Quantum Programming

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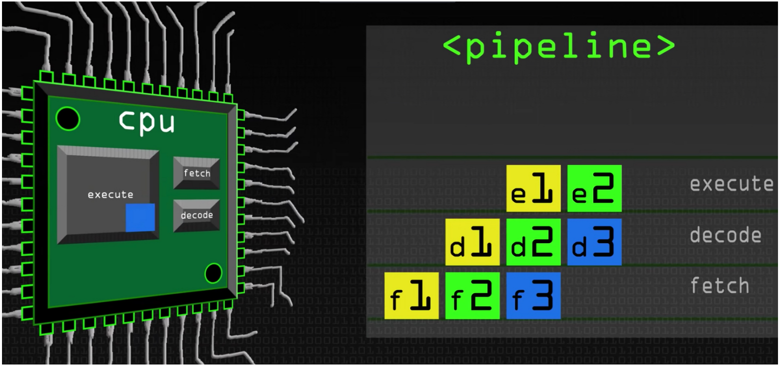
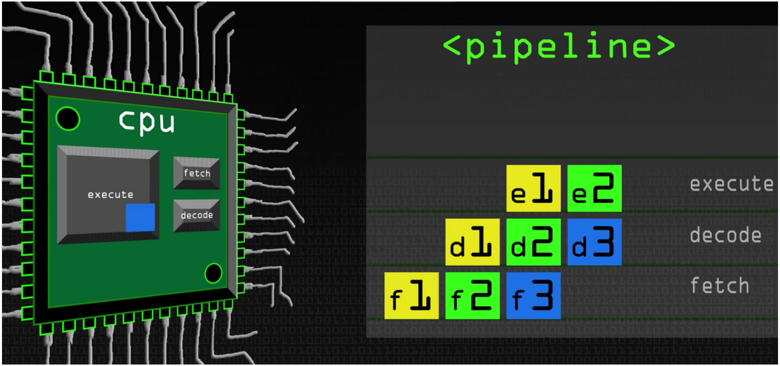
Outline

Von Neumann CPU



- Three main building blocks of Von Neumann computer IO, CPU and MEM (connected via system bus)



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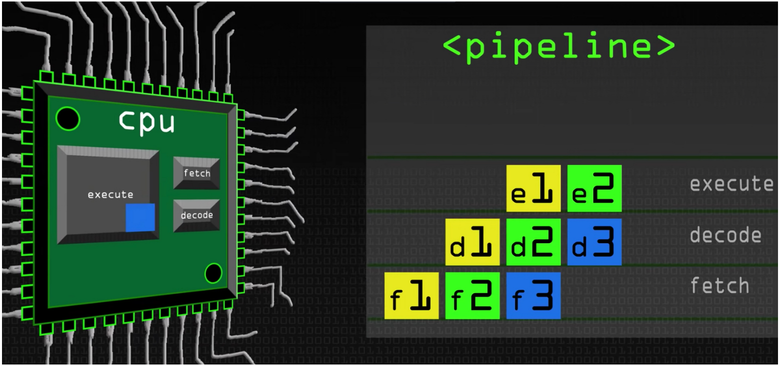
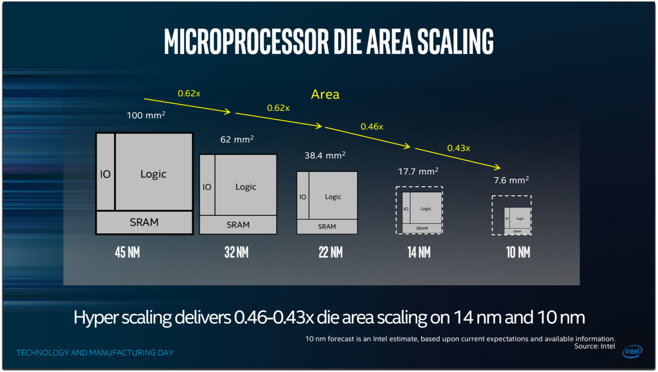
- Programs and data are stored in the same memory (Address Register and Data Register)

Von Neumann CPU



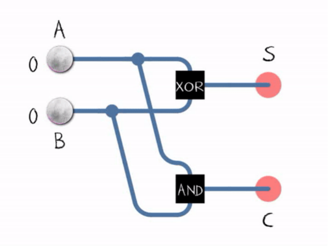
- Three main building blocks of Von Neumann computer IO, CPU and MEM (connected via system bus)
- Programs and data are stored in the same memory (Address Register and Data Register)
- CPU cycle: fetch → decode → execute (pipline model offers speed up)

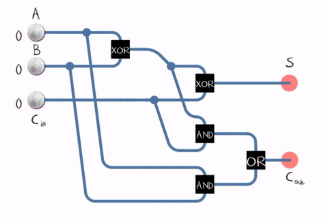
Moore's Law

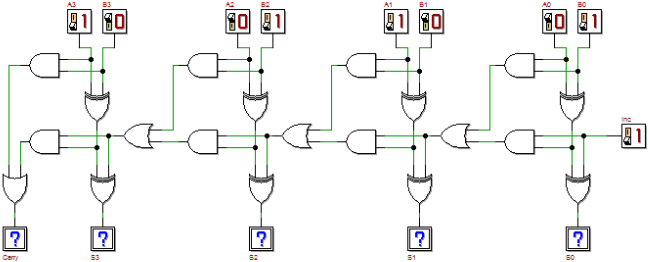


- Moore's Law: Number of transistors in a dense integrated circuit doubles about every two years
- Quantum tunneling imposes physical limits to the size of the transistors used in microprocessors

Classical Adder Circuit

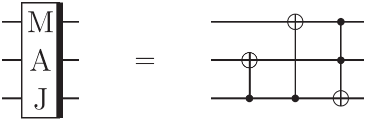


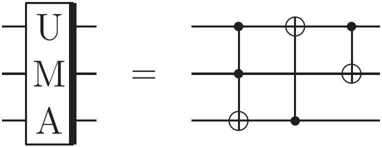




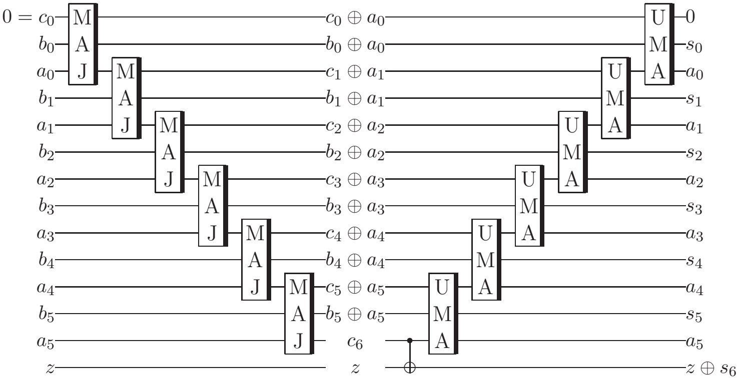
Quantum Adder Circuit

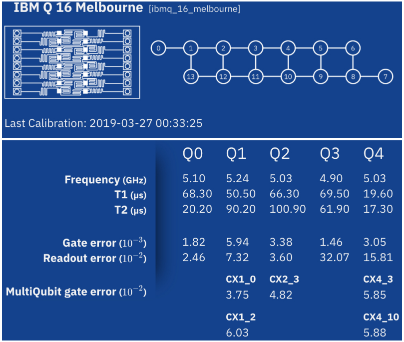
- $a_{n-1} \ldots a_1 a_0 + b_{n-1} \ldots b_1 b_0$?
- $c_0 = 0$; $c_{i+1} = MAJ(a_i, b_i, c_i) \to s_i = a_i \oplus b_i \oplus c_i$ and $s_n = c_n$
- In a reversible ripple-carry adder, we must then erase the carry bits, working our way back down.





6-bit Quantum Ripple Adder

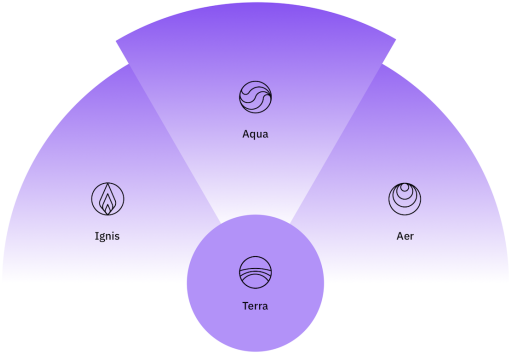



IBM Q 16 Melbourne [ibmq_16_melbourne]

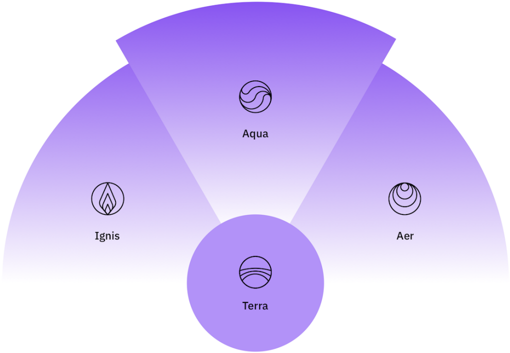
Last Calibration: 2019-03-27 00:33:25

	Q0	Q1	Q2	Q3	Q4
Frequency (GHz)	5.10	5.24	5.03	4.90	5.03
T1 (μs)	68.30	50.50	66.30	69.50	19.60
T2 (μs)	20.20	90.20	100.90	61.90	17.30
Gate error (10^{-3})	1.82	5.94	3.38	1.46	3.05
Readout error (10^{-2})	2.46	7.32	3.60	32.07	15.81
MultiQubit gate error (10^{-2})		**CX1_0** 3.75	**CX2_3** 4.82		**CX4_3** 5.85
		CX1_2 6.03			**CX4_10** 5.88

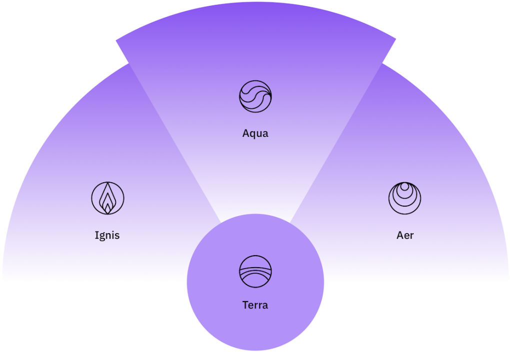
- Superconducting transmon qubits (a variant of superconducting charge qubits where states represent the presence or absence of excess Cooper pairs in the island) [PhysRevA.76.042319]
- Measurement, control and coupling of the transmons is performed by means of microwave resonators



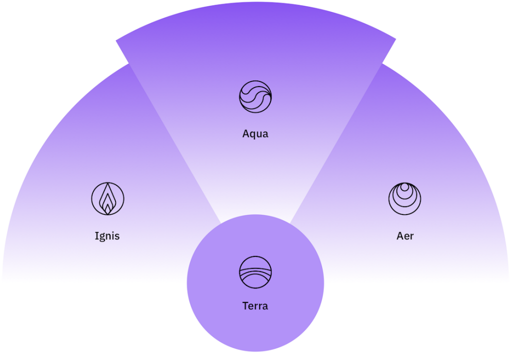
- Qiskit is an open-source framework for working with noisy quantum computers at the level of pulses, circuits, and algorithms.



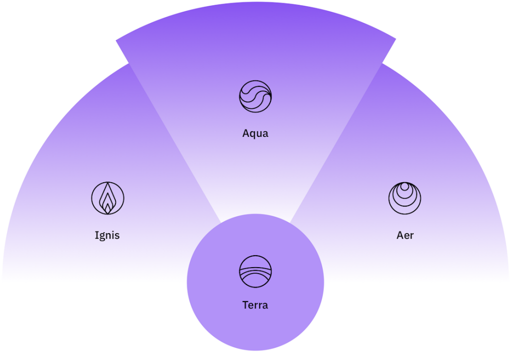
- Terra, the 'earth' element, provides a bedrock for composing quantum programs at the level of circuits and pulses



- Aer, the 'air' element, provides simulators, emulators and debuggers

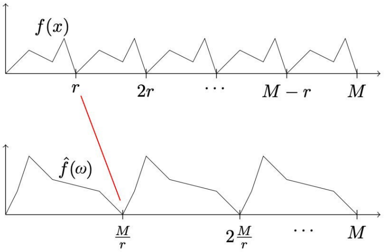


- Ignis, the 'fire' element, is dedicated to fighting noise and errors



- Aqua, the 'water' element, is dedicated to material simulation, algorithm optimization and AI

Coding ...



- The quantum Fourier transform is the classical discrete Fourier transform applied to the vector of amplitudes of a quantum state,

$$\hat{Q_{FT}} : \sum \alpha_j |j > \rightarrow \sum \tilde{\alpha_k} |k >; \tilde{\alpha_k} = 1/\sqrt{N} \sum_j e^{(2i\pi/N)^{jk}} \alpha_j$$

- $\hat{Q_{FT}}^k = \begin{pmatrix} 1 & \omega^k & \omega^{2k} & \dots & \omega^{(N-1)k} \end{pmatrix}; \omega_n = e^{2i\pi/N}$

- Unitary: $Q_{FT} Q_{FT}^{\dagger} = 1$
- Linear shift: a linear shift of a state-vector causes a relative phase shift of its Fourier transform

$$Q_{FT}^4 \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$Q_{FT}^4 \begin{pmatrix} 1/2 \\ i/2 \\ -1/2 \\ i/2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

- Period/Wavelength relationship: if f is periodic with period r, then $\hat{f}$ is periodic with M/r

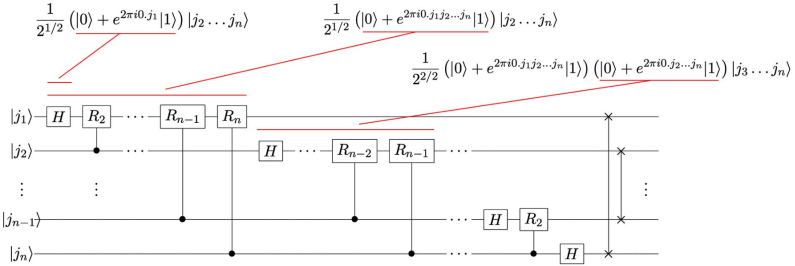
- Take $N = 2^n$ and $|0>, |1> ... |2^n - 1>$ computational basis for n-qubits
- In binary representation:

$$j = j_1 j_2 \cdots j_n = j_1 2^{n-1} + j_2 2^{n-2} + \cdots + j_n$$

$$0.j_l j_{l+1} \cdots j_m = j_l/2 + j_{l+1}/4 + \cdots + j_m/2^{m-l+1}$$

$$\begin{aligned}
|j\rangle &\rightarrow \frac{1}{2^{n/2}} \sum_{k=0}^{2^n-1} e^{2\pi i jk/2^n} |k\rangle \\
&= \frac{1}{2^{n/2}} \sum_{k_1=0}^{1} \cdots \sum_{k_n=0}^{1} e^{2\pi i j \left(\sum_{l=1}^{n} k_l 2^{-l}\right)} |k_1 \ldots k_n\rangle \\
&= \frac{1}{2^{n/2}} \sum_{k_1=0}^{1} \cdots \sum_{k_n=0}^{1} \bigotimes_{l=1}^{n} e^{2\pi i j k_l 2^{-l}} |k_l\rangle \\
&= \frac{1}{2^{n/2}} \bigotimes_{l=1}^{n} \left[\sum_{k_l=0}^{1} e^{2\pi i j k_l 2^{-l}} |k_l\rangle \right] \\
&= \frac{1}{2^{n/2}} \bigotimes_{l=1}^{n} \left[|0\rangle + e^{2\pi i j 2^{-l}} |1\rangle \right] \\
&= \frac{\left(|0\rangle + e^{2\pi i 0.j_n}|1\rangle\right)\left(|0\rangle + e^{2\pi i 0.j_{n-1}j_n}|1\rangle\right)\cdots\left(|0\rangle + e^{2\pi i 0.j_1 j_2 \cdots j_n}|1\rangle\right)}{2^{n/2}}
\end{aligned}$$

- The QFT can be implemented with a series of the controlled-R gate $R_n = \begin{pmatrix} 1 & 0 \\ 0 & e^{2i\pi/2^n} \end{pmatrix}$



Coding ...

Additional material

6-bit Quantum Ripple Adder