MATH 231-01: Formula Sheet for Midterm 1

1. Vectors

$$\mathbf{v} = \langle v_1, v_2, v_3 \rangle, \quad \|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3 = \|\mathbf{u}\| \, \|\mathbf{v}\| \cos\theta$$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}, \quad \|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\| \, \|\mathbf{v}\| \sin\theta$$

$$\mathrm{proj}_{\mathbf{u}} \mathbf{v} = \frac{\mathbf{v} \cdot \mathbf{u}}{\|\mathbf{u}\|^2} \mathbf{u}$$

2. Lines and Planes

If $P = (x_0, y_0, z_0)$, then:

$$\text{line through } P \text{ with direction } \mathbf{d}: \quad \langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t\mathbf{d}$$

$$\text{plane through } P \text{ with normal } \mathbf{n}: \quad \mathbf{n} \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$$

3. Vector Functions

$$\mathbf{v}(t) = \mathbf{r}'(t), \quad \mathbf{a}(t) = \mathbf{r}''(t), \quad \text{speed } = \|\mathbf{v}(t)\|$$

$$\text{arc length: } s = \int_a^b \|\mathbf{r}'(t)\| \, dt$$

$$\text{curvature: } \kappa = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$$

4. Multivariable Functions

$$f_x = \frac{\partial f}{\partial x}, \quad f_y = \frac{\partial f}{\partial y}$$

$$\text{plane tangent to the graph: } f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) = z - f(x_0, y_0)$$

$$\text{linear approximation: } f(x, y) \approx f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

5. Gradient and Directional Derivatives

$$\nabla f = \langle f_x, f_y, f_z \rangle, \quad D_{\mathbf{u}} f = \nabla f \cdot \mathbf{u}$$

∇f points in direction of fastest increase of f

∇f is orthogonal to the level surface/curve

6. Chain Rule

If $z = f(x, y)$, $x = g(s, t)$, $y = h(s, t)$, then

$$\frac{\partial z}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s}, \qquad \frac{\partial z}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$$