In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
from skimage import measure
```

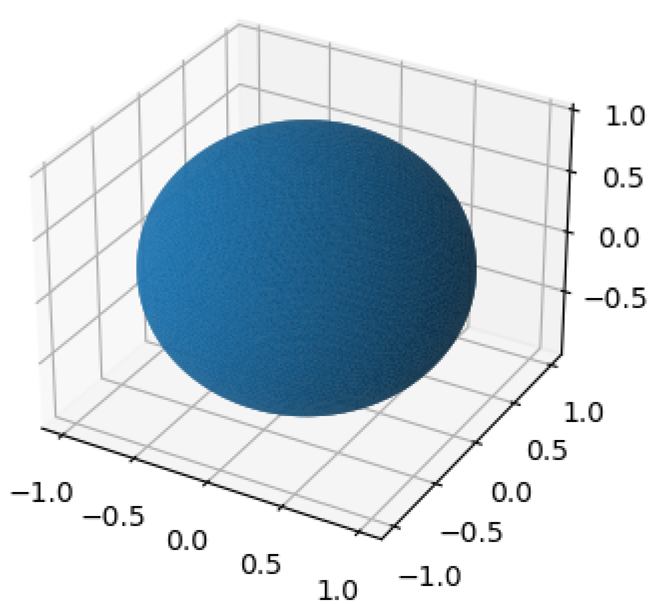
CAD Extra Credit Homework by Sasha Bakker

Initialize Coordinates

In [2]:
```
x, y, z = np.mgrid[-2:2:101j, -2:2:101j, -2:2:101j]
l, m, n = np.shape(x)
```

Sphere

In [3]:
```
sphere = np.sqrt(x ** 2 + y ** 2 + z ** 2) - 1

verts, faces, normals, values = measure.marching_cubes(sphere, 0, spacing=(0.04,

verts[:,0]=verts[:,0]-np.mean(verts[:,0]) # translate the coordinates of mesh ver
verts[:,1]=verts[:,1]-np.mean(verts[:,1]) # translate the coordinates of mesh ver
verts[:,2]=verts[:,2]-np.mean(verts[:,2]) # translate the coordinates of mesh ver

fig = plt.figure(dpi=100)
ax = fig.add_subplot(111, projection='3d')
ax.plot_trisurf(verts[:, 0], verts[:,1], faces, verts[:, 2])
plt.show()
```



Check the radius of the sphere:

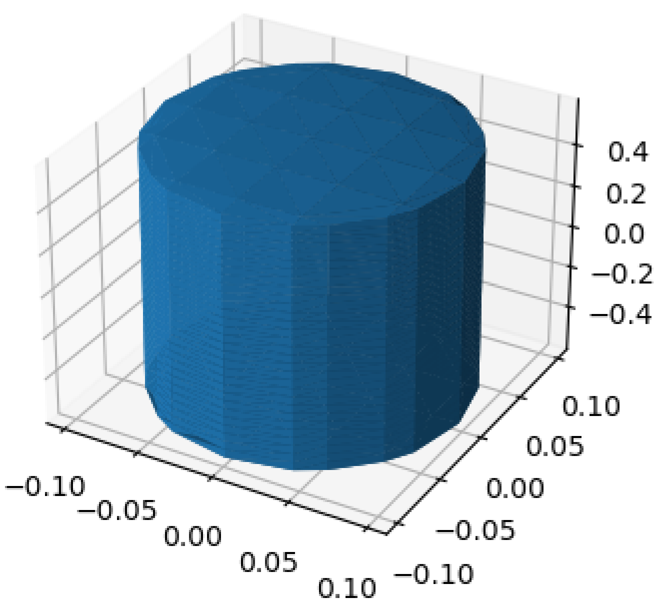
In [4]:
```
print(max(verts[:,0]))
print(min(verts[:,0]))
```

```
1.0004764334766438
-0.9995235665233562
```

The sphere is centered at (0,0) with radius $r = 1$. This means the domain of the sphere in x should be (-1,1). We see that this is true because the min and max values in x are $\min(x) = -1$ and $\max(x) = 1$, within the uncertainty due to the mesh discretization of ± 0.04.

Cylinder

In [5]:
```
cylinder = x**2 + y**2 - 0.1**2
plane1 = z - 0.7
plane2 = z + 1/2

# Cylinder with lids
sqrcil = np.zeros((l,m,n))

for i in range(l):
    for j in range(m):
        for k in range(n):
            sqrcil[i,j,k] = np.max( np.array( [cylinder[i,j,k], plane1[i,j,k] ]
            sqrcil[i,j,k] = np.max( np.array( [sqrcil[i,j,k], -plane2[i,j,k] ]  )

verts, faces, normals, values = measure.marching_cubes(sqrcil, 0, spacing=(0.04,

verts[:,0]=(verts[:,0]-np.mean(verts[:,0])) # translate the coordinates of mesh w
verts[:,1]=(verts[:,1]-np.mean(verts[:,1])) # translate the coordinates of mesh w
verts[:,2]=(verts[:,2]-np.mean(verts[:,2])) # translate the coordinates of mesh w

# PLOTTING
fig = plt.figure(dpi=100)
ax = fig.add_subplot(111, projection='3d')
ax.plot_trisurf(verts[:, 0], verts[:,1], faces, verts[:, 2])
plt.show()
```



Check the radius of the cylinder:

In [6]:
```
print(max(verts[:,0]))
print(min(verts[:,0]))
```

```
0.09800003051757811
-0.09800003051757811
```

The cylinder is centered at (0,0) with radius $r = 0.1$. This means the domain of the sphere in x

should be (-0.1,0.1). We see that this is true because the min and max values in x are $\min(x) = -0.1$ and $\max(x) = 0.1$, within the uncertainty due to the mesh discretization of ± 0.04.

Check the height of the cylinder:

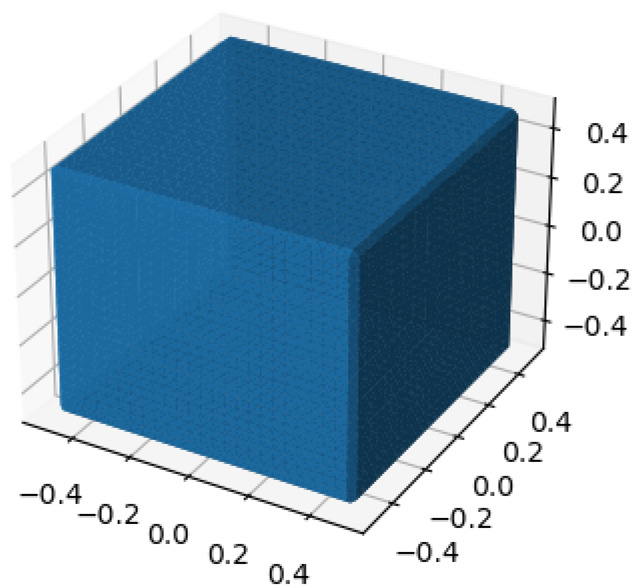
In [7]:
```
print(max(verts[:,2]) - min(verts[:,2]))
```

```
1.186666717529297
```

The cylinder needs to have a height $h = 1.2$ such that, when it intersects with a unit cube, a height of 0.2 of the cylinder sticks out the top of the cube. We see that the height is correct because $\max(x) - \min(x) = 1.2$, within the uncertainty due to the mesh discretization of ± 0.04.

Unit Cube

In [8]:
```python
unitcube = np.zeros((l, m, n))

for i in range(l):
    for j in range(m):
        for k in range(n):
            unitcube[i,j,k] = np.max( np.array([ np.abs(x[i,j,k]) , np.abs(y[i,j,

verts, faces, normals, values = measure.marching_cubes(unitcube, 0, spacing=(0.04

verts[:,0]=(verts[:,0]-np.mean(verts[:,0])) # translate the coordinates of mesh v
verts[:,1]=(verts[:,1]-np.mean(verts[:,1])) # translate the coordinates of mesh v
verts[:,2]=(verts[:,2]-np.mean(verts[:,2])) # translate the coordinates of mesh v

# PLOTTING
fig = plt.figure(dpi=100)
ax = fig.add_subplot(111, projection='3d')
ax.plot_trisurf(verts[:, 0], verts[:,1], faces, verts[:, 2])
plt.show()
```



Check the length of the cube:

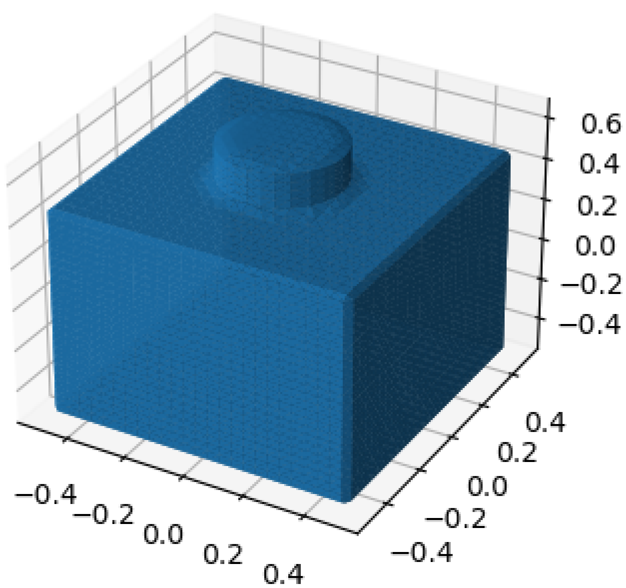
In [9]:
```python
print(max(verts[:,0]) - min(verts[:,0]))
print(max(verts[:,1]) - min(verts[:,1]))
print(max(verts[:,2]) - min(verts[:,2]))
```

```
1.0
1.0
1.0
```

The cube is centered at (0,0) with length $L = 1$. This means the domain of the cube in x, y, and z should be (-0,5, 0.5). We see that this is true because the length computed in each dimension is $\max(x) - \min(x) = 1$, within the uncertainty due to the mesh discretization of ± 0.04.

Cylinder Intersection with Cube (Min/Max)

In [10]:
```
cylinder = x**2 + y**2 - 0.2**2
plane1 = z - 0.7
plane2 = z + 1/2
sqrcil = np.zeros((l,m,n))
unitcube = np.zeros((l, m, n))
unioncube = np.zeros((l,m,n))


for i in range(l):
    for j in range(m):
        for k in range(n):
            sqrcil[i,j,k] = np.max( np.array( [cylinder[i,j,k], plane1[i,j,k] ]
            sqrcil[i,j,k] = np.max( np.array( [sqrcil[i,j,k], -plane2[i,j,k] ]  )
            unitcube[i,j,k] = np.max( np.array([ np.abs(x[i,j,k]) , np.abs(y[i,j,
            unioncube[i,j,k] = np.min( np.array([unitcube[i,j,k], sqrcil[i,j,k] ]

verts, faces, normals, values = measure.marching_cubes(unioncube, 0, spacing=(0.0

verts[:,0]=(verts[:,0]-np.mean(verts[:,0])) # translate the coordinates of mesh w
verts[:,1]=(verts[:,1]-np.mean(verts[:,1])) # translate the coordinates of mesh w
verts[:,2]=(verts[:,2]-np.mean(verts[:,2])) # translate the coordinates of mesh w

# PLOTTING
fig = plt.figure(dpi=100)
ax = fig.add_subplot(111, projection='3d')
ax.plot_trisurf(verts[:, 0], verts[:,1], faces, verts[:, 2])
plt.show()
```

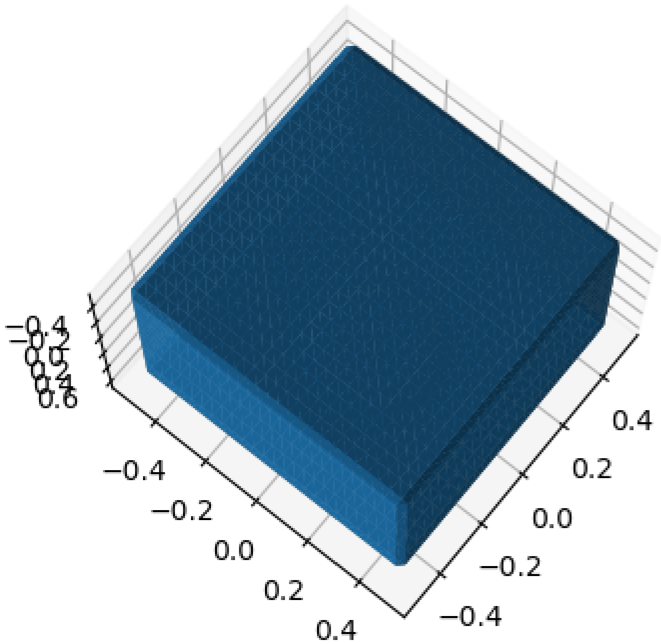


In [11]:
```
print(max(verts[:,2]) - min(verts[:,2]))
```

```
1.2000000000000002
```

The maximum height in the domain z is 1.2 just as we would expect, such that the height of the cylinder component is 0.2 and the height of the cube component is 1.

In [12]:
```
# PLOTTING
fig = plt.figure(dpi=100)
ax = fig.add_subplot(111, projection='3d')
ax.plot_trisurf(verts[:, 0], verts[:,1], faces, verts[:, 2])
ax.view_init(-109, -39)
plt.show()
```



We see that the bottom of the surface is completely flat.

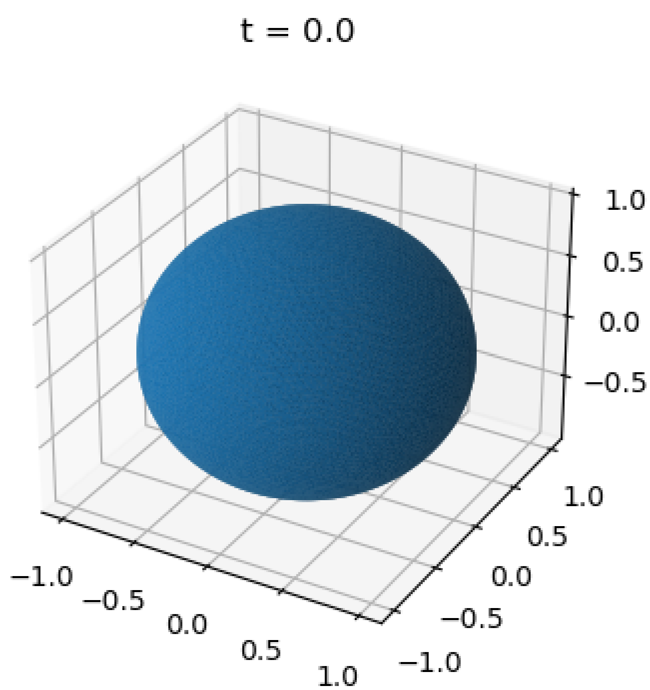
Metamorphasis

$$f(x, y, x, t) = f_1(x, y, z) \cdot (1 - t) + f_2(x, y, z) \cdot t$$

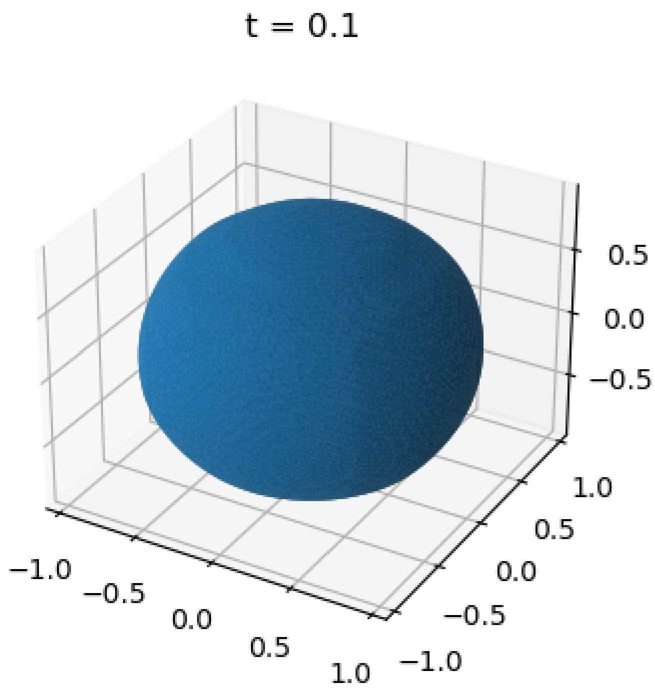
$\Delta t = 0.1$

In [13]:
```
def compute_meta(f1, f2, t):

    return f1 * (1-t) + f2 * t

def plot_meta(meta, t):

    verts, faces, normals, values = measure.marching_cubes(meta, 0, spacing=(0.04

    verts[:,0]=(verts[:,0]-np.mean(verts[:,0])) # translate the coordinates of me
    verts[:,1]=(verts[:,1]-np.mean(verts[:,1])) # translate the coordinates of me
    verts[:,2]=(verts[:,2]-np.mean(verts[:,2])) # translate the coordinates of me

    # PLOTTING
    fig = plt.figure(dpi=100)
    ax = fig.add_subplot(111, projection='3d')
    ax.plot_trisurf(verts[:, 0], verts[:,1], faces, verts[:, 2])
    ax.set_title(f"t = {t}")
    plt.show()
```

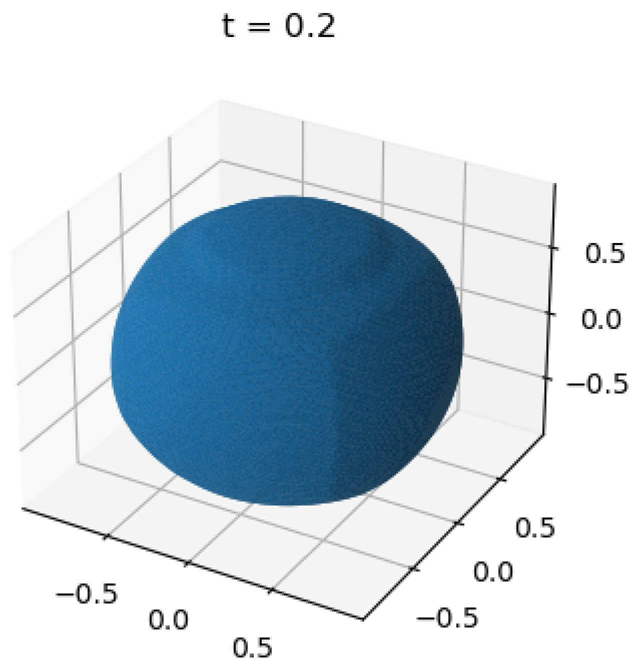
In [14]:
```
t = 0.0
meta = compute_meta(sphere, unioncube, t)
plot_meta(meta, t)
```



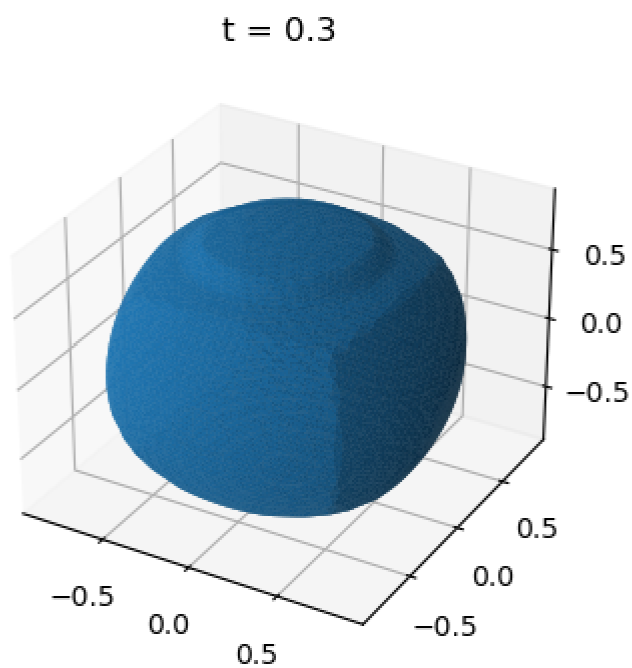
In [15]:
```
t = 0.1
meta = compute_meta(sphere, unioncube, t)
plot_meta(meta, t)
```



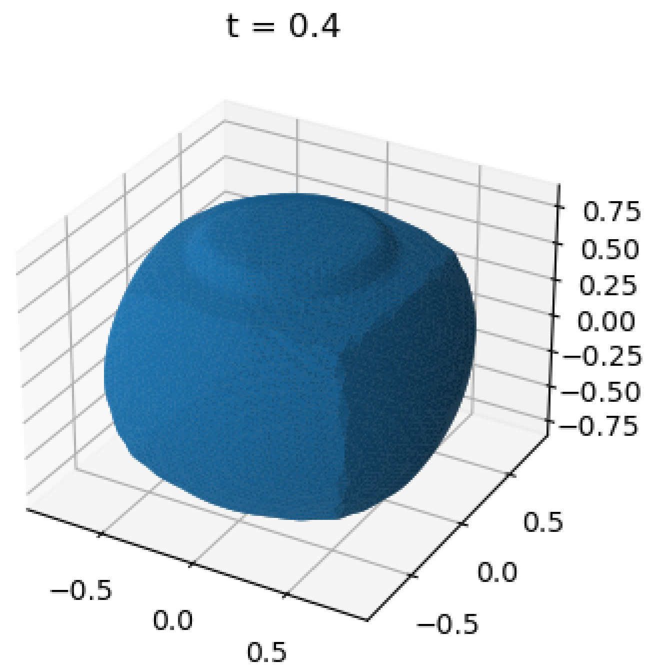
In [16]:
```
t = 0.2
meta = compute_meta(sphere, unioncube, t)
plot_meta(meta, t)
```



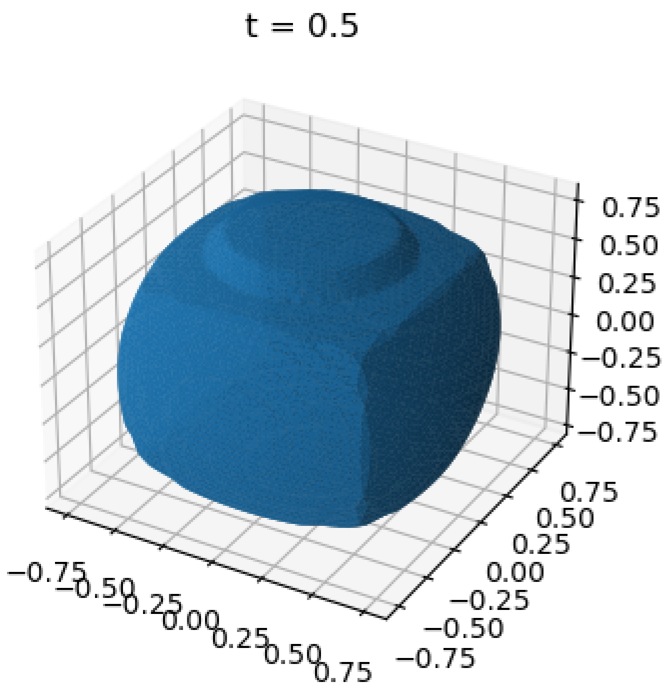
In [17]:
```
t = 0.3
meta = compute_meta(sphere, unioncube, t)
plot_meta(meta, t)
```



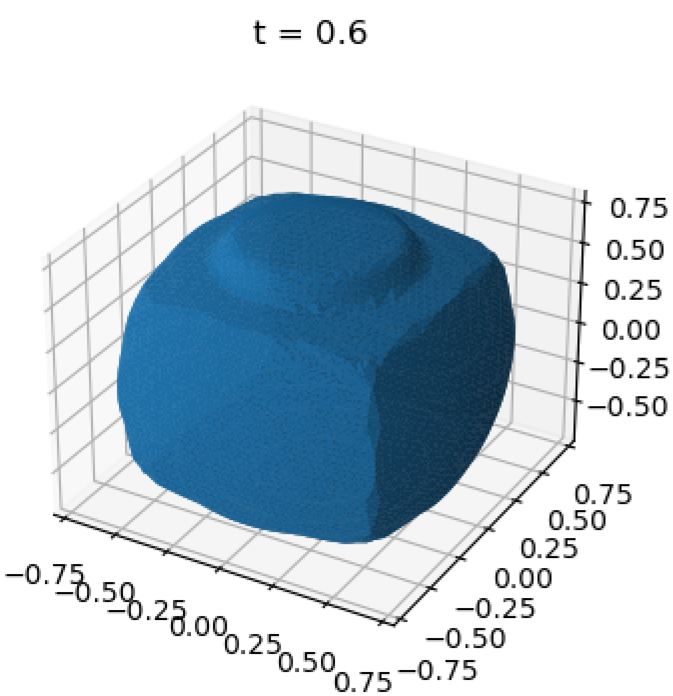
In [18]:
```
t = 0.4
meta = compute_meta(sphere, unioncube, t)
plot_meta(meta, t)
```



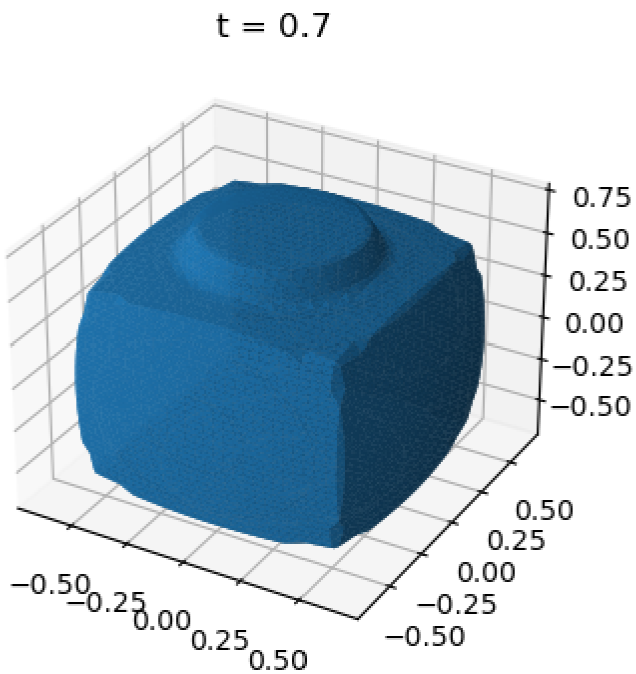
In [19]:
```
t = 0.5
meta = compute_meta(sphere, unioncube, t)
plot_meta(meta, t)
```



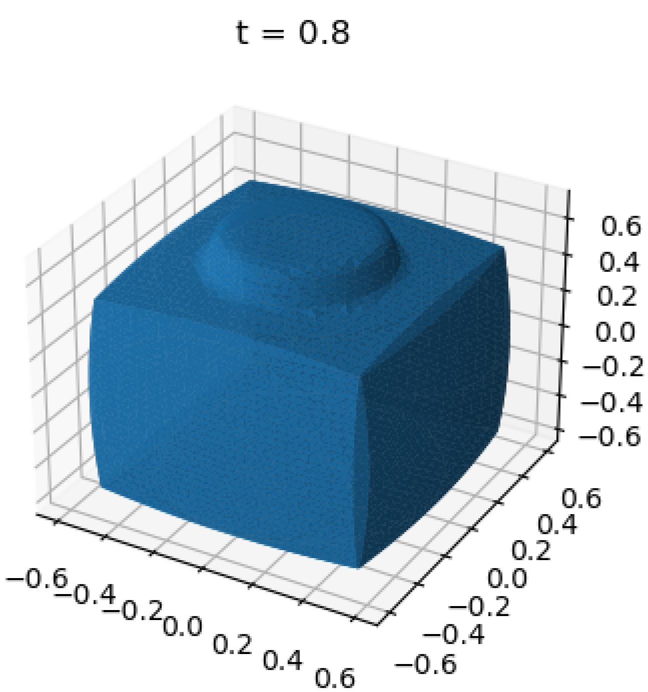
In [20]:
```
t = 0.6
meta = compute_meta(sphere, unioncube, t)
plot_meta(meta, t)
```



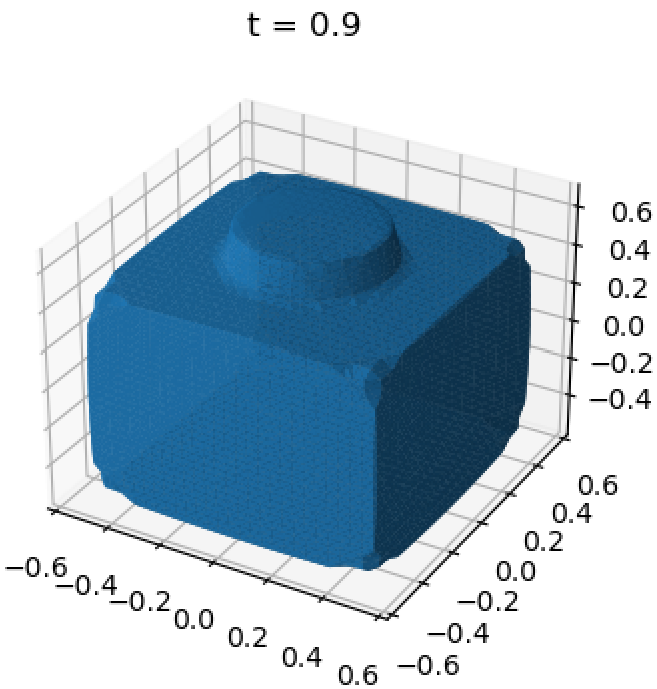
In [21]:
```
t = 0.7
meta = compute_meta(sphere, unioncube, t)
plot_meta(meta, t)
```



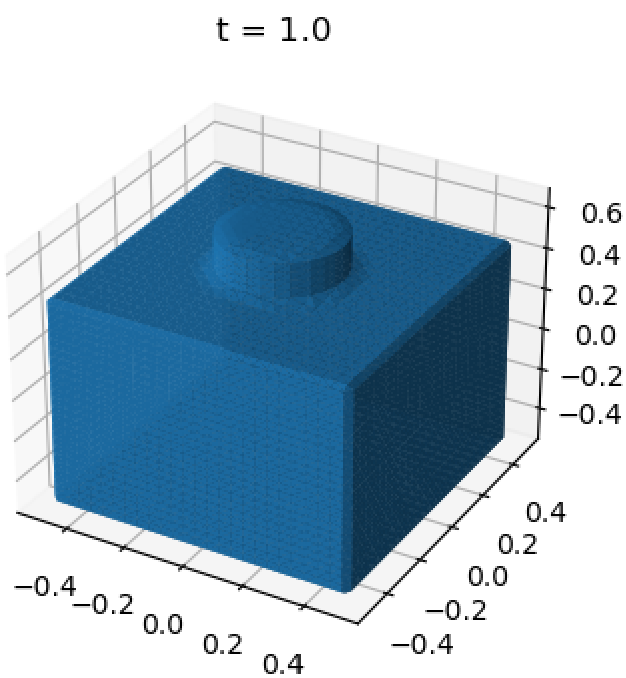
In [22]:
```
t = 0.8
meta = compute_meta(sphere, unioncube, t)
plot_meta(meta, t)
```



In [23]:
```
t = 0.9
meta = compute_meta(sphere, unioncube, t)
plot_meta(meta, t)
```



In [24]:
```
t = 1.0
meta = compute_meta(sphere, unioncube, t)
plot_meta(meta, t)
```



In [ ]:
```

```