Семінар 15. Ізоморфізм графів

19 червня 2023

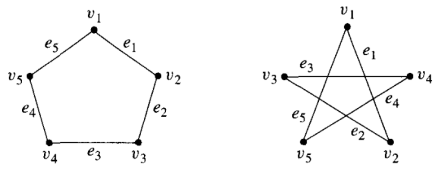


Figure 11.4.1

Call this graph G. Now consider the graph G' represented in Figure 11.4.2.

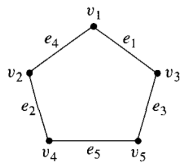
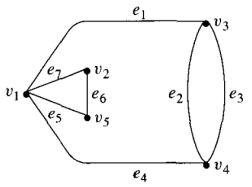


Figure 11.4.2

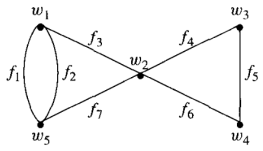
• Definition

Let G and G' be graphs with vertex sets $V(G)$ and $V(G')$ and edge sets $E(G)$ and $E(G')$, respectively. ***G* is isomorphic to *G'*** if, and only if, there exist one-to-one correspondences g: $V(G) \to V(G')$ and h: $E(G) \to E(G')$ that preserve the edge-endpoint functions of G and G' in the sense that for all $v \in V(G)$ and $e \in E(G)$,

$$v \text{ is an endpoint of } e \quad \Leftrightarrow \quad g(v) \text{ is an endpoint of } h(e). \qquad 11.4.1$$



G

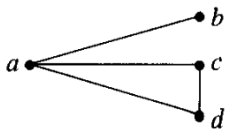


G'

• Definition

If G and G' are simple graphs, then **G is isomorphic to G'** if, and only if, there exists a one-to-one correspondence g from the vertex set $V(G)$ of G to the vertex set $V(G')$ of G' that preserves the edge-endpoint functions of G and G' in the sense that for all vertices u and v of G,

$$\{u, v\} \text{ is an edge in } G \iff \{g(u), g(v)\} \text{ is an edge in } G'. \qquad 11.4.2$$



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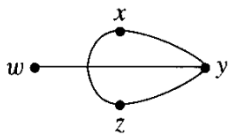


G'

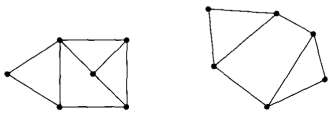
Theorem 11.4.1

Each of the following properties is an invariant for graph isomorphism, where n, m, and k are all nonnegative integers:

1. has n vertices;
2. has m edges;
3. has a vertex of degree k;
4. has m vertices of degree k;
5. has a circuit of length k;
6. has a simple circuit of length k;
7. has m simple circuits of length k;
8. is connected;
9. has an Euler circuit;
10. has a Hamiltonian circuit.

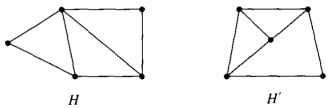
Show that the following pairs of graphs are not isomorphic by finding an isomorphic invariant that they do not share.

a.



G $\qquad$ G'

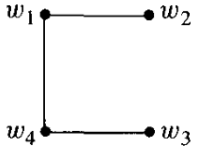
b.



H $\qquad$ H'

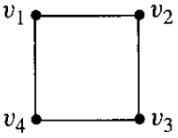
6.

G



G'

7.



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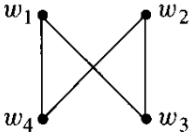
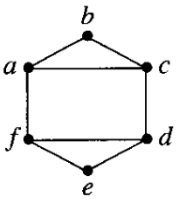


G'

8.



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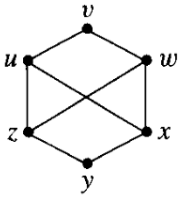
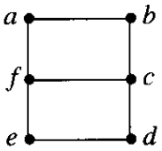


G'

9.



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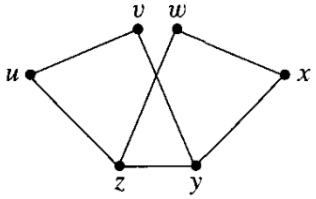
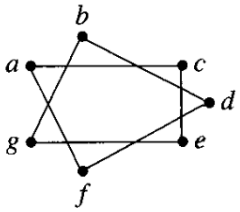


G'

10.



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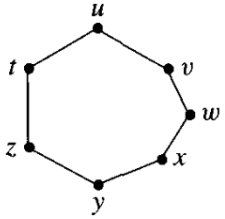
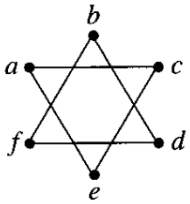


G'

11.



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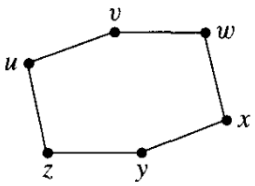
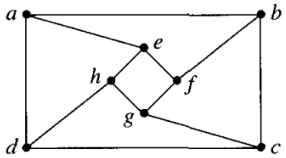


G'

12.



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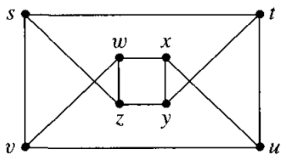
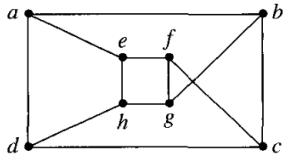


G'

13.



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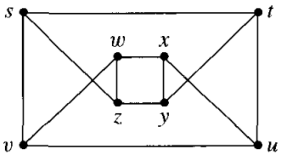

G'

14. Draw all nonisomorphic simple graphs with three vertices.