

Table of Laplace Transforms

$$L[k] = \frac{k}{s} \quad (1)$$

$$L[t^n] = \frac{n!}{s^{n+1}} \quad (2)$$

$$L[e^{at}] = \frac{1}{s-a} \quad (3)$$

$$L[t^n e^{at}] = \frac{n!}{(s-a)^{n+1}} \quad (4)$$

$$L[\cos(kt)] = \frac{s}{s^2 + k^2} \quad (5)$$

$$L[\sin(kt)] = \frac{k}{s^2 + k^2} \quad (6)$$

$$L[e^{at} \cos(kt)] = \frac{s-a}{(s-a)^2 + k^2} \quad (7)$$

$$L[e^{at} \sin(kt)] = \frac{k}{(s-a)^2 + k^2} \quad (8)$$

$$L[t \sin(kt)] = \frac{2ks}{(s^2 + k^2)^2} \quad (9)$$

$$L[t \cos(kt)] = \frac{s^2 - k^2}{(s^2 + k^2)^2} \quad (10)$$

$$L[t \sin(kt + a)] = \frac{s \sin(a) + k \cos(a)}{s^2 + k^2} \quad (11)$$

$$L[t \cos(kt + a)] = \frac{s \cos(a) - k \sin(a)}{s^2 + k^2} \quad (12)$$

$$L[u(t-a)] = \frac{e^{-as}}{s}, a \geq 0 \quad (13)$$

$$L[\delta(t-a)] = e^{-as}, a \geq 0 \quad (14)$$

Differentiation Properties

If $L[y(t)] = Y(s)$, then:

$$L[y'(t)] = sY(s) - y(0) \quad (1)$$

$$L[y''(t)] = s^2Y(s) - sy(0) - y'(0) \quad (2)$$

Shifting Theorems

First Shifting Theorem If $L[f(t)] = F(s)$, then:

$$L[e^{at}f(t)] = F(s-a) \quad (1)$$

Second Shifting Theorem If $L[f(t)] = F(s)$, then:

$$L[u(t-a) \cdot f(t-a)] = e^{-as} \cdot L[f(t)] \quad (1)$$

$$= e^{-as}F(s) \quad (2)$$

$$L^{-1}[e^{-as}F(s)] = u(t-a) \cdot f(t-a) \quad (3)$$

Laplace Transforms of Piecewise Functions

Suppose $f(t)$ is a piecewise-defined function:

$$f(t) = \begin{cases} f_1(t) & 0 \leq t < a_1 \\ f_2(t) & a_1 \leq t < a_2 \\ f_3(t) & a_2 \leq t < a_3 \\ \dots & \dots \\ f_n(t) & a_{n-1} \leq t < a_n \end{cases}$$

$f(t)$ can be written in terms of step functions as:

$$f(t) = f_1(t) + (f_2(t) - f_1(t)) \cdot u(t-a_1) + (f_3(t) - f_2(t)) \cdot u(t-a_2) + \dots + (f_n(t) - f_{n-1}(t)) \cdot u(t-a_{n-1})$$