Report on Development of Algorithm for Coding of Skip Jump Depth (SJD) Hidden Markov Models for Estimation of Time Variant Hidden Markov Models

A. Theoretical Background and Relevance in the context of our Proposed Research Objectives:

- Among the topologies used for HMM, ordering of states from low to high is a Left-Right model while arriving at any hidden state from any other state is an ergodic model. Usually it is observed in a few engineering problems that issues related to estimation and ordering of states are critical in recognition of the nature of system dynamics. These topologies are parameterized by a term called the Jump Depth (JD). JD relates to the maximum number of states (excluding the present state) that can be jumped to from a particular state. For instance, a JD of 1 implies jump from a state to itself (or) to its immediate neighbor. However, for JD >1, the transition is called “skip transition”. A skip transition is one which results the sequence to skip over one or more states.

- In this context, **it is important to note that in time variant models there are possibilities that some states may never be visited by a particular sequence. Hence, if we observe that if such a transition exists, the estimated parameters of the skipped state should not be updated based on the particular data sequence.** Such a capability of identifying and modifying the estimate of the ‘B’ (observation probability matrix) does not exist in generic algorithms developed thus far.

- In our research problem, we had observed during our initial studies (please refer to previous report submitted during our review at LIMAT with regard to non-stationary CDHMM) that the values for the ‘B’ matrix (probability density estimate) for lack of PD (‘no’ PD) in a specified phase window of a phi-q (5° and 10°) sequence was extremely large while those related to PD was substantially small, leading to confusion in the plot of ‘B matrix versus various state sequences. Hence, it becomes essential that to avoid any confusion/ misinterpretation that those estimates pertaining to ‘no’ PD window is considered as trivial and skipped. Thus, this procedure of Skip Jump Depth (SJD) HMM becomes an appropriate candidate as a technique for this proposed research.

B. Macro Level Algorithm for Implementation of Skip Jump Depth HMM

Step 1: Obtaining the Initial State Probabilities (π_i) for λ = (A, B, π)

where ‘A’ is the transition probability matrix

‘B’ is the Initial information available at the nodes using first observation sequence

‘π’ is the initial state probability vector

Compute $$tmp_i = \frac{1}{JD^*} + 0.1RND(\ ); \quad i \in [1, JD^*]$$

$$\pi_i = \frac{tmp_i}{\sum_{j=1}^{JD^*} tmp_i}$$

The reference paper [1] assumes uniform distribution.

Step 2: Computing Transition Probabilities (a_{ij}):

- $$temp_{ij} = \frac{1}{JD} + 0.1RND(\,);\; i \in [1, N]\, and\; j \in [i, i + JD]^1$$
- $$a_{ij} = \frac{tmp_{ij}}{\sum_{j=i}^{i+JD} tmp_{ij}}$$

Step 3: Procedure for Initialization of B Matrix (AVGINIT)

- **All training sequences are divided into 'N' equal time segments corresponding to 'N' states**
- **For each state the tokens (state labels) that were observed during the corresponding time segments are accumulated and probability of observing the symbol (state label) is computed from the resulting average**
- **To avoid difficulties during computation the minimum observable probability is set to zero**

Step 4: Training of HMM Using Forward, Backward and Baum- Welch Algorithm

- **Forward Algorithm: With initial probability sequence, A & B what is the probability of occurrence in the context of A & B?**
- **Backward Algorithm: Reserve approach (either forward or backward depending on threshold given by**

  $$PV(i) = \pi_i + \sum_{j=1}^{i=1} PV(j) \frac{a_{ii}}{1 - a_{jj}}$$
- **Using this with forward and backward probability estimation find the probability of visiting node (with all the earlier initial estimates)**
- **Estimated values of (A, B, π) in this context with the available/ estimated through skip jump states**

Step 5: Skip- Train Procedure

- **Compute the probability of visiting a node:**

  $$PV(i) = \pi_i + \sum_{j=1}^{i=1} PV(j) \frac{a_{ii}}{1 - a_{jj}}$$

If PV(i) $\geq$ 0 then update state 'i'

Else Do not update state 'i'

Step 6: Updating Probability for Non-skipped State Transitions

- **'i' is updated, if PV(i) $<$ 0**
- **θ - PV(i) needs to be distributed over the rest of non-skipped nodes which are connected to the current nodes**
- **End the loop while Step 5 and Step 6 are complete**

C. Coding and Hand Computation Tasks Implemented/ Completed:

- **Forward Algorithm**
- **Backward Algorithm**
- **Baum- Welch Algorithm**
- **Computation and validation for Rain- Dry based dataset specified in reference paper for all states considered**

D. Pending Work and Novelties Perceived in the Proposed Research Work:

- **Execution of loops for computation of (now checked with hand computation for existing data of the reference paper) A, B, π according to θ**
- **Novelties in the choice of distribution (Cauchy, Poisson etc)**

Reference:

[1] Solomon Lemer and Baruch Mazor, "Issues related to the estimation of time variant hidden markov models", S10.17, pp. 553-556.