MATH 231-01: Homework Assignment 2

15 September 2025

Due: 22 September 2025 by 10:00pm Eastern time, submitted on Moodle as a single PDF.

Instructions: Write your solutions on the following pages. If you need more space, you may add pages, but make sure they are in order and label the problem number(s) clearly. You should attempt each problem on scrap paper first, before writing your solution here. Excessively messy or illegible work will not be graded. You must show your work/reasoning to receive credit. You do not need to include every minute detail; however the process by which you reached your answer should be evident. You may work with other students, but please write your solutions in your own words.

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Score:

1. Use the cross product to find the area of the triangle with vertices $A = (3, 0, 0)$, $B = (3, 3, 0)$, and $C = (0, 1, 3)$.

2. Suppose $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are unit vectors in $\mathbb{R}^3$ such that $\mathbf{v}$ and $\mathbf{w}$ make an angle of $\pi/6$, and $\mathbf{u}$ and $\mathbf{v} \times \mathbf{w}$ make an angle of $2\pi/3$ (both in radians). Find the volume of the parallelepiped generated by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

3. Solve Problem 60 on p. 825 of the textbook (Section 10.4).

4. Prove that the cross product is anticommutative. In other words, given arbitrary vectors $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$, use the definition of the cross product to show that $\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v})$.

5. For many years, I've been trying to prove that $0 = 1$. Here's my latest attempt:

Let $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ denote the standard basis vectors in $\mathbb{R}^3$. Then

$$\mathbf{0} = \mathbf{i} \times \mathbf{i} = (\mathbf{i} \times \mathbf{i}) \times \mathbf{j} = \mathbf{i} \times (\mathbf{i} \times \mathbf{j}) = \mathbf{i} \times \mathbf{k} = -\mathbf{j}.$$

Since $\|\mathbf{0}\| = 0$ and $\| -\mathbf{j}\| = 1$, it follows that $0 = 1$. $\square$

Where did I go wrong? Explain.

6. Let L_1 be the line with vector equation

$$\langle x, y, z\rangle = \langle 1, 0, 1\rangle + t\langle 2, -1, 0\rangle.$$

Let L_2 be the line with symmetric equations

$$\frac{x+1}{3} = \frac{1-y}{2} = z - 1.$$

(a) Show that L_1 and L_2 intersect.

(b) Find an equation for the plane that contains L_1 and L_2.

7. Find an equation for the line that contains the point $P = (2, 0, 1)$ and is parallel to to the planes given by the equations $x + 2y + 3z = 6$ and $2x - y + z = 4$.

8. Let $A = (-1, 0, 2)$ and $B = (0, 3, 0)$. The collection of all points $P = (x, y, z)$ with equal distance to A and B forms a plane. Find an equation of the form

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

that represents this plane.