Formula Sheet (DRAFT)

1. Optimization

Second derivative test: Let f be a twice-differentiable function, and let

$$H_f = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}.$$

If $\nabla f(x, y) = \mathbf{0}$, then:

(a) f has a relative maximum at (x, y) if $\det(H_f(x, y)) > 0$ with $f_{xx}(x, y) < 0$ or $f_{yy}(x, y) < 0$;
(b) f has a relative minimum at (x, y) if $\det(H_f(x, y)) > 0$ with $f_{xx}(x, y) > 0$ or $f_{yy}(x, y) > 0$;
(c) f has a saddle point at (x, y) if $\det(H_f(x, y)) < 0$;
(d) no conclusion may be drawn if $\det(H_f(x, y)) = 0$.

2. Double and triple integration

Polar coordinates:

$$x = r\cos\theta, \qquad y = r\sin\theta, \qquad dA = r\,dr\,d\theta$$

Cylindrical coordinates:

$$x = r\cos\theta, \qquad y = r\sin\theta, \qquad z = z, \qquad dV = r\,dz\,dr\,d\theta$$

Spherical coordinates:

$$x = \rho\sin\phi\cos\theta, \qquad y = \rho\sin\phi\sin\theta, \qquad z = \rho\cos\phi, \qquad dV = \rho^2\sin(\phi)\,d\rho\,d\theta\,d\phi$$

3. Vector fields

Line integrals:

$$\int_C f\,ds = \int_a^b f(\mathbf{r}(t))\|\mathbf{r}'(t)\|dt, \qquad \int_C \mathbf{F}\cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)dt$$

Surface integrals:

If Σ is parametrized by $\mathbf{r}$:

$$\int_\Sigma f\,dS = \iint_R f(\mathbf{r}(s,t))\|\mathbf{r}_s\times\mathbf{r}_t\|dA$$

If Σ is the graph of g:

$$\int_\Sigma f\,dS = \iint_R f(x, y, g(x, y))\sqrt{\|\nabla g\|^2 + 1}\,dA$$

Flux:

$$\int_\Sigma \mathbf{F}\cdot\mathbf{n}\,dS$$