

Формула включень-виключень

$A, B, C, A_1, A_2, \dots, A_n$ - скінченні множини

- $|A \cup B| = |A| + |B| - |A \cap B|$
- $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$
- $|\bigcup_{i=1}^n A_i| = \sum_{i=1}^n |A_i| - \sum_{1 \leq i < j \leq n} |A_i \cap A_j| + \sum_{1 \leq i < j < k \leq n} |A_i \cap A_j \cap A_k| - \dots + (-1)^{n-1} |\bigcap_{i=1}^n A_i|$

• Definition

A collection of nonempty sets $\{A_1, A_2, \dots, A_n\}$ is a **partition** of a set A if, and only if,

1. $A = A_1 \cup A_2 \cup \dots \cup A_n$;
2. A_1, A_2, \dots, A_n are mutually disjoint.

22. a. Is $\{\{a, d, e\}, \{b, c\}, \{d, f\}\}$ a partition of $\{a, b, c, d, e, f\}$?
- b. Is $\{\{w, x, v\}, \{u, y, q\}, \{p, z\}\}$ a partition of $\{p, q, u, v, w, x, y, z\}$?
- c. Is $\{\{5, 4\}, \{7, 2\}, \{1, 3, 4\}, \{6, 8\}\}$ a partition of $\{1, 2, 3, 4, 5, 6, 7, 8\}$?
- d. Is $\{\{3, 7, 8\}, \{2, 9\}, \{1, 4, 5\}\}$ a partition of $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$?
- e. Is $\{\{1, 5\}, \{4, 7\}, \{2, 8, 6, 3\}\}$ a partition of $\{1, 2, 3, 4, 5, 6, 7, 8\}$?

25. Let \mathbf{Z} be the set of all integers and let

$$A_0 = \{n \in \mathbf{Z} \mid n = 4k, \text{ for some integer } k\},$$

$$A_1 = \{n \in \mathbf{Z} \mid n = 4k + 1, \text{ for some integer } k\},$$

$$A_2 = \{n \in \mathbf{Z} \mid n = 4k + 2, \text{ for some integer } k\}, \text{ and}$$

$$A_3 = \{n \in \mathbf{Z} \mid n = 4k + 3, \text{ for some integer } k\}.$$

Is $\{A_0, A_1, A_2, A_3\}$ a partition of \mathbf{Z} ? Explain your answer.

30. Let $A = \{1, 2, 3\}$, $B = \{u, v\}$, and $C = \{m, n\}$. List the elements of each of the following sets:

a. $A \times (B \times C)$ b. $(A \times B) \times C$ c. $A \times B \times C$

8. For all sets A , B , and C ,

$$(A - B) \cup (C - B) = (A \cup C) - B.$$

9. For all sets A , B , and C ,

$$(A - B) \cap (C - B) = (A \cap C) - B.$$

H 10. For all sets A and B , $A \cup (A \cap B) = A$.

11. For all sets A , $A \cup \emptyset = A$.

12. For all sets A , B , and C , if $A \subseteq B$ then $A \cap C \subseteq B \cap C$.

13. For all sets A , B , and C , if $A \subseteq B$ then $A \cup C \subseteq B \cup C$.

14. For all sets A and B , if $A \subseteq B$ then $B^c \subseteq A^c$.

H 15. For all sets A , B , and C , if $A \subseteq B$ and $A \subseteq C$ then

$$A \subseteq B \cap C.$$

16. For all sets A , B , and C ,

$$A \times (B \cup C) = (A \times B) \cup (A \times C).$$

17. For all sets A , B , and C ,

$$A \times (B \cap C) = (A \times B) \cap (A \times C).$$