CS1800 Homework 3 Solutions

Problem 1: Beatles Set Representation

Given:

$$A = \{\text{paul, george}\}, \quad B = \{\text{ringo, george}\}, \quad U = \{\text{john, paul, ringo, george}\}$$

The bit string representations:

$$A = 0110, \quad B = 0011$$

i. $A \cup B$

$$A \cup B = 0110 \text{ OR } 0011 = 0111$$

ii. $A \cap B$

$$A \cap B = 0110 \text{ AND } 0011 = 0010$$

iii. A^C

$$A^C = 1001$$

Problem 2: Set Operations (Listing)

Given:

$$A = \{2, 4, 6, 8\}, \quad B = \{1, 3, 5\}, \quad U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

i. $\{x - 1 \in U \mid x \in A\}$

$$\{x - 1 \mid x \in A\} = \{1, 3, 5, 7\}$$

ii. $\{x \in B \mid x \text{ is even}\}$

$$\emptyset$$

iii. $\{x \in A \mid x + 3 \in U\}$

$$\{6\}$$

iv. $A \cap B$

$$\emptyset$$

v. $A \cup B$

$$A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$$

vi. $B - A$

$$B - A = \{1, 3, 5\}$$

vii. $(A \cap B^C)^C$

$$(A \cap B^C)^C = \{1, 3, 5, 7, 9\}$$

viii. $A \triangle B$

$$A \triangle B = \{1, 2, 3, 4, 5, 6, 8\}$$

Problem 3: Set Operations (Shading)

i. $A \cup (B - C)$

ii. $(A \cup B) - C$

iii. $A^C \cap B^C$

iv. $((A^C \cup B^C) \cup C^C)^C$

v. $(B^C \cap B) \cup (C \cap A)$

vi. $(C - A) \cup (A - B) \cup (B - C)$

Problem 4: Set Algebra

i. $A \cap A$

$$A \cap A = A$$

ii. $(A^C \cap B^C)^C \cap U$

$$(A^C \cap B^C)^C \cap U = (A \cup B)$$

iii. $(A \cup A) \cap (B \cup A^C)$

$$(A \cup A) \cap (B \cup A^C) = A \cap B$$

Problem 5: Set Builder Notation

i. Express the set:

$$S = \{n \in \mathbb{Z} \mid n \in \mathbb{N}, -5 \leq n < 7\}$$

The list:

$$S = \{0, 1, 2, 3, 4, 5, 6\}$$

ii. Express the set B of integers whose fourth power is either 16 or 81:

$$B = \{x \in \mathbb{Z} \mid x^4 = 16 \text{ or } x^4 = 81\}$$

iii. The list for B:

$$B = \{-3, -2, 2, 3\}$$

Problem 6: Digital Circuit

i. Express Y in terms of A, B, C using logical operators.

ii. Simplify Y using logic identities.

iii. Draw the simplified logic circuit.