Restricted Boltzmann Machine

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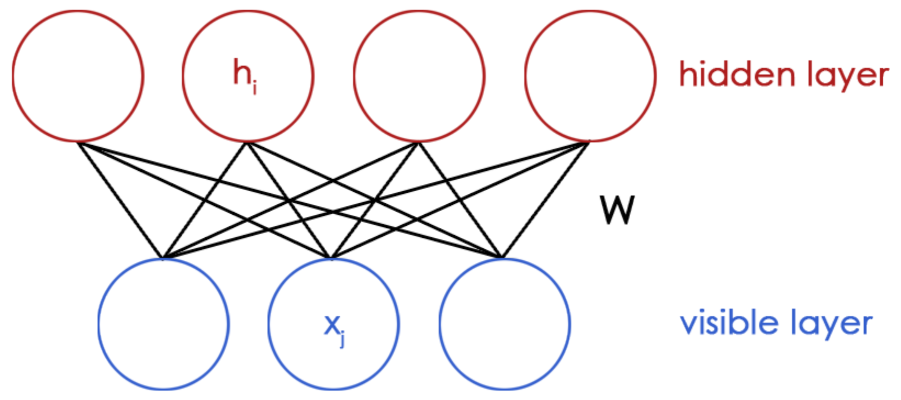
Definition

A Restricted Boltzman Machine is an undirected graph which can be decomposed in two layers.



It is *restricted* because no connection between nodes of the same layer is supposed. The matrix W characterizes the connections between the two layers. For simplification, we suppose that the variables $\mathbf{x}$ and $\mathbf{h}$ are binary.

Energy and Probability

We introduce the two bias vectors c and b to define the energy of this graph:

$$E(x, h) = -h^\top W x - c^\top x - b^\top h$$

We can write the joint probability of $\mathbf{x}$ and $\mathbf{h}$ as:

$$p(x, h) = \exp(-E(x, h))/Z$$

Z is the partition parameter which can be computed by calculating all the possibles values of $\mathbf{x}$ and $\mathbf{h}$. In practice, this parameter is intractable.
Inference We can prove that given $\mathbf{x}$, h_j follows a Bernoulli:

$$p(h_j = 1|x) = sigm(b_j + W_{j.}x)$$

$W_{j.}$ is the j-th row of W and $sigm$ defines the sigmoid function.

Contrastive Divergence

To train the RBM with our data $\mathbf{x}^{(t)}$, we would like to minimize the average loss: $\frac{1}{T}\sum_t l(f(\mathbf{x}^{(t)})) = \frac{1}{T}\sum_t -\log p(\mathbf{x}^{(t)})$
We will apply a stochastic gradient descent. We derive each term of the sum with respect to our model parameter θ as follow (where $\mathbb{E}_h$ and $\mathbb{E}_{x,h}$ are expectations of h and (x, h) respectively):

$$\frac{\partial -\log p(\mathbf{x}^{(t)})}{\partial\theta} = \mathbb{E}_h\left[\frac{\partial E(\mathbf{x}^{(t)}, h)}{\partial\theta}|\mathbf{x}^{(t)}\right] - \mathbb{E}_{x,h}\left[\frac{\partial E(\mathbf{x}, h)}{\partial\theta}\right]$$

The first term is called the *positive phase* and the second term the *negative phase*. Because of the difficulty to compute the seconde term, we will use an algorithm called *Contrastive Divergence*. The key points of the algorithm are:

- We estimate the expectation $\mathbb{E}_{x,h}$ by sampling a single point $\tilde{\mathbf{x}}$.
- To do so, we use Gibbs sampling in chain (we apply it k times).
- We initialize our Gibbs sampling with $\mathbf{x}^{(\mathbf{t})}$.

Future Research

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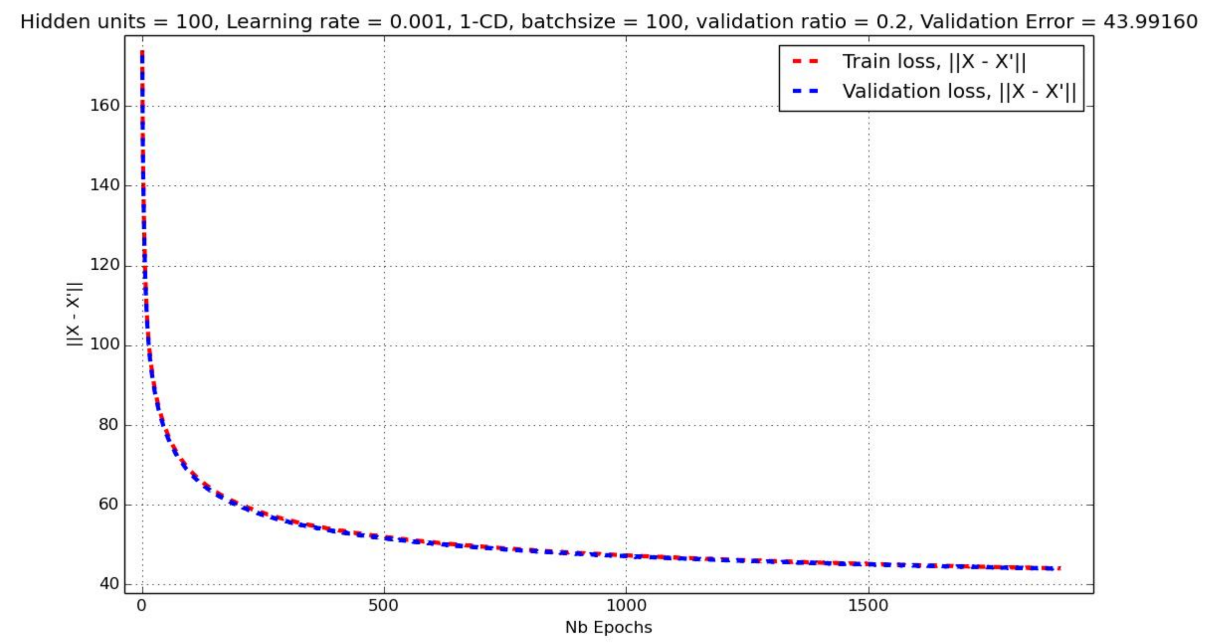
Results

We implemented the algorithm and applied it to the MNIST dataset.



Conclusion

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References

[1] Asja Fischer and Christian Igel. Training restricted boltzmann machines: An introduction. *Pattern Recognition*, 47(1):25–39, 2014.

[2] Geoffrey Hinton. A practical guide to training restricted boltzmann machines. *Momentum*, 9(1):926, 2010.

[3] Tijmen Tieleman. Training restricted boltzmann machines using approximations to the likelihood gradient. In *Proceedings of the 25th international conference on Machine learning*, pages 1064–1071. ACM, 2008.