CS1800 Discrete Structures	Prof Higger & Prof Hamlin
Fall 2024	Nov 19, 2024

HW8: Function Growth, Sequences & Series

Due: Nov 26, 2024 @ 11:59 PM

Instructions:

- HW instructions
- academic integrity and collaboration

Problem 1 [36 pts]: Sequences & Series

For each of the following sequences:

- Identify if the sequence is arithmetic, geometric or quadratic[1]. Justify your response.
- Give an expression for a_k, the k-th term in the sequence. Use the convention that the first term shown in each sequence below corresponds to $k = 0$.
- If the sequence is arithmetic or geometric, compute the sum of the first 11 terms in the sequence (i.e. $a_0 + a_1 + \ldots + a_{10}$)

i 18, 72, 288, 1152, 4608, 18432, ...

ii -1, 1, 7, 17, 31, 49, ...

iii 0, -1, -2, -3, -4, -5, ...

Problem 2 [18 pts (3 each)]: Function Growth True/False

Tell whether the statements are true or false, no justification is needed.

i $5n^3 + 2n = O(n^4)$

ii $n^5 = O(7n + 1)$

iii If a function $2n^3 + 6n = O(h(n))$, then $2n^3 + 6n > h(n)$ for every value of n.

[1] its possible a sequence can be neither arithmetic, geometric or quadratic, but each of these examples is of one of these three types

iv $4n + 1 = \Omega(n^2)$

v $6n^2 + 4 = O(6n^2 + 4)$

vi $5 \log_2 n = \Theta(7 \log_{10} n + 1)$[2]

Problem 3 [22 pts (14, 8) pts]: Function Growth

i Organize the following functions into seven columns. Items in the same column should have the same asymptotic growth rates (big-O). If a column is to the left of another column, all its growth rates should be slower than those of the column(s) to its right.

n^2, $n!$, $\log_2 n$, $n \log_2 n$, $3n$, $5n^2 + 3$, 2^n, 10000, $n \log_3 n$, $100n, 3log_3 n$

ii Identify the simplest[3] function $f(n)$ which has $3n + 4n^2 + 3n! = O(f(n))$.

Problem 4 [24 pts (6 each)]: Demonstrating Function Growth

Recall that $f(x) = O(g(x))$ means that:

$$\exists c, x_0 \in \mathbb{R} \text{ such that } x \geq x_0 \rightarrow 0 \leq f(x) \leq cg(x).$$

Using this definition, show that each of the statements below is true, or explain in one sentence why the statement is false. To prove a statement is true, find "witness values" c, x_0 such that $0 \leq f(x) \leq cg(x)$ for $x \geq x_0$. For clarity, write out this inequality for each true statement.

i $2^x = O(3^x)$

ii $5x^3 + x = O(x^3)$

iii $x^4 = O(\ln(x))$

iv $4x + 7 = O(x^2)$

[2] Recall that you can change the base of logarithms using:

$$\log_a b \cdot \log_b x = \log_a x$$

[3] having the fewest operations