MATH 231-01: Homework Assignment 8

3 November 2025

Due: 10 November 2025 by 10:00pm Eastern time, submitted on Moodle as a single PDF.

Instructions: Write your solutions on the following pages. If you need more space, you may add pages, but make sure they are in order and label the problem number(s) clearly. You should attempt each problem on scrap paper first, before writing your solution here. Excessively messy or illegible work will not be graded. You must show your work/reasoning to receive credit. You do not need to include every minute detail; however the process by which you reached your answer should be evident. You may work with other students, but please write your solutions in your own words.

Name:

Score:

1. Find the volume of the region that lies above the solid triangle with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(1, 2, 0)$ and below the paraboloid $z = 6 - x^2 - y^2$.

2. Complete Problem 42 in Section 13.6 of the textbook (p. 1066).

3. Complete Problem 46 in Section 13.6 of the textbook (p. 1067).

4. Let R be the region bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = y$ and $z = 0$. Evaluate the triple integral

$$\iiint_R z dV.$$

5. Find the volume of the region that lies outside the cylinder $x^2 + y^2 = 1$ and inside the sphere $x^2 + y^2 + z^2 = 4$.

6. Let R be the region that lies above the plane $z = 1$, below the sphere $x^2+y^2+z^2 = 4$, and within the first octant. Evaluate the triple integral

$$\iiint_R \frac{1}{x^2 + y^2 + z^2} dV.$$

7. Let R be the region that lies outside the sphere $x^2 + y^2 + (z-1)^2 = 1$, inside the sphere $x^2 + y^2 + z^2 = 4$, and above the plane $z = 0$. Suppose R has uniform density $\rho(x, y, z) = 1$ and center of mass $(\overline{x}, \overline{y}, \overline{z})$. Find $\overline{z}$.