CS1800 Discrete Structures	Prof Higger & Prof Hamlin
Fall 2024	Nov 8, 2024

HW7: Induction

Due: Nov 15, 2024 @ 11:59 PM

Instructions:

- HW instructions
- academic integrity and collaboration

Problem 1 [20 pts]: Upper triangular matrix

The diagonal of a square matrix refers to all elements in a line from the top-left to the bottom-right of the matrix. For example, in the 3×3 matrix below, all diagonal entries are 1 where off-diagonal entries are 0:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Note that the only entry in a 1×1 matrix is on the diagonal.

The upper-diagonal of a matrix are all entries which are on the diagonal or above. For example, in the 3×3 matrix below, all upper-diagonal entries are 1 where non-upper-diagonal entries are 0:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Use induction to show that an $n \times n$ matrix has $n(n+1)/2$ upper-diagonal entries.

Problem 2 [20 pts]: A partial series $\frac{1}{k(k+1)}$

Consider the series:

$$\frac{1}{1*2} + \frac{1}{2*3} + \frac{1}{3*4} + \dots \frac{1}{n(n+1)} = \sum_{k=1}^{n} \frac{1}{k(k+1)}$$

Show that:

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = 1 - \frac{1}{n+1}$$

for $n = 1, 2, 3, \ldots$

Problem 3 [20 pts]: A partial series k^3

Prove, using mathematical induction, that

$$\sum_{k=0}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Problem 4 [20 pts]: Function Growth ($n < 2^n$)

Use induction to show that:

$$n < 2^n$$

for all $n \in \mathbb{Z}^+$[1]

[1] $\mathbb{Z}^+$ is the set of all positive integers: $1, 2, 3, \ldots$