MATH 231-01: Homework Assignment 7

27 October 2025

Due: 3 November 2025 by 10:00pm Eastern time, submitted on Moodle as a single PDF.

Instructions: Write your solutions on the following pages. If you need more space, you may add pages, but make sure they are in order and label the problem number(s) clearly. You should attempt each problem on scrap paper first, before writing your solution here. Excessively messy or illegible work will not be graded. You must show your work/reasoning to receive credit. You do not need to include every minute detail; however the process by which you reached your answer should be evident. You may work with other students, but please write your solutions in your own words.

Name:

Score:

1. Let R be the region bounded by the parabola $x = y^2$ and the line $y = x - 2$. Evaluate the double integral $\iint_R xydA$.

2. Let R be the triangular region with vertices $(0,0)$, $(1,1)$, and $(-1,2)$. Evaluate the double integral $\iint_R xydA$.

3. Evaluate the iterated integral $\int_0^2 \int_0^{\sqrt{4-x^2}} \sqrt{x^2+y^2} dy dx$.

4. Let R be the lamina occupying the region bounded by the parabolas $x = y^2$ and $y = x^2$. Assuming R has density given by $\rho(x, y) = \sqrt{x}$, find its center of mass.

5. In this exercise, you will determine the value of the integral

$$I = \int_{-\infty}^{\infty} e^{-x^2} dx.$$

To get you started, observe that

$$I^2 = \Big(\int_{-\infty}^{\infty} e^{-x^2} dx \Big) \Big(\int_{-\infty}^{\infty} e^{-y^2} dy \Big) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} dx dy.$$

(a) Show that $\displaystyle\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} dx dy = \int_{0}^{2\pi} \int_{0}^{\infty} e^{-r^2} r dr d\theta.$

(b) Evaluate the iterated integral $\displaystyle\int_{0}^{2\pi} \int_{0}^{\infty} e^{-r^2} r dr d\theta$ and state the value of I.