CS1800 Homework 2 Solutions

Problem 1: English to Logic

Given:

- a: The alarm is ringing
- b: The battery is low
- c: The system is on

i. The alarm is ringing and the system is on

$$a \wedge c$$

ii. The alarm is not ringing but the system is on

$$\neg a \wedge c$$

iii. The system is not on and the alarm is not ringing

$$\neg c \wedge \neg a$$

iv. There is no way that the alarm is ringing

$$\neg a$$

v. The alarm is ringing or the battery is low

$$a \vee b$$

vi. Despite the fact that the alarm is ringing, the battery is low

$$a \wedge b$$

Problem 2: Arithmetic & Logic

i. $(7 \geq 7) \wedge (8 < 5)$

$$\begin{aligned} 7 \geq 7 &\text{ is True} \\ 8 < 5 &\text{ is False} \\ \text{True} \wedge \text{False} &= \text{False} \end{aligned}$$

$$\text{False}$$

ii. $(7 \geq 7) \vee (8 < 5)$

$$\begin{aligned} 7 \geq 7 &\text{ is True} \\ 8 < 5 &\text{ is False} \\ \text{True} \vee \text{False} &= \text{True} \end{aligned}$$

$$\text{True}$$

iii. $(6 = 3 + 3) \wedge (10 < 1)$

$$\begin{aligned} 6 = 3 + 3 &\text{ is True} \\ 10 < 1 &\text{ is False} \\ \text{True} \wedge \text{False} &= \text{False} \end{aligned}$$

$$\text{False}$$

iv. $\neg(9 = 8)$

$$\begin{aligned} 9 = 8 &\text{ is False} \\ \neg\text{False} &= \text{True} \end{aligned}$$

$$\text{True}$$

v. $\neg((6 = 3 + 3) \vee (10 < 1))$

$$\begin{aligned} 6 = 3 + 3 &\text{ is True} \\ 10 < 1 &\text{ is False} \\ \text{True} \vee \text{False} &= \text{True} \\ \neg\text{True} &= \text{False} \end{aligned}$$

$$\text{False}$$

Problem 3: Tarski World Predicates & Quantifiers

i. $\textbf{star}(c) \land \neg\textbf{shade}(c)$

"c is a star and c is not shaded."

False

ii. $\exists x(\textbf{circ}(x) \land \textbf{shade}(x))$

"There exists an x such that x is a circle and x is shaded."

True

iii. $\forall x(\textbf{square}(x) \to \neg\textbf{shade}(x))$

"For all x, if x is a square, then x is not shaded."

True

iv. $\forall x\forall y(\textbf{star}(x) \land \neg\textbf{shade}(x) \land \textbf{next to}(x, y)) \to (\textbf{shade}(y) \land \textbf{circ}(y))$

"For all x and y, if x is a star and not shaded, and x is next to y, then y is shaded and is a circle."

False

v. $\exists x\forall y(\textbf{next to}(x, y))$

"There exists an x such that for all y, x is next to y."

False

vi. $\forall y\exists x(\textbf{shade}(x) \land \textbf{next to}(x, y))$

"For all y, there exists an x such that x is shaded and x is next to y."

True

Problem 4: Converse, Inverse, Contrapositive

Given the statement:

$$\forall x(\neg \text{shade}(x) \to \text{circle}(x))$$

i. True or False?

The statement is **True**.

ii. Contrapositive:

$$\forall x(\neg \text{circle}(x) \to \text{shade}(x))$$

The contrapositive is **True**.

iii. Converse:

$$\forall x(\text{circle}(x) \to \neg \text{shade}(x))$$

The converse is **False**.

iv. Inverse:

$$\forall x(\text{shade}(x) \to \neg \text{circle}(x))$$

The inverse is **False**.

v. True conditional statement whose converse is False:

"If an object is a rectangle, then it is not shaded." The converse is False if there is an unshaded circle.

vi. True conditional statement whose contrapositive is False:

This situation is not possible. If the conditional is True, the contrapositive must also be True.

Problem 5: Vending Machine

Given:

- E: The machine is empty.
- S: The user made a selection.
- P: The user paid a quarter.
- V: The machine will vend a soda.
- R: The machine will return the quarter.

i. Truth Table for V and R:

E	S	P	V	R
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	0
1	0	0	0	0
1	0	1	0	1
1	1	0	0	0
1	1	1	0	1

ii. Expression for V:

$$V = \neg E \land S \land P$$

iii. Expression for R:

$$R = E \land P$$

Problem 6: English to Logic (Part 2)

Given:

- h: They have hamsters.
- w: They have whales.
- d: They have dinosaurs.

i. The zoo doesn't have any hamsters or whales, but it does have dinosaurs.

$$\neg h \land \neg w \land d$$

ii. The zoo has at least two of the three groups of animals.

$$(h \land w) \lor (h \land d) \lor (w \land d)$$

iii. The zoo has exactly two of the three groups of animals.

$$(h \land w \land \neg d) \lor (h \land d \land \neg w) \lor (w \land d \land \neg h)$$

iv. The zoo has, at most, one of these groups of animals.

$$(\neg h \land \neg w \land d) \lor (\neg h \land w \land \neg d) \lor (h \land \neg w \land \neg d)$$