1. Suppose that $S = \{a, b, c, d, e\}$ and R is a binary relation on S such that $a\ R\ b$, $b\ R\ c$, and $d\ R\ e$. List all of the following that must be true if R is (a) reflexive (but not symmetric or transitive), (b) symmetric (but not reflexive or transitive), (c) transitive (but not reflexive or symmetric), and (d) an equivalence relation.

$c R b \quad c R c \quad a R c \quad b R a \quad a R d \quad e R a \quad e R d \quad c R a$

Чи є R відношенням еквівалентності, знайдіть класи еквівалентності

3. $A = \{0, 1, 2, 3, 4\}$
$R = \{(0, 0), (0, 4), (1, 1), (1, 3), (2, 2), (3, 1), (3, 3), (4, 0), (4, 4)\}$

4. $A = \{a, b, c, d\}$
$R = \{(a, a), (b, b), (b, d), (c, c), (d, b), (d, d)\}$

5. $A = \{1, 2, 3, 4, \ldots, 20\}$. R is defined on A as follows:

$$\text{For all } x, y \in A, \quad x\, R\, y \quad \Leftrightarrow \quad 4 \mid (x - y).$$

6. $A = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$. R is defined on A as follows:

$$\text{For all } x, y \in A, \quad x\, R\, y \quad \Leftrightarrow \quad 3 \mid (x - y).$$

7. $A = \{(1,3), (2,4), (-4,-8), (3,9), (1,5), (3,6)\}$. R is defined on A as follows: For all $(a, b), (c, d) \in A$,

$$(a, b)\ R\ (c, d) \quad \Leftrightarrow \quad ad = bc.$$

8. $X = \{a, b, c\}$ and $A = \mathscr{P}(X)$. R is defined on A as follows:

For all sets U and V in $\mathscr{P}(X)$, $\quad U\ R\ V \quad \Leftrightarrow \quad N(U) = N(V)$.

(That is, the number of elements in U equals the number of elements in V.)

9. $X = \{-1, 0, 1\}$ and $A = \mathscr{P}(X)$. R is defined on $\mathscr{P}(X)$ as follows: For all sets S and T in $\mathscr{P}(X)$,

$S\ R\ T \quad \Leftrightarrow \quad$ the sum of the elements in S equals the sum of the elements in T.

10. A is the set of all strings of length 4 in a's and b's. R is defined on A as follows: For all strings s and t in A,

$s\ R\ t \quad \Leftrightarrow \quad$ the first two characters of s equal the first two characters of t.

Доведіть, що R відношення еквівалентності, опишіть класи еквівалентності

17. A is the set of all students at your college.

a. R is the relation defined on A as follows: For all x and y in A,

$x\ R\ y \quad \Leftrightarrow \quad x$ has the same major (or double major) as y.

(Assume "undeclared" is a major.)

b. S is the relation defined on A as follows: For all $x, y \in A$,

$x\ S\ y \quad \Leftrightarrow \quad x$ is the same age as y.

***H* 27.** Let A be the set of all straight lines in the Cartesian plane. Define a relation $\|$ on A as follows:

$$\text{For all } l_1 \text{ and } l_2 \text{ in } A, \quad l_1 \parallel l_2 \quad \Leftrightarrow \quad l_1 \text{ is parallel to } l_2.$$

25. Define P on the set $\mathbf{R} \times \mathbf{R}$ of ordered pairs of real numbers as follows: For all $(w, x), (y, z) \in \mathbf{R} \times \mathbf{R}$,

$$(w, x)\ P\ (y, z) \quad \Leftrightarrow \quad w = y.$$

23. I is the relation defined on $\mathbf{R}$ as follows:

$$\text{For all } x, y \in \mathbf{R}, \quad x\ I\ y \quad \Leftrightarrow \quad x - y \text{ is an integer.}$$