ABSTRACT

The Boids Flocking Algorithm, conceptualized by Craig Reynolds, is a computational model inspired by the collective motion observed in natural phenomena such as bird flocks and fish schools. It relies on three simple rules – alignment, cohesion, and separation – to guide independent agents in a virtual space, creating behaviors that resemble those found in the natural world. The term "Boid" is a blend of "bird" and "oid", emphasizing its application to entities resembling birds in flight. It was originally developed to explore patterns and behaviours in simple agents and its capacity to simulate group behaviors makes it invaluable in understanding complex interactions within dynamic systems.

The objective of our project is to implement the BOIDS algorithm for sanitation and hygiene operations. This algorithm can potentially be applied in autonomous navigation, surveillance , monitoring and search and rescue operations. By studying this algorithm and analyzing this behavior, we would like to expand the scope into the health sector and clean rooms by using these robot swarms for clearing the area of infectants. These swarms can cover larger areas swiftly and with meticulous attention to detail, ensuring that every corner is reached, and every surface is treated. This would ensure that the path along its trajectory is completely covered by the swarm of robots, as opposed to an approach where the group of robots are allowed to explore the area individually. This would also be helpful in applications where the robot has a limited range of coverage and effectiveness. By harnessing the power of swarm robotics, we can achieve a higher level of cleanliness and safety, effectively neutralizing a wide array of pathogens and ensuring a safer environment for everyone.

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SECTION 1

MATHEMATICAL MODEL

In the project we test and analyze the flocking behavior of the Boids algorithm. We use 2 algorithms to demonstrate this behavior. In the first approach, we use the Boids algorithm, similar to what is done in [1]. The second approach implements flocking using potential fields. The second approach was included as it allowed us to prove properties like separation, and convergence to common velocities. There also are a few other works that analyze the Boids algorithm too [4].

APPROACH 1 :

Done by SAI MANIKANTA BADIGA , JONATHAN RANJITH THOMAS

The mathematical model, similar to that implemented in [2], is as follows:

The set of Boid agents are described as: $B = \{b_i, i = 1,2,3,4,\ldots,n\}$

The Boids within the Boid's field of view f_a are denoted as:

$N_i = \{b_i \in B; \forall b_j : |b_j - b_i| \leq L, j = 1,2,3, \ldots f_a\}$, where L is the radius of the robot's field of view and hence the range where the alignment, cohesion and separation are effective

l_i is the position of Boid b_i

v_i is the velocity of b_i

The following terms a_i, c_i, s_i denote the alignment, cohesion and separation of Boid b_i respectively

$$a_i = - v_i + \frac{1}{f_a} \sum_{\forall b_j \in N_i} v_j$$

$$c_i = \frac{1}{f_a} \sum_{\forall b_j \in N_i} l_j$$

$s_i = \frac{1}{f_a} \sum_{\forall b_j \in N_i} \frac{p_j - p_i}{d(bi,bj)}$, where the function $d\big(b_i, b_j\big)$is the separation between the Boid's b_i and b_j

$l_j = l_i + \ v_i + X_a a_i + X_c c_i + \ X_s s_i$, where l_j denotes the next position of Boid b_i and X_a is the constant of alignment, X_c is the constant of cohesion and X_s is the constant separation.

We found that this model was difficult to prove due to the non-linearity and dynamic nature of this algorithm.

APPROACH 2:

Done by YASH PALLIWAL, VENKATA SAI DEEPAK MUTTA

This approach is based on [3]. In a system consisting of N robots, defined by the dynamics as follows.

$$\dot{r}_i = v_i$$

$$\dot{v}_i = a_i \quad i = 1,\cdots, N$$

$$\theta_i = arctan2(\dot{y}_i, \dot{x}_i)$$

Where r_i is the position of the robot, v_i is the velocity of the robot and a_i is the acceleration of the robot and θ_i denotes the heading of the robot.

Each robot is controlled by controlling the acceleration input by using 2 different components.

$$a_i = \ a_{r_i} + a_{\theta_i}$$

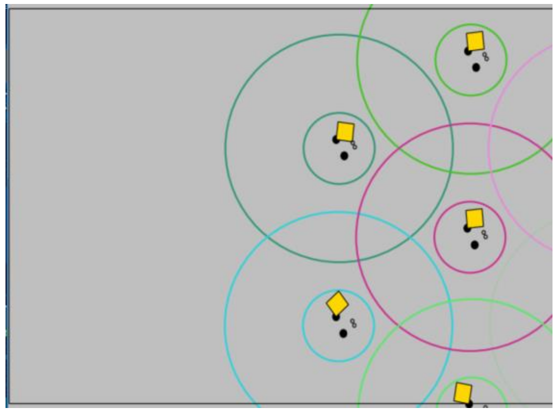
Where a_{r_i} is the component from potential function and a_{θ_i} is the component that controls heading. In this approach by using potential functions and dynamics we were able to theoretically prove that,

1. At equilibrium robots reach the stable distances to their neighbours. In other words, the potential function converges to its minima.
2. At equilibrium state, all the robots reach to a common heading and velocity. In other words the sum of velocities of all the robots combined is going to be minimum.

Both of the above-mentioned characteristics are analysed in detail in the section 3 under the theoretical analysis. Before we conduct a theoretical analysis, we must consider some constraints and assumptions in the system of N robots.

ASSUMPTIONS

- The environment of operation is deterministic.
- Each robot has its own set of sensors which are identical to every other robot. These sensors have a limited range within which they can sense all the other robots.
- For approach 1, the robots have no communication capabilities, and the robots only know about the other robot's position and velocity information when they are within their sensory range.
- For approach 2 robots know the position and velocity details of all other available robots



The large circle in the above figure shows the sensory range for each robot.

SECTION 2

Theoretical analysis

Property 1 (separation distance)

Done by YASH PALIWAL and JONATHAN RANJITH THOMAS

As defined in the mathematical model, the robots maintain a specified distance between each other and flock together using the concept of potential fields. In order to explain the stability of the above defined model we calculated the equilibrium point for the potential function [equilibrium distance that all the robots maintain between others avoiding any sort of collision]. By taking a gradient for the above defined potential function and equalizing it to zero, we can find the minima for the function that can be treated as a point of stability.

$$v_{ij}(r_{ij}) = \frac{1}{||r_{ij}||^2} + \log(||r_{ij}||^2)$$

Using this potential function, we obtain the gradient of it as follows.

$$\nabla v_{ij}(r_{ij}) = \frac{2}{||r_{ij}||} - \frac{2}{||r_{ij}||^3}$$

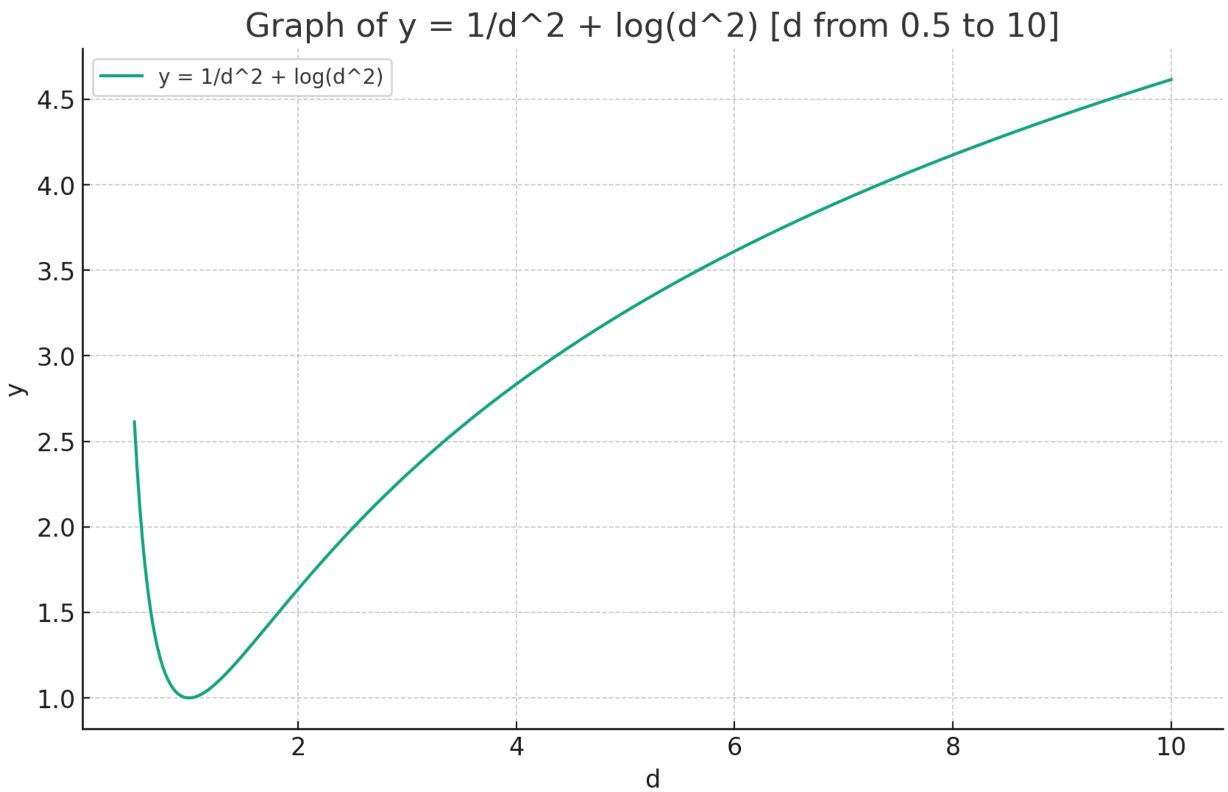
Equating this equation to zero, we find the value of the minima.

$$\frac{2}{||r_{ij}||}\left\{1 - \frac{1}{||r_{ij}||^2}\right\} = 0$$

Upon simplification, we get the value

$$||r_{ij}|| = 1$$

This means that a robot maintains a region of unit circle around it beyond which no other robot ventures.



Property 2 (velocity convergence)

Done by Sai Manikanta Badiga and Venkata Sai Deepak Mutta

The system of N boids with dynamics specified below following the control law in equation 2, with initial conditions in Ω, converges to one of the minima of v_t and a common orientation.

$$a_{i= -\nabla_{r_i} V_i + a_{\theta_i}} \qquad\qquad a_{\theta_i} = -\sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{\theta_i - \theta_j}{||r_{ij}||^2} (\hat{z} \times v_i)$$

Eq 4: Control law Eq 5: for a_{θ_i}

In order to prove that the velocity is going to converge to a minima, we take the time derivative and solve it using the equations listed above.

$$\dot{v}_t = \sum_{i=1}^{N} \nabla_{r_i} v_i \cdot v_i + \sum_{i=1}^{N} v_i^T a_i + \sum_{i=1}^{N} \theta_i \dot{\theta}_i$$

Upon solving the equation using the control law and value for a_{θ_i} we obtain the following equation.

$$\dot{v}_t = \sum_{i=1}^{N} v_i^T \left(- \sum_{j \in N_i} \frac{\theta_i - \theta_j}{||r_{ij}||^2} (\hat{z} \times v_i) \right) + \sum\nolimits_{i=1}^{N} \theta_i \dot{\theta}_i$$

Now, going back to the control equations and angular velocity, we have one more equation described in the paper.

$$\dot{\theta}_i = -\sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{\theta_i - \theta_j}{||r_{ij}||^2}$$

Expanding this equation, we get.

$$\dot{\theta}_i = -\sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{\theta_i}{||r_{ij}||^2} + \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{\theta_j}{||r_{ij}||^2}$$

$$= -\left(\frac{1}{||r_{i1}||^2}, \quad \dots, \quad \frac{1}{||r_{ii-1}||^2}, \quad \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{1}{||r_{ij}||^2}, \quad \frac{1}{||r_{ii+1}||^2}, \quad \dots, \frac{1}{||r_{in}||^2} \right) \begin{bmatrix} \theta_1 \\ \cdot \\ \theta_i \\ \cdot \\ \theta_N \end{bmatrix}$$

Using the above equation and solving for $\dot{v}_t$, we get the below format.

$$\begin{aligned} \dot{V}_t &= -\begin{bmatrix} \theta_1 \\ \vdots \\ \theta_N \end{bmatrix}^T \begin{bmatrix} \sum_{\substack{j=1 \\ j\neq 1}}^{N} \|r_{1j}\|^{-2} & -\|r_{12}\|^{-2} & \cdots & -\|r_{1N}\|^{-2} \\ \vdots & \vdots & & \vdots \\ -\|r_{N1}\|^{-2} & -\|r_{N2}\|^{-2} & \cdots & \sum_{\substack{j=1 \\ j\neq N}}^{N} \|r_{Nj}\|^{-2} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_N \end{bmatrix} \\ &= -\theta^T L_w \theta, \end{aligned}$$

$\dot{v}_t \leq 0$, Ω is positively invariant. Applying LaSalle's invariant principle on the dynamics of the robot in Ω we conclude that all trajectories converge to the largest invariant set in $\{(r_{ij}, v_i, \theta_i) \mid V^{\cdot} t = 0, i, j = 1, ..., N\}$. Equality $\dot{v}_t = 0$ holds only at configurations where all agents have the same constant heading, $\theta_1 = \cdots = \theta_N = \overline{\theta}$. Let $S\theta$ be the

set where all orientations are the same. In this set of robots we have the heading angles as follows.

$$\tan\overline{\theta} = k = \frac{\dot{y}_i}{\dot{x}_i} \quad i = 1, \dots, N$$

To find a stable point we take gradient and equalize to zero.

Differentiating $\frac{\dot{y}_i}{\dot{x}_i} = k$ we get:

$$\frac{d}{dt}\left(\frac{\dot{y}_i}{\dot{x}_i}\right) = 0 \Rightarrow \frac{\ddot{y}_i}{\ddot{x}_i} = \frac{\dot{y}_i}{\dot{x}_i} = k = \frac{a_{y_i}}{a_{x_i}} = \frac{(\nabla_{r_i} V_i)_y}{(\nabla_{r_i} V_i)_x}.$$

This means that the potential force applied on the i^{th} robot is in alignment with its velocity. Now the stability can be seen in 2 different cases where potential is greater than zero and potential is less than or equal to zero.

In case 1, $-\nabla_{r_i} V_i \cdot v_i \leq 0$. If $V_{v_i} = \frac{1}{2} v_i^T v_i$, then $\dot{V}_{v_i} = v_i \dot{v}_i = -\nabla_{r_i} V \cdot v_i \leq 0$. Now $\dot{v}_i = a_i = -\nabla_{r_i} V_i$. These dynamics suggest that v_i will converge to the largest invariant set in $S_{v_i} = \{(r_i, v_i) | \dot{V}_{v_i} = 0\}$. We find that the potential and velocities are aligned, and S_{v_i} will only contain configurations that have potential=0 or velocity =0. The reason for this is since the product is negative or zero, the values of individual terms must have opposite signs which violates the rule of alignment for potential and velocities.

In case 2, $-\nabla_{r_i} V_i \cdot v_i > 0$. Here, the potential function is less than zero. This means that it is monotonically decreasing and eventually converges to its minima.

From these cases we can conclude that irrespective of scenarios the net total velocities of robots will converge to a minimum.

SECTION 3

IMPLEMENTATION

The implementation for both the algorithms is done using the Robotarium package in python. During the implementation we performed 2 simulations, one for each approach. We also made use of some additional libraries like matplotlib for data plotting and generating graphs for analyzing the simulation data.

<u>APPROACH 1:</u>

<u>Implemented by Jonathan and Sai manikanta Badiga</u>

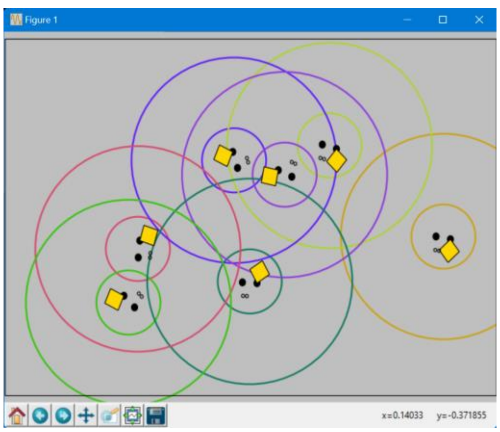


Fig 2. Random initialization of robots

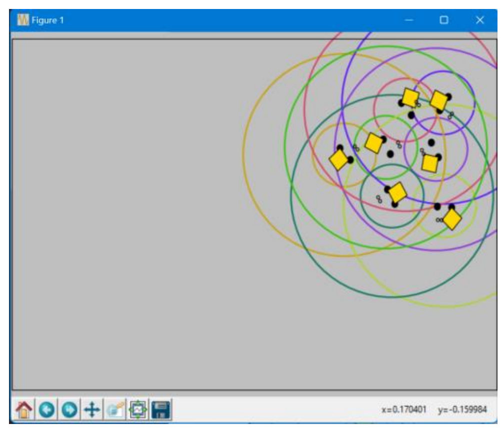


fig 3. Obstacle avoidance from wall

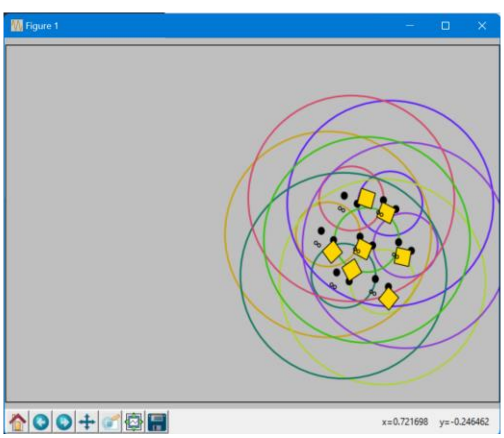
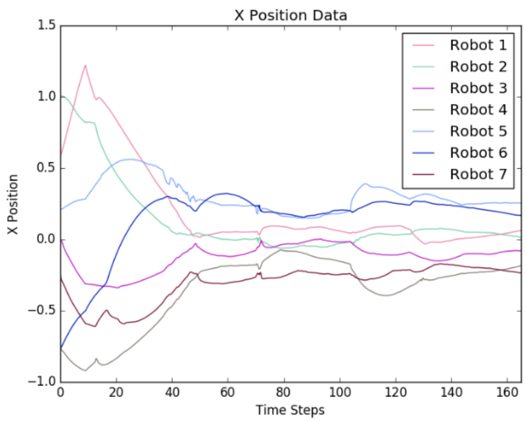
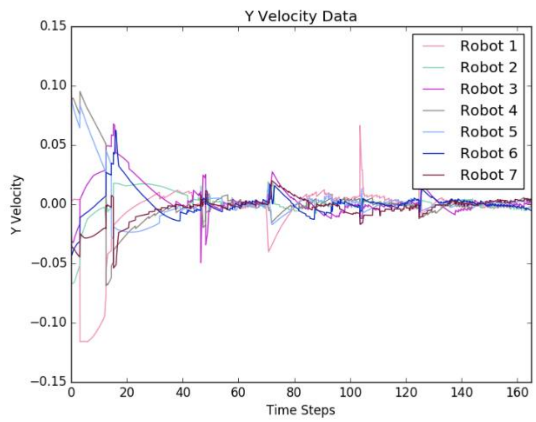
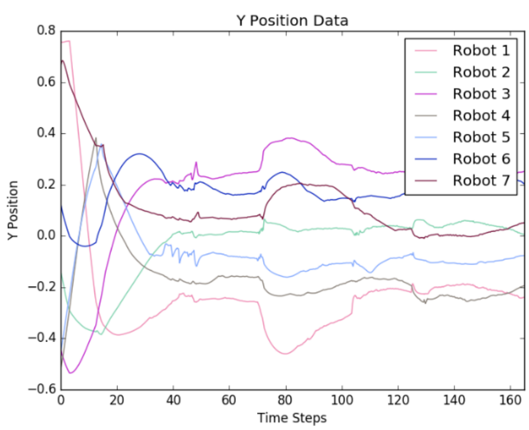
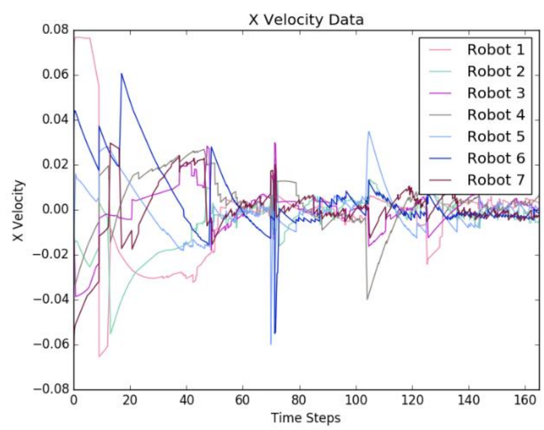


Fig 4. Coordinated flock movement.









The above plots show the positions and velocities of robots along x and y axes over all timesteps.

The spikes in graphs for velocity data are due to abnormal change in velocity when encountered with the walls. This is obtained because of the obstacle avoidance behavior of the flock.

From the graphs above for position data we can see that the coordinates for x and y axes are parallel to each other, and this proves that the robots are reaching a point of convergence.

APPROACH 2:

Implemented by Yash and Deepak

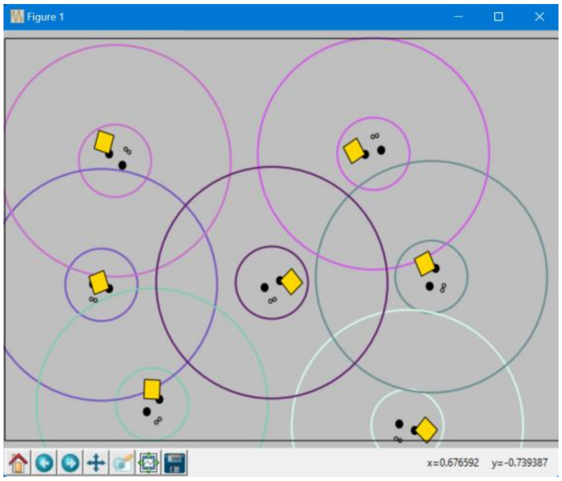


Fig 6. Random initialization of robot

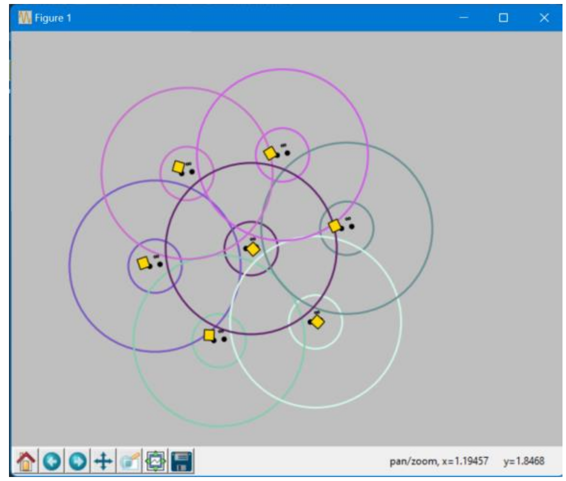


fig 7. Coordinated flock movement while maintaining the distance between robots.

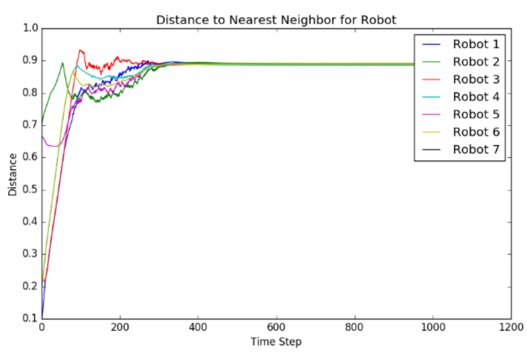


fig 8. Convergence of distance between neighboring robots

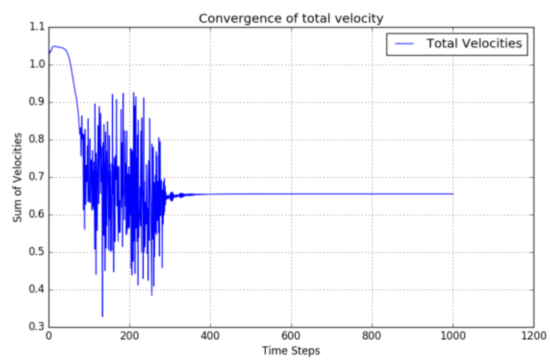


fig 9. Convergence of net total velocities to a minima

From the above shown graphs we can see that the robots are reaching a convergence in terms of the inter robot distance and the final velocities. According to our theoretical analysis the stable distance of separation for the potential function is 1 which we were able to see in the simulations.

Videos of implementation:

Approach 1:

https://drive.google.com/file/d/1Bz3fsWjCbDzAS1rCxRTkgZRkG36jTzro/view?usp=sharing

Approach 2:

https://drive.google.com/file/d/1QzzhDfIMTStB-udIou6gR6RJY4wsfC_w/view?usp=sharing

References:

1. https://vanhunteradams.com/Pico/Animal_Movement/Boids-algorithm.html
2. Choi T, Ahn C. Artificial life based on boids model and evolutionary chaotic neural networks for creating artworks. Swarm Evol Comput 2019;47:80–9
3. Tanner, H. G., Jadbabaie, A., & Pappas, G. J. (2003). Stability of Flocking Motion. University of Pennsylvania. Technical Report No: MS-CIS-03-03.
4. R. Olfati-Saber, "Flocking for multi-agent dynamic systems: algorithms and theory," in *IEEE Transactions on Automatic Control*, vol. 51, no. 3, pp. 401-420, March 2006, doi: 10.1109/TAC.2005.864190.