Семінар 11. Відображення

6 червня 2023

• Definition

A **function f from a set X to a set Y** is a relation* between elements of X, called **inputs,** and elements of Y, called **outputs,** with the property that each input is related to one and only one output. The notation $f\colon X \to Y$ means that f is a function from X to Y. X is called the **domain** of f, and Y is called the **co-domain** of f.

Given an input element x in X, there is a unique output element y in Y that is related to x by f. We say that "f sends x to y" and write $x \xrightarrow{f} y$ or $f\colon x \to y$. The unique element y to which f sends x is denoted

$\boldsymbol{f(x)}$ and is called **f of x,** or
the output of f for the input x, or
the value of f at x, or
the image of x under f.

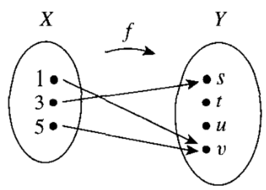
The set of all values of f taken together is called the *range of f* or the *image* of *X under f*. Symbolically,

$$\textbf{range of } f = \textbf{image of } X \textbf{ under } f = \{y \in Y \mid y = f(x), \text{ for some } x \text{ in } X\}.$$

Given an element y in Y, there may exist elements in X with y as their image. If $f(x) = y$, then x is called **a preimage of y** or **an inverse image of y.** The set of all inverse images of y is called *the inverse image of y*. Symbolically,

$$\textbf{inverse image of } y = \{x \in X \mid f(x) = y\}.$$

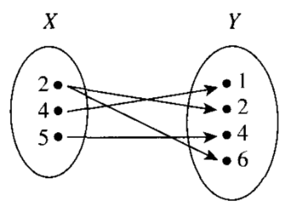
1. Let $X = \{1, 3, 5\}$ and $Y = \{s, t, u, v\}$. Define $f\colon X \to Y$ by the following arrow diagram.



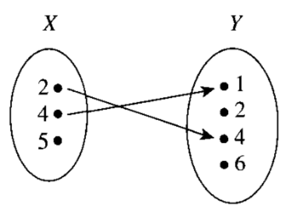
a. Write the domain of f and the co-domain of f.
b. Find $f(1)$, $f(3)$, and $f(5)$.
c. What is the range of f?
d. Is 3 an inverse image of s? Is 1 an inverse image of u?
e. What is the inverse image of s? of u? of v?
f. Represent f as a set of ordered pairs.

3. Let $X = \{2, 4, 5\}$ and $Y = \{1, 2, 4, 6\}$. Which of the following arrow diagrams determine functions from X to Y?

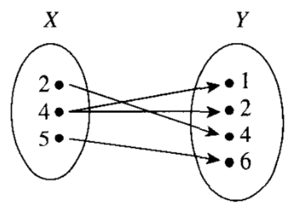
a.



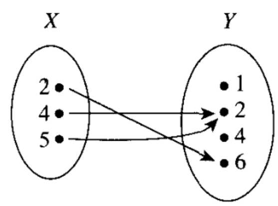
b.



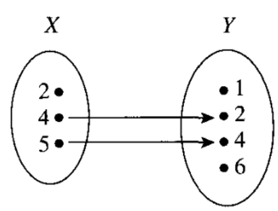
c.



d.



e.



• Definition

Let F be a function from a set X to a set Y. F is **one-to-one** (or **injective**) if, and only if, for all elements x_1 and x_2 in X,

$$\text{if } F(x_1) = F(x_2), \text{ then } x_1 = x_2.$$

Or, equivalently,

$$\text{if } x_1 \neq x_2, \text{ then } F(x_1) \neq F(x_2).$$

Symbolically,

$$F\colon X \to Y \text{ is one-to-one} \quad \Leftrightarrow \quad \forall x_1, x_2 \in X, \text{ if } F(x_1) = F(x_2) \text{ then } x_1 = x_2.$$

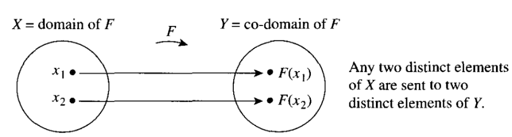


Figure 7.2.1(a) A One-to-One Function Separates Points

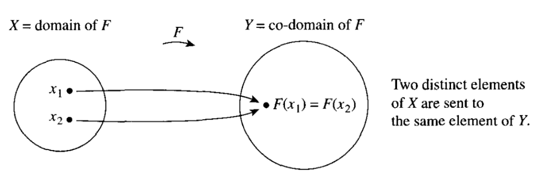


Figure 7.2.1(b) A Function That Is Not One-to-One Collapses Points Together

• Definition

Let F be a function from a set X to a set Y. F is **onto** (or **surjective**) if, and only if, given any element y in Y, it is possible to find an element x in X with the property that $y = F(x)$.

Symbolically:

$$F: X \to Y \text{ is onto} \quad \Leftrightarrow \quad \forall y \in Y, \exists x \in X \text{ such that } F(x) = y.$$

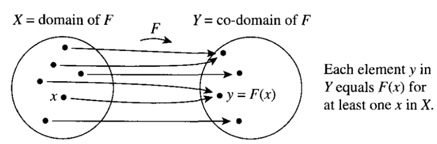


Figure 7.2.3(a) A Function That Is Onto

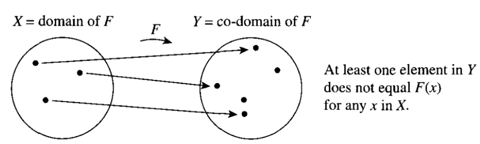


Figure 7.2.3(b) A Function That Is Not Onto

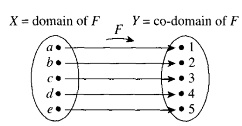
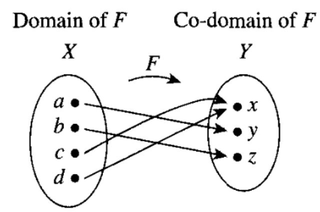


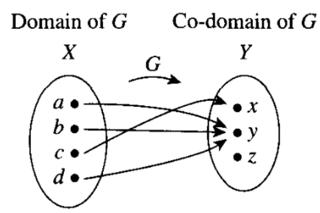
Figure 7.2.5 An Arrow Diagram for a One-to-One Correspondence

• Definition

A **one-to-one correspondence** (or **bijection**) from a set X to a set Y is a function $F: X \to Y$ that is both one-to-one and onto.

7. Let $X = \{a, b, c, d\}$ and $Y = \{x, y, z\}$. Define functions F and G by the arrow diagrams below.

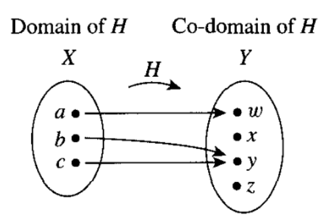


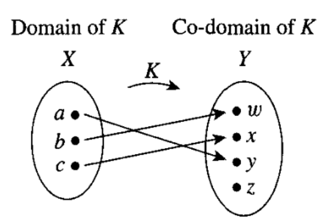


a. Is F one-to-one? Why or why not? Is it onto? Why or why not?

b. Is G one-to-one? Why or why not? Is it onto? Why or why not?

8. Let $X = \{a, b, c\}$ and $Y = \{w, x, y, z\}$. Define functions H and K by the arrow diagrams below.





a. Is H one-to-one? Why or why not? Is it onto? Why or why not?

b. Is K one-to-one? Why or why not? Is it onto? Why or why not?

In each of 16–19 a function f is defined on a set of real numbers. Determine whether or not f is one-to-one and justify your answer.

16. $f(x) = \dfrac{x+1}{x}$, for all real numbers $x \neq 0$

17. $f(x) = \dfrac{x}{x^2+1}$, for all real numbers x

18. $f(x) = \dfrac{3x-1}{x}$, for all real numbers $x \neq 0$

19. $f(x) = \dfrac{x+1}{x-1}$, for all real numbers $x \neq 1$

Означення

Дві множини A, B (не обов‘язково скінченні) називають рівнопотужними, якщо існує бієкція з однієї в іншу

- Скінченні множини
- Натуральні числа і парні натуральні числа
- Натуральні числа і цілі числа
- Інтервал (0, 1) і інтервал (1, + inf)
- Відрізок [0, 1] і відрізок [1, + inf)