Table of Laplace Transforms

$$L[k] = \frac{k}{s} \tag{1}$$

$$L[t^n] = \frac{n!}{s^{n+1}} \tag{2}$$

$$L[e^{at}] = \frac{1}{s-a} \tag{3}$$

$$L[t^n e^{at}] = \frac{n!}{(s-a)^{n+1}} \tag{4}$$

$$L[\cos(kt)] = \frac{s}{s^2+k^2} \tag{5}$$

$$L[\sin(kt)] = \frac{k}{s^2+k^2} \tag{6}$$

$$L[e^{at}\cos(kt)] = \frac{s-a}{(s-a)^2+k^2} \tag{7}$$

$$L[e^{at}\sin(kt)] = \frac{k}{(s-a)^2+k^2} \tag{8}$$

$$L[t\sin(kt)] = \frac{2ks}{(s^2+k^2)^2} \tag{9}$$

$$L[t\cos(kt)] = \frac{s^2-k^2}{(s^2+k^2)^2} \tag{10}$$

$$L[t\sin(kt+a)] = \frac{s\sin(a)+k\cos(a)}{s^2+k^2} \tag{11}$$

$$L[t\cos(kt+a)] = \frac{s\cos(a)-k\sin(a)}{s^2+k^2} \tag{12}$$

$$L[u(t-a)] = \frac{e^{-as}}{s}, a \geq 0 \tag{13}$$

$$L[\delta(t-a)] = e^{-as}, a \geq 0 \tag{14}$$

Differentiation Properties
If $L[y(t)] = Y(s)$, then:

$$L[y'(t)] = sY(s) - y(0) \tag{1}$$

$$L[y''(t)] = s^2Y(s) - sy(0) - y'(0) \tag{2}$$

Shifting Theorems

First Shifting Theorem If $L[f(t)] = F(s)$, then:

$$L[e^{at}f(t)] = F(s-a) \tag{1}$$

Second Shifting Theorem If $L[f(t)] = F(s)$, then:

$$L[u(t-a)\cdot f(t-a)] = e^{-as}\cdot L[f(t)] \tag{1}$$

$$= e^{-as}F(s) \tag{2}$$

$$L^{-1}[e^{-as}F(s)] = u(t-a)\cdot f(t-a) \tag{3}$$

Laplace Transforms of Piecewise Functions
Suppose $f(t)$ is a piecewise-defined function:

$$f(t) = \begin{cases} f_1(t) & 0 \leq t < a_1 \\ f_2(t) & a_1 \leq t < a_2 \\ f_3(t) & a_2 \leq t < a_3 \\ ... & ... \\ f_n(t) & a_{n-1} \leq t < a_n \end{cases}$$

f(t) can be wrtten in termss of step functions as:

$$f(t) = f_1(t) + (f_2(t) - f_1(t))\cdot u(t-a_1) + (f_3(t) - f_2(t)) \cdot u(t-a_2) + ... + (f_n(t) - f_{n-1}(t))\cdot u(t-a_{n-1})$$

Convolution Theorem

$$L[f(t) * g(t)] = L[f(t)]\cdot L[g(t)] \tag{1}$$

$$= F(s)\cdot G(s) \tag{2}$$

and

$$L^{-1}[F(s)\cdot G(s)] = f(t) * g(t), \text{ where} \tag{1}$$

$$f(t) * g(t) = \int_0^t f(t-\tau)g(\tau)d\tau \tag{2}$$