to calculate the tractor and trailer yaw rate errors. The trailer yaw rate error and actual yaw rate is fed to the MRAC controller to obtain adaptive control variable. The differential braking control unit uses the error of the tractor and trailer yaw rates to generate required corrective yaw moments for individual wheels. Two PID controllers are used to achieve this, and the PID controllers use the MRAC's output to generate corrective yaw moments. The output from the differential braking control is fed to the brake actuation module, from where the corrective brake pressures are applied to the plant model.

Torque distribution strategy

The torque distribution strategy is formulated in consideration of two cases, understeer ($e_\gamma < 0$) and oversteer ($e_\gamma > 0$), it could be seen that positive corrective yaw moment yields higher yaw rate, while negative yaw moment would lower yaw rates. Thus, for positive yaw rates ($e_\gamma > 0$), negative yaw moment needs to be applied and vice-versa. The braking strategy is formulated to reduce the yaw rate error magnitudes [5].

Table 2. Differential braking control strategy [5]

Tractor			Trailer		
γ_1	$e_{\gamma 1}$	Braked Wheels	γ_2	$e_{\gamma 2}$	Braked Wheels
$\gamma_1 > 0$	$e_{\gamma 1} > 0$	R1	$\gamma_2 > 0$	$e_{\gamma 2} > 0$	R3
	$e_{\gamma 1} < 0$	L2		$e_{\gamma 2} < 0$	L3
	$e_{\gamma 1} = 0$	-		$e_{\gamma 2} = 0$	-
$\gamma_1 < 0$	$e_{\gamma 1} > 0$	R2	$\gamma_2 < 0$	$e_{\gamma 2} > 0$	R3
	$e_{\gamma 1} < 0$	L1		$e_{\gamma 2} < 0$	L3
	$e_{\gamma 1} = 0$	-		$e_{\gamma 2} = 0$	-
$\gamma_1 = 0$	$e_{\gamma 1} > 0$	L2	$\gamma_2 = 0$	$e_{\gamma 2} > 0$	R3
	$e_{\gamma 1} < 0$	R2		$e_{\gamma 2} < 0$	L3
	$e_{\gamma 1} = 0$	-		$e_{\gamma 2} = 0$	-

γ – yaw rate, e_γ – yaw rate error, L – left wheels and R – right wheels.

Results

The model was simulated for a double lane change procedure at a speed of 100 kmph, with a road friction coefficient μ=0.7. The model was simulated for both with and without MRAC at various loading levels. From a basic research it was understood that the maximum loading for a two-axle tractor with a single axle trailer system was 9000 kg. So, varying load levels from 0-9000 kg were chosen and the control strategy was evaluated. Results have been provided for two distinct loading levels of 2250 kg and 3375 kg on the trailer. It must be noted that this load is in addition to the weight of the trailer. Simulation results for trajectory of the tractor, yaw rate of tractor and trailer and hitch articulation rate are plotted for various loading conditions.

Tractor trajectory for various trailer loads

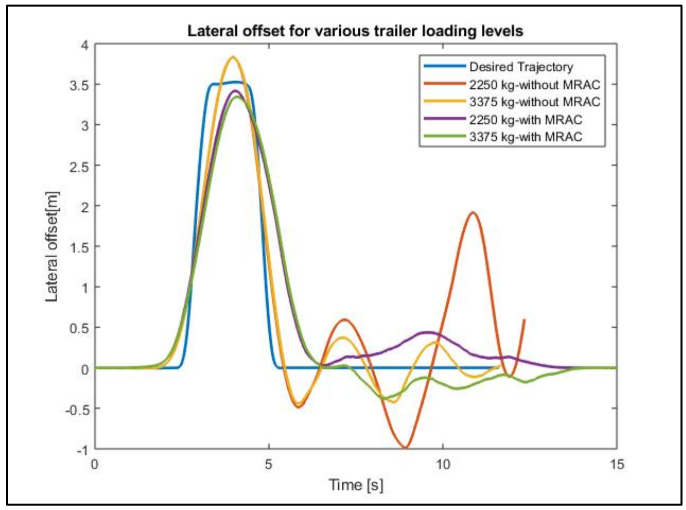

Figure 4 Lateral offset for various trailer loading conditions

Figure 4 shows plots for trajectory of tractor for two different payloads – 2250 kg and 3375 kg with and without MRAC. From the figure we can see that the system oscillates violently at speeds of 100 kmph for both the loading conditions when it is not controlled by the MRAC controller. But these oscillations are significantly reduced when simulated with MRAC along with the differential braking strategy. Thus the controller is able to control the yaw rate of the vehicle near the desired yaw rate and stabilize the system quickly.

Yaw rate of tractor for various trailer loads

From Figure 4, it was evident that the MRAC controlled the tractor yaw rate optimally to improve its performance on the double lane change maneuver. Figure 5 shows the yaw rate of tractor for various trailer loading conditions at 100 kmph. From this plot we can see that the tractor yaw rate oscillates with higher amplitudes when it is not controlled by MRAC controller. The system with 2250 kg without MRAC goes unstable. The system with MRAC controller for both the payloads almost gives similar performance which validates the application of MRAC control on the system. Depending on the deviation from the desired yaw rates, the MRAC controller generates corresponding adjusting parameter which in-turn generates the required differential braking force and controls the system. It is easy to observe this effect from Figure 5. For instance, if we compare for tractor yaw rate with 2250 kg payload with and without MRAC controller, we can see initially, when the yaw rates are within the limits of the desired ones, both the tractors almost have similar values of yaw rates but once, the yaw rate of tractor starts deviating from the desired ones, the MRAC controller brakes the wheels as per the control strategy to make the tractor follow the desired

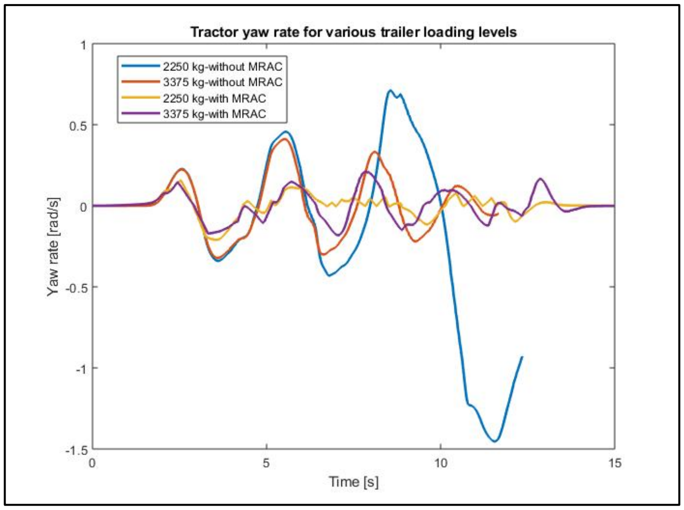

Figure 5 Tractor yaw rate for various trailer loading conditions

yaw rate values. In case of the one without controller, the yaw rates go on increasing as the tractor maneuvers through the DLC test track and the system becomes unstable after a certain threshold which depends on the friction conditions of the road surface.

Yaw rate of trailer for different loads

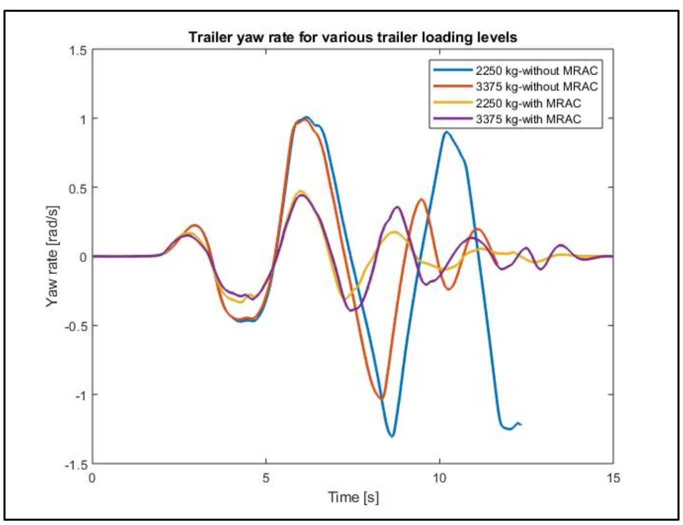

Figure 6 Trailer yaw rate for various loading conditions

The trailer yaw rates for controlled and uncontrolled systems also show the similar behavior as that of the tractor yaw rates. Even here, the MRAC controlled system, shows stable performance for both the loading conditions. The controller keeps the actual yaw rate of the trailer almost close to the desired yaw rates by application of differential force at various axles. Thus, the system behaves in a stable manner as compared to the uncontrolled system which goes unstable after a certain point due to excess yaw rates.

Hitch rate for various trailer loads

Figure 7 shows the hitch rate during the double lane change maneuver. Hitch rate shows same trend as the yaw rates of tractor and trailer since

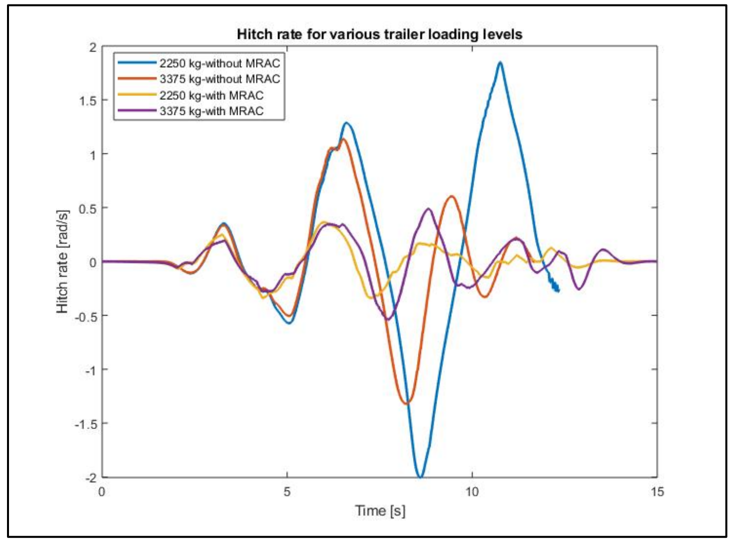

Figure 7 Hitch articulation rate for various trailer loads

it is dependent on both the yaw rates. The hitch rate for 2250 kg of trailer load shows higher oscillations (without MRAC) and again after a certain point it goes unstable. The MRAC controlled systems show almost similar performance which is stable for different loading conditions.

Conclusion

In this work, MRAC and a differential braking control strategy is implemented to prevent yaw plane instabilities in a tractor-semitrailer system. The MRAC and differential braking strategy prevents the instabilities by tracking the yaw rates of the tractor and the trailer and keeping them as close as possible to the desired yaw rates that are generated from the reference model. The MRAC controller is used as an upper-level controller which always monitors the yaw rates of two units and gives the magnitude of differential force required to provide the corrective yaw moment to the system. This magnitude of differential force is fed into the lower-level controller which generates the required brake pressure at each individual wheel to keep the performance of the system within desired limits. The differential braking strategy is implemented on the TruckSim model through Simulink co-simulation and the results obtained are plotted in the previous section of this paper. From the simulation results at 100 kmph it can be observed that the vehicle without MRAC is going unstable after it reaches the friction limit. But in case of the MRAC controlled system, the controller controls the excessive change in yaw rate by applying brakes to different wheels which helps stabilize the system in wide range of road friction conditions. Another important observation is the performance of the MRAC controlled tractor-trailer systems for different trailer loading conditions. The MRAC controlled system gives almost same performance for different trailer loading conditions. Thus, it is can be concluded through simulations done in this paper that a MRAC could be an effective and robust control system for preventing yaw instabilities in tractor-trailer combination for a wide range of trailer loading conditions and different road friction conditions.

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Definitions/Abbreviations

$\boldsymbol{m_T, m_R}$	Mass of the tractor and trailer
$\boldsymbol{V_G, V_{GR}}$	C.G. velocity of tractor and trailer
$\boldsymbol{J_T, J_R}$	Moment of Inertia of tractor and trailer
$\boldsymbol{\psi, \theta}$	Yaw and angle of tractor

Appendix

Using Cramer's rule, from the steady state equations, the yaw rate gain can be found as,

$$numerator := \begin{bmatrix} -\frac{Qyb}{V} & Qyd & 0 & -Qyt \\ -\frac{Qpb}{V} & Qpd & 0 & -Qpt \\ -\frac{Qtb}{V} & 0 & 0 & -Qtt \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$denominator := \begin{bmatrix} -\frac{Qyb}{V} & -(Qyr - m \cdot V) & 0 & -Qyt \\ -\frac{Qpb}{V} & -(mr \cdot V \cdot (c + ar) + Qpr) & 0 & -Qpt \\ -\frac{Qtb}{V} & mr \cdot V \cdot ar - Qtr & 0 & -Qtt \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$Steady\ state\ yaw\ rate\ = \frac{\det(numerator)}{\det(denominator)}$$

$$Steady\ state\ yaw\ rate\ = \frac{\det(numerator)}{\det(denominator)} * Road\ wheel\ steer\ angle$$

Similarly, for hitch angle articulation,

$$numerator1 := \begin{bmatrix} -\frac{Qyb}{V} & -(Qyr - m \cdot V) & 0 & Qyd \\ -\frac{Qpb}{V} & -(mr \cdot V \cdot (c + ar) + Qpr) & 0 & Qpd \\ -\frac{Qtb}{V} & mr \cdot V \cdot ar - Qtr & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$Steady\ state\ hitch\ articulation\ angle\ = \frac{\det(numerator1)}{\det(denominator)}$$

$$\boldsymbol{Steady\ state\ hitch\ articulation\ angle\ = \frac{\det(numerator1)}{\det(denominator)} * Road\ wheel\ steer\ angle}$$