

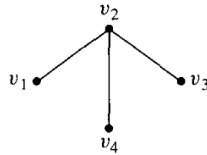
Семінар 12. Графи

9 червня 2023

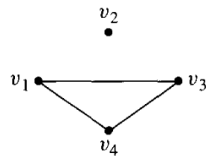
25. **a.** In a group of 15 people, is it possible for each person to have exactly 3 friends? Explain. (Assume that friendship is a symmetric relationship: If x is a friend of y , then y is a friend of x .)
- b.** In a group of 4 people, is it possible for each person to have exactly 3 friends? Why?
26. In a group of 25 people, is it possible for each to shake hands with exactly 3 other people? Explain.
28. Suppose a graph has vertices of degrees 0, 2, 2, 3, and 9. How many edges does the graph have?
29. Suppose a graph has vertices of degrees 1, 1, 4, 4, and 6. How many edges does the graph have?

- b. Show that for all integers $n \geq 1$, the number of edges of K_n is $\frac{n(n-1)}{2}$.

Definition: If G is a simple graph, the **complement** of G , denoted G' , is obtained as follows: The vertex set of G' is identical to the vertex set of G . However, two distinct vertices v and w of G' are connected by an edge if, and only if, v and w are not connected by an edge in G . For example, if G is the graph

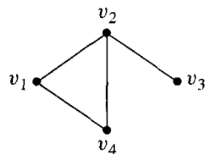


then G' is



39. Find the complement of each of the following graphs.

a.



b. v_1 — v_2

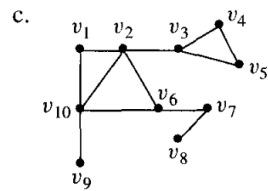
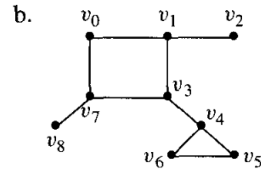
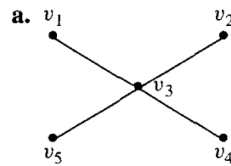
v_4 — v_3

40. a. Find the complement of the graph K_4 , the complete graph on four vertices. (See Example 11.1.8.)
 b. Find the complement of the graph $K_{3,2}$, the complete bipartite graph on $(3, 2)$ vertices. (See Example 11.1.9.)

42. Let G be a simple graph with n vertices. What is the relation between the number of edges of G and the number of edges of the complement G' ?
 43. Show that at a party with at least two people, there are at least two mutual acquaintances or at least two mutual strangers.

H ★ 45. In a group of two or more people, must there always be at least two people who are acquainted with the same number of people within the group? Why?

6. An edge whose removal disconnects the graph of which it is a part is called a **bridge**. Find all bridges for each of the following graphs.



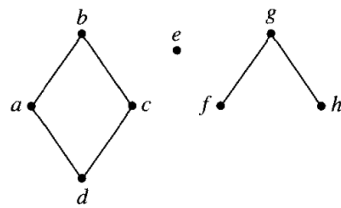
Задача

Доведіть, що серед 6 людей є завжди або 3 знайомих між собою, або 3 попарно незнайомих

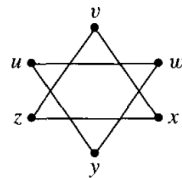
7. Given any positive integer n , (a) find a connected graph with n edges such that removal of just one edge disconnects the graph; (b) find a connected graph with n edges that cannot be disconnected by the removal of any single edge.

8. Find the number of connected components for each of the following graphs.

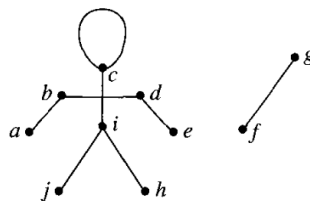
a.



b.



c.



d.

