CS1800 Homework 3 Solutions

Problem 1: Beatles Set Representation

Given:

$$A = \{paul, george\}, \quad B = \{ringo, george\}, \quad U = \{john, paul, ringo, george\}$$

Bit string representation:

$$A = 0101, \quad B = 0011$$

i. $A \cup B$ (Union)

Union of two sets is represented using the OR operator:

$$A \cup B = 0111 \quad \text{(OR)}$$

ii. $A \cap B$ (Intersection)

Intersection of two sets is represented using the AND operator:

$$A \cap B = 0001 \quad \text{(AND)}$$

iii. A^C (Complement of A)

The complement of A is represented using the NOT operator:

$$A^C = 1010 \quad \text{(NOT)}$$

Problem 2: Set Operations (Listing)

Given:

$$A = \{2, 4, 6, 8\}, \quad B = \{1, 3, 5\}, \quad U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

i. $\{x - 1 \in U \mid x \in A\}$

Subtracting 1 from each element in A:

$$\{1, 3, 5, 7\}$$

ii. $\{x \in B \mid x$ **is even**$\}$

Since no element in B is even:

$$\emptyset$$

iii. $\{x \in A \mid x + 3 \in U\}$

Checking elements in A for which $x + 3$ is in U:

$$\{2, 4, 6\}$$

iv. $A \cap B$

No common elements between A and B:

$$A \cap B = \emptyset$$

v. $A \cup B$

The union of A and B:

$$A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$$

vi. $B - A$

Elements in B but not in A:

$$B - A = \{1, 3, 5\}$$

vii. $(A \cap B^C)^C$

Finding the complement of B in U, intersecting with A, and then taking the complement:

$$(A \cap B^C)^C = \{1, 3, 5, 7, 9\}$$

viii. $A \triangle B$

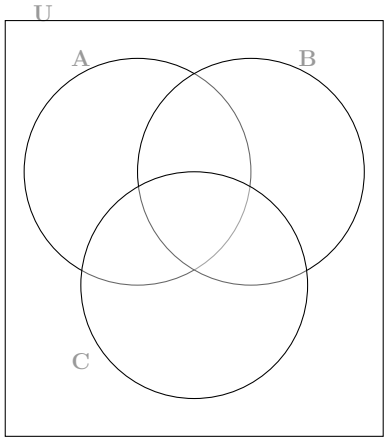
The symmetric difference between A and B:

$$A \triangle B = \{1, 2, 3, 4, 5, 6, 8\}$$

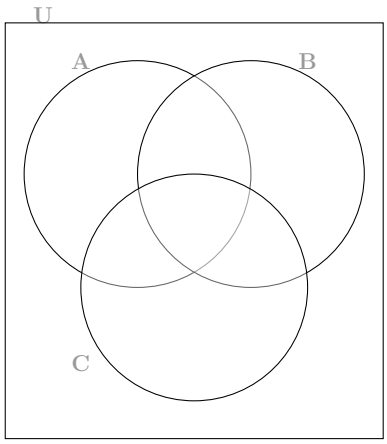
Problem 3: Set Operations (Shading)

For this problem, the regions of the Venn diagrams are shaded based on the following set expressions:

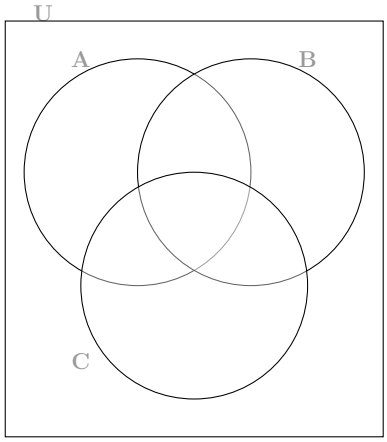
i. $A \cup (B - C)$



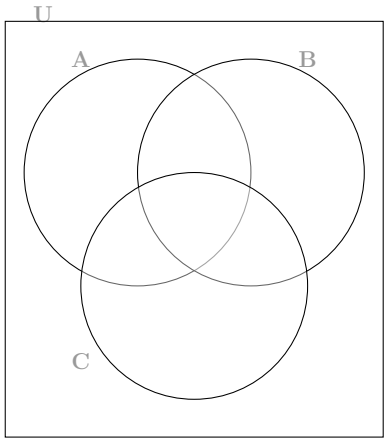
ii. $(A \cup B) - C$



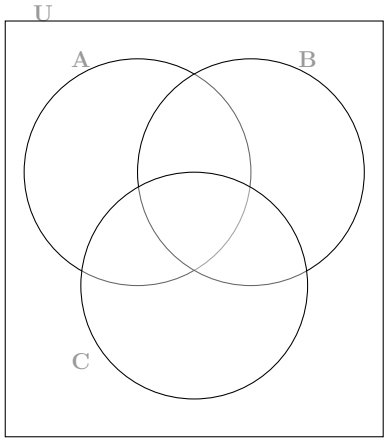
iii. $A^C \cap B^C$



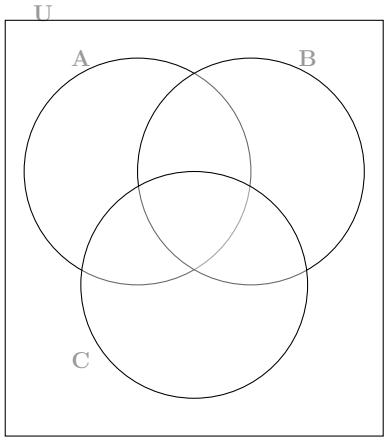
iv. $((A^C \cup B^C) \cup C^C)^C$



v. $(B^C \cap B) \cup (C \cap A)$



vi. $(C - A) \cup (A - B) \cup (B - C)$



Problem 4: Set Algebra

i. $A \cap A$

Using the Idempotent Law:

$$A \cap A = A$$

ii. $(A^C \cap B^C)^C \cap U$

Applying De Morgan's Law and simplifying:

$$(A^C \cap B^C)^C = A \cup B, \quad A \cup B \cap U = A \cup B$$

Answer: $A \cup B$

iii. $(A \cup A) \cap (B \cup A^C)$

Using the Idempotent Law and simplifying:

$$A \cup A = A, \quad B \cup A^C = U, \quad A \cap U = A$$

Answer: A

Problem 5: Set Builder Notation

i. Set $S = \{n \in \mathbb{Z} \mid n \in \mathbb{N} \text{ and } -5 \leq n < 7\}$

Listing the elements:

$$S = \{0, 1, 2, 3, 4, 5, 6\}$$

ii. Express the set B of integers whose fourth power is either 16 or 81.

Using set-builder notation:

$$B = \{x \in \mathbb{Z} \mid x^4 = 16 \text{ or } x^4 = 81\}$$

iii. Listing the elements of B:

$$B = \{-3, -2, 2, 3\}$$

Problem 6: Digital Circuit

i. Expression for Y in terms of A, B, and C:

From the circuit:

$$Y = \neg((A \wedge B) \vee \neg C)$$

ii. Simplifying Y using logic identities:

First, apply Demorgan's Law:

$$Y = \neg(A \wedge B)\neg C$$

Second, apply involution:

$$Y = \neg(A \wedge B) \wedge C$$

Third, apply De Morgan's Law:

$$Y = (\neg A \vee \neg B) \wedge C$$

iii. Drawing the circuit: