HEADER

BEHAVIOR

Stress (σ): $\sigma = \frac{F}{A_0}$

Strain (ε): $\varepsilon = \frac{\Delta L}{L_0}$

Young's Modulus (E): $E = \frac{\sigma}{\varepsilon}$ (in elastic region)

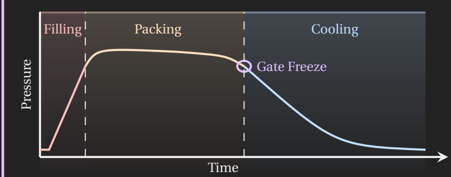
Shear Modulus (G): $G = \frac{\tau}{\gamma}$

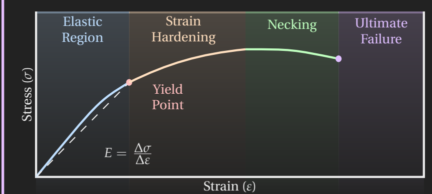
Poisson's Ratio (ν): $\nu = -\frac{\varepsilon_\perp}{\varepsilon_\parallel}$

True Stress (σ_{true}): $\sigma_{\text{true}} = \frac{F}{A_{\text{instantaneous}}}$

True Strain ($\varepsilon_{\text{true}}$): $\varepsilon_{\text{true}} = \ln \frac{L}{L_0}$

Stress Relaxation: $\sigma(t) = \sigma_0 e^{-\frac{t}{\tau}}$





RHEOLOGY

Shear rate: $\dot\gamma = \frac{u_0}{h}$ **Shear stress:** $\tau = \eta\dot\gamma$

Viscosity (η): $\eta = \frac{\tau}{\dot\gamma}$

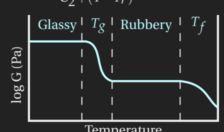
Deborah Number (De): $De = \frac{\lambda}{t_{\text{obs}}}$ where λ = relaxation time

Storage Modulus (G'): $G' = \frac{\sigma_0}{\gamma_0}\cos(\delta)$

Loss Modulus (G''): $G'' = \frac{\sigma_0}{\gamma_0}\sin(\delta)$

Stress Relaxation: $\sigma(t) = \sigma_0 e^{-t/\tau}$

Time-Temperature Superposition: $\log(a_T) = \frac{-C_1(T-T_r)}{C_2+(T-T_r)}$



PROCESSING

Capillary Rheometer: $\dot\gamma = \frac{4Q}{\pi R^3}, \quad \tau = \frac{(P_0-P_L)R}{2L}$

$\dot\gamma$ = shear rate, Q = volumetric flow rate, R = die radius, L = die length, and $(P_0 - P_L)$ = pressure drop.

Rotational Rheometer – Cone & Plate:

$$\dot\gamma = \frac{\Omega}{\theta_0}, \quad \tau = \frac{3T}{2\pi R^3}, \quad \eta = \frac{\tau}{\dot\gamma} = \frac{3T\theta_0}{2\pi R^3\Omega}$$

Ω = angular velocity, θ_0 = cone angle, T = torque, R = plate radius.

Rotational Rheometer – Parallel Plates:

$$\dot\gamma = \frac{r\Omega}{H}, \quad \tau = \frac{3T}{2\pi R^3}, \quad \eta = \frac{\tau}{\dot\gamma} = \frac{3TH}{2\pi R^3 r\Omega}$$

r = radial position, H = gap height.

Gibbs Free Energy for Blends:

$\Delta G = \Delta H - T\Delta S, \quad \Delta H = \nu(\delta_1 - \delta_2)^2\phi_1\phi_2$

ν = molar volume, δ = solubility parameters, ϕ = volume fractions.

Melt Flow Index (MFI): Mass of polymer extruded under standard conditions $[g/10\,\text{min}]$

DIMENSIONAL ANALYSIS

Buckingham Pi Theorem: Every system with m physical quantities reduced to m-n dimensionless groups, n = number basic dimensions

Basic Dimensions [MLTΘ]:
Length (L): m Mass (M): kg Time (T): s Temp (Θ): K

Matrix Method: (1) Select n dimension core matrix (repeating vars)
(2) Form [L,M,T] rows x vars cols matrix
(3) Solve for Pi groups: $\Pi_1 = f(\Pi_2, \Pi_3, \ldots)$

Key Dimensionless Numbers:

Deborah Number: De = $\frac{\text{Material relax time}}{\text{Process time}}$
De → 0: Viscous fluid, De → ∞: Elastic solid

Biot Number: Bi = $\frac{\text{Surface convection}}{\text{Internal conduction}} = \frac{hL}{k}$
Bi ≪ 1: Tc ≈ Ts (uniform temp)
Bi ≫ 1: Ts ≈ T∞ (surface controlled)

Scaling Laws:
Length: L ∝ size
Area: A ∝ L^2
Volume/Weight: V,W ∝ L^3
Stress: σ = W/A ∝ L

Similarity Types:
Geometric: shape ratios same
Kinematic: velocity ratios same
Dynamic: force ratios same

MATRIX TRANSFORMATION METHOD EXAMPLE

Consider pipe flow with pressure drop (Δp), diameter (D), length (L), viscosity (η), density (ρ), velocity (u)

Step 1: List Variables with Dimensions

Variable	Dimension	Variable	Dimension
D	L	Δp	$ML^{-1}T^{-2}$
ρ	ML^{-3}	L	L
u	LT^{-1}	η	$ML^{-1}T^{-1}$

Step 2: Select Core Matrix (n=3 basic dimensions)
Choose D, ρ, u as repeating variables

Step 3: Form Dimensional Matrix

	D	ρ	u	Δp	L	η
L	1	−3	1	−1	1	−1
M	0	1	0	1	0	1
T	0	0	−1	−2	0	−1

Step 4: Solve for Π Groups
For Π_1 using Δp:
$[D^a\rho^b u^c \Delta p] = [M^0L^0T^0]$

$$L: \quad a - 3b + c - 1 = 0$$
$$M: \quad b + 1 = 0$$
$$T: \quad -c - 2 = 0$$

Solving gives: $\Pi_1 = \frac{\Delta p}{\rho u^2}$

Similarly for remaining groups:

$$\Pi_2 = \frac{L}{D}$$
$$\Pi_3 = \frac{\eta}{D\rho u} = \frac{1}{Re}$$

Final Result:
$f(\frac{\Delta p}{\rho u^2}, \frac{L}{D}, \frac{\eta}{D\rho u}) = 0$

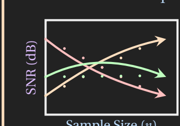
DESIGN OF EXPERIMENTS

Seven-Step Process:
1. Identify Factors/Metrics
2. Formulate Objective
3. Design Experiment
4. Run Trials
5. Analyze Results
6. Select Setpoints
7. Iterate/Validate

Control Factors:
Temperature, Pressure, Time
Material Properties, Geometry
Process Parameters

Noise Factors:
Manufacturing Variance
Environmental Conditions
User/Operation Variations

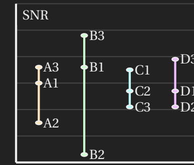
Signal-to-Noise Ratio Analysis Signal-to-Noise Ratio (SNR) measures the relationship between response values and their variation across sample size.



$$\eta_L = -10\log_{10}\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right)$$
$$\eta_S = -10\log_{10}\left(\frac{1}{n}\sum_{i=1}^{n}y_i^2\right)$$
$$\eta_N = 10\log_{10}\left(\frac{\bar y^2}{s^2}\right)$$

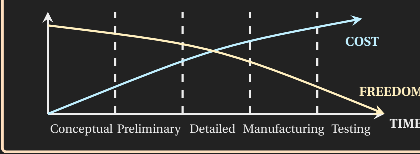
L9 Orthogonal Array Experimental design matrix enabling efficient study of multiple factor effects with minimal trials.

	A	B	C	D
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1



Response Surface Analysis Maps relationship between input factors and system response for optimization.

$$y = \beta_0 + \sum_{i=1}^{k}\beta_i x_i + \sum_{i=1}^{k}\beta_{ii}x_i^2 + \sum_{i<j}^{k}\beta_{ij}x_i x_j + \varepsilon$$

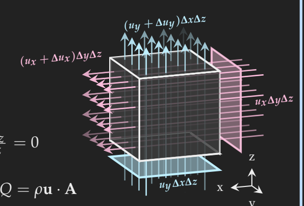


CONTINUITY

MASS CONSERVATION

Cartesian:

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0$$



Cylindrical:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{\partial v_z}{\partial z} = 0$$

Mass Flow Rate: $\dot m = \rho Q = \rho\mathbf{u}\cdot\mathbf{A}$

Continuity (Incompressible): $\nabla\cdot\mathbf{u} = 0$

Continuity (Compressible): $\frac{\partial}{\partial x}(\rho u_x) + \frac{\partial}{\partial y}(\rho u_y) + \frac{\partial}{\partial z}(\rho u_z) = 0$

MOMENTUM CONSERVATION

$$\rho\left(\frac{\partial u_x}{\partial t} + u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2}\right) + \rho g_x$$

$$\rho\frac{D\mathbf{u}}{Dt} = -\nabla p + \eta\nabla^2\mathbf{u} + \rho\mathbf{g}$$

Non-Newtonian Viscosity:

$$\eta = \eta(\dot\gamma) = \eta\left(\sqrt{\frac{1}{2}\sum_{i,j}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2}\right)$$

ENERGY CONSERVATION

$$\rho c_v\left(\frac{\partial T}{\partial t} + u_x\frac{\partial T}{\partial x} + u_y\frac{\partial T}{\partial y} + u_z\frac{\partial T}{\partial z}\right)$$
$$= k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + 2\mu\left(\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right)$$
$$+\mu\left(\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)^2 + \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)^2\right) + \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)^2\Bigg) + \dot Q$$

$$\rho C_p\frac{DT}{Dt} = -\nabla\cdot\mathbf{q} + \dot Q + \dot Q_{\text{viscous heating}}$$

Material Derivative:
$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T$

Fourier's Law:
$\mathbf{q} = -k\nabla T$

Heat Conduction:
$-\nabla\cdot\mathbf{q} = k\nabla^2 T = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)$

Simple Shear Heating: $\dot Q_{\text{viscous heating}} = \eta\left(\frac{\partial u}{\partial y}\right)^2$

ORDER OF MAGNITUDE

From continuity: $(U/L_x) \sim (V/L_y)$. If $L_x \gg L_y$, then $U \gg V$.

Momentum in x-dir: $\rho U^2/L_x \sim \Delta p/L_x \sim \mu U/L_x^2 \sim \mu U/L_y^2$.

Dominance depends on $\frac{L_x}{L_y}$ and Re $= \frac{\rho U L_x}{\mu}$. For small Re and large L_x/L_y, viscous terms $(\mu U/L_y^2)$ dominate over inertial $(\rho U^2/L_x)$.

Second derivative: $\partial^2 u/\partial x^2 \sim U/L_x^2$ vs. squared gradient: $(\partial u/\partial x)^2 \sim (U/L_x)^2$. Scaling guides term retention.

Geometric Parameters:
Width Ratio:
$\frac{W}{L} \sim O(1)$
Channel Aspect:
$\frac{h}{W} \ll 1$
Flow Analysis:
Continuity Scaling:
$\frac{U}{L_x} \sim \frac{V}{L_y}$
Momentum Scaling:
$\rho\frac{U^2}{L_x} \sim \frac{\Delta P}{L_x} \sim \mu\frac{U}{L_y^2}$

Critical Numbers:
Re$\left(\frac{L_y}{L_x}\right)$
Re$\left(\frac{L_y}{L_x}\right)$
Process-Specific Scaling:
Injection Molding:
$Re \ll 1, \quad \epsilon \ll 1$
Film Casting:
$We = \frac{\rho v^2 h}{\sigma} \gg 1$

SOLVING THE GOVERNING EQUATIONS

FUNDAMENTAL ASSUMPTIONS AND REDUCTIONS

Assumption / Approach	Resulting Simplification
Steady State: $\frac{\partial}{\partial t} = 0$	$\cancel{\frac{\partial\rho}{\partial t}} + \nabla\cdot(\rho\mathbf{u}) = 0$ $\rho\cancel{\frac{\partial\mathbf{u}}{\partial t}} + \rho(\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \mu\nabla^2\mathbf{u}$
Constant Material Properties: $\rho = \text{const}, \mu = \text{const}$	$\frac{\partial\rho}{\partial t} = 0,\ \frac{\partial\rho}{\partial x_i} = 0$ $\implies \nabla\cdot\mathbf{u} = 0$
Fully Developed Flow: $\frac{\partial u_x}{\partial x} = 0$	$\cancel{\frac{\partial u_x}{\partial x}} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0$
Dimensions of Geometry: $\frac{\partial}{\partial z} = 0,\ u_z = 0$	$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \cancel{\frac{\partial u_z}{\partial z}} = 0$ with $\cancel{u_z} = 0$
Symmetry (Independent of Width): $\frac{\partial}{\partial z} = 0$	$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \cancel{\frac{\partial u_z}{\partial z}} = 0$
Dimensional Analysis (Low Re)	$\rho(\mathbf{u}\cdot\nabla)\mathbf{u} = \cancel{\rho(\mathbf{u}\cdot\nabla)}\mathbf{u} = 0 \implies 0 = -\nabla p + \mu\nabla^2\mathbf{u}$
Constant Material Properties (Reiterated)	$\nabla\cdot\mathbf{u} = 0, 0 = -\nabla p + \mu\nabla^2\mathbf{u}$
Lubrication Approximation: $\frac{\partial p}{\partial x} \gg \frac{\partial p}{\partial y}$	$-\nabla p = -\frac{\partial p}{\partial x}\hat\imath - \cancel{\frac{\partial p}{\partial y}}\hat\jmath$

LUBRICATION

Key Parameters:
$\epsilon = \frac{h}{L} \ll 1$ (thin film)
$Re = \frac{\rho U h}{\mu}$

Velocity Profile:
$u(y) = \frac{1}{2\mu}\left(\frac{dp}{dx}\right)(y^2 - hy)$

Simplified Momentum:
$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}$

Flow Rate:
$q = -\frac{h^3}{12\mu}\left(\frac{dp}{dx}\right)$

SCALE ANALYSIS AND TERM REDUCTION

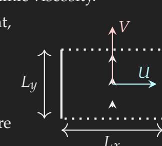
Inertial terms: $\rho U\frac{U}{L_x} \approx \rho V\frac{U}{L_y}$ **Viscous terms:** $\mu\frac{U}{L_y^2}$

Here, U and V are the velocities in the x and y directions, respectively. L_x and L_y are the lengths in the x and y directions. ρ is the fluid density, and μ is the dynamic viscosity.

To see which terms are more important, we use the Reynolds number (Re)

$Re = \frac{\rho U L_y}{\mu}$

If $Re\frac{L_y}{L_x} \ll 1$, viscous forces are more significant than inertial forces.



TEXTBOOK

EXAMPLE 9.1: FLOW IN A TUBE

Consider the classical problem of pressure drop during flow in a smooth straight pipe, ignoring the inlet effects. The first step is to list all possible variables or quantities that are related to the problem under consideration. In this case, we have:

Target quantity: Pressure drop Δp
Geometric variables: Pipe diameter D, and pipe length L
Physical or material properties: viscosity η, density ρ
Process variable: average fluid velocity u

If we choose D, u, and ρ as the repeating variables, the dimensional matrix is written as:

$$\begin{array}{c|cccccc} & D & u & \rho & \Delta p & L & \eta \\ M & 0 & 0 & 1 & 1 & 0 & 1 \\ L & 1 & 1 & -3 & -1 & 1 & -1 \\ T & 0 & -1 & 0 & -2 & 0 & -1 \end{array} \tag{1}$$

After reducing the core matrix to an identity matrix, the dimensional matrix becomes:

$$\begin{array}{c|cccccc} & D & u & \rho & \Delta p & L & \eta \\ M & 1 & 0 & 0 & 0 & 1 & 1 \\ L & 0 & 1 & 0 & 2 & 0 & 1 \\ T & 0 & 0 & 1 & 1 & 0 & 1 \end{array} \tag{9.8}$$

which results in the 3 dimensionless groups,

$$\Pi_1 = \frac{\Delta p}{u^2 \rho} = Eu \text{ (Euler number)} \tag{9.9}$$

$$\Pi_2 = \frac{L}{D} \tag{2}$$

$$\Pi_3 = \frac{\eta}{Du\rho} = Re^{-1} \text{ (Reynolds number)} \tag{3}$$

The following relationship can be written:

$$f\left(Eu, Re, \frac{L}{D}\right) = 0 \tag{9.10}$$

which, of course, by itself cannot produce the nature of the relation; however, the form of the function f can be generated experimentally.

DIMENSIONLESS VARIABLES

Name	Symbol	Definition	Meaning
Biot	Bi	$\frac{hL}{k}$	Convection / Conduction
Brinkman	Br	$\frac{\eta u^2}{k\Delta T}$	Viscous / Conduction
Capillary	Ca	$\frac{\tau R}{\sigma_s}$	Deviatoric / Surface
Damköhler	Da	$\frac{c\Delta H_r}{\rho C_p T_0}$	Reaction / Internal
Deborah	De	$\frac{\lambda}{t}$	Relaxation / Process
Fourier	Fo	$\frac{\alpha t}{L^2}$	Process / Thermal
Graetz	Gz	$\frac{uL}{\alpha}\left(\frac{d}{L}\right)$	Convection / Conduction
Nusselt	Nu	$\frac{hL}{k_{fluid}}$	Convective / Conductive
Péclet	Pe	$\frac{UL}{\alpha}$	Advection / Diffusion
Prandtl	Pr	$\frac{\nu}{\alpha}$	Momentum / Thermal
Reynolds	Re	$\frac{\rho u L}{\eta}$	Inertia / Viscous
Schmidt	Sc	$\frac{\nu}{D}$	Mechanical / Diffusion
Weissenberg	We	$\lambda\dot{\gamma}$ or $\frac{N_1}{\tau}$	Elastic / Viscous

EXAMPLES

PRESSURE-DRIVEN FLOW THROUGH SLIT

Assumptions
The following assumptions are made:
1. The flow is steady, fully developed, and entrance effects are ignored.
2. The fluid is Newtonian and incompressible.
3. The flow is unidirectional, with only one non-zero velocity component u_z.

Governing Equations
The continuity equation for an incompressible flow reduces to
$\frac{\partial u_z}{\partial z} = 0.$

The z-momentum equation for a Newtonian, incompressible flow (Navier-Stokes equations) is given by
$-\frac{\partial p}{\partial z} + \mu \frac{\partial^2 u_z}{\partial y^2} = 0.$

The x- and y-momentum components reduce to
$-\frac{\partial p}{\partial x} = -\frac{\partial p}{\partial y} = 0.$

This indicates that the pressure depends only on z. Since u_z does not vary with z, the pressure gradient $\frac{\partial p}{\partial z}$ is constant:
$\frac{\partial p}{\partial z} = \frac{\Delta p}{L}.$

Simplified Momentum Equation
Substituting the constant pressure gradient into the z-momentum equation:
$\frac{1}{\mu}\frac{\Delta p}{L} = \frac{\partial^2 u_z}{\partial y^2}.$

Boundary Conditions
No-slip boundary conditions are applied:
$u_z\left(\pm\frac{h}{2}\right) = 0.$

Solution for Velocity Profile
Integrating the simplified momentum equation twice and applying the boundary conditions, the velocity profile is obtained as
$u_z(y) = \frac{h^2}{8\mu}\frac{\Delta p}{L}\left[1 - \left(\frac{2y}{h}\right)^2\right].$

Mean Velocity
The mean velocity in the channel is calculated as
$\bar{u}_z = \frac{2}{h}\int_0^{h/2} u_z(y)\, dy = \frac{h^2}{12\mu}\frac{\Delta p}{L}.$

Volumetric Flow Rate
The volumetric flow rate is then given by
$Q = hW\bar{u}_z = \frac{Wh^3}{12\mu}\frac{\Delta p}{L},$
where W is the width of the channel.

SHEAR FLOW AND VISCOUS HEATING

Assumptions
The following assumptions are made:
1. The material is at 210°C.
2. The plate has a surface area of 100 cm^2 and is moving at a constant speed of $u_w = 1.0$ cm/s.
3. The gap between the plates is $h = 0.1$ cm.
4. The fluid follows the apparent shear viscosity ($\eta_a = \mu$) vs. shear rate ($\dot{\gamma}$) relationship provided.

Governing Equations
The viscous heating per unit volume is governed by the equation:
$\Phi_v = \mu\left(\frac{\partial u}{\partial y}\right)^2,$
where:
Φ_v is the viscous heating per unit volume,
μ is the shear viscosity,
$\frac{\partial u}{\partial y}$ is the velocity gradient.

Shear Rate Calculation
The velocity gradient (shear rate) is given by:
$\dot{\gamma} = \frac{\partial u}{\partial y} = \frac{u_w}{h}.$
Substituting the given values:
$\dot{\gamma} = \frac{1.0\,\text{cm/s}}{0.1\,\text{cm}} = 10\,\text{s}^{-1}.$

Viscosity from Graph
From the given graph, the viscosity μ corresponding to $\dot{\gamma} = 10\,\text{s}^{-1}$ is approximately:
$\mu = 0.2\,\text{Pa}\cdot\text{s}.$

Viscous Heating Calculation
Substituting μ and $\dot{\gamma}$ into the viscous heating formula:
$\Phi_v = \mu\dot{\gamma}^2 = (0.2)(10)^2 = 20\,\text{W/m}^3.$

Adiabatic Boundary Condition
With adiabatic boundaries, the heat generated internally is retained, leading to a uniform temperature rise throughout the fluid domain due to symmetry and constant thermal properties.
Conclusion on Temperature Rise
The temperature rise is uniform throughout the fluid domain due to the adiabatic boundary conditions and uniform generation of viscous heating.

VISCOUS HEATING

First, let's find μ from the plot and known parameters.
What is $\dot{\gamma}$ (shear rate)?

$$\dot{\gamma} = \frac{u_0}{h} = \frac{1.0\,(\text{cm/sec})}{0.1\,(\text{cm})} = 10\,\text{sec}^{-1}$$

Say,

$$\eta = 1.2 \times 10^5\ \text{g/cm.sec.}$$

Now let's simplify Φ_v:

$$\Phi_v = 2\left[\left(\frac{\partial u_x}{\partial x}\right)^2 + \left(\frac{\partial u_y}{\partial y}\right)^2 + \left(\frac{\partial u_z}{\partial z}\right)^2\right]$$

$$+\left[\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right)^2 + \left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right)^2 + \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right)^2\right].$$

Only u_x is not zero, and it is a function of y.

$$\Rightarrow \Phi_v = 2\left(\frac{\partial u_x}{\partial y}\right)^2 \quad \Rightarrow \quad \Phi_v = 2\left(\frac{u_0}{h}\right)^2 = 2 \times 10^2\ \text{sec}^{-2}.$$

$$\Rightarrow \mu\Phi_v = 1.2 \times 10^7\ \left[1.2 \times 10^5 \times 2 \times 10^2\right]\ \text{g/cm.sec.}$$

Units:

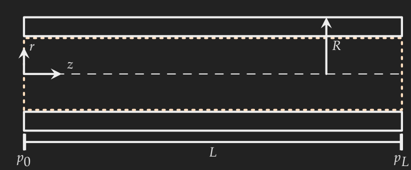
$$\text{Energy per unit time per unit volume:} \quad \frac{\text{g.cm}}{\text{sec}^2}\cdot\frac{\text{cm}}{\text{sec}}\cdot\frac{1}{\text{cm}^3}$$

UNIFORM TEMPERATURE RISE

Since $\frac{\partial u_x}{\partial y}$ (or $\dot{\gamma}$) is uniform throughout the flow domain, so is the viscosity (μ). Thus, $\mu\Phi_v$ should be uniform $\Rightarrow T$ is uniform.

FLOW THROUGH CYLINDRICAL PIPE

Tube flow is encountered in several polymer processes, such as in extrusion dies and sprue and runner systems inside injection molds. When deriving the equations for pressure-driven flow in tubes, also known as Hagen-Poiseuille flow, we assume that the flow is steady, fully developed, with no entrance effects, and axis-symmetric.



Assuming $u_z = u_z(r)$, $u_r = u_\theta = 0$ and $p = p(z)$, the only non-vanishing component of the rate-of-deformation tensor is the rz-component. For generalized Newtonian flow, τ_{rz} is the only non-zero component of the viscous stress. The z-momentum equation then reduces to:

$$\frac{1}{r}\frac{d}{dr}(r\tau_{rz}) = \frac{dp}{dz},$$

where τ_{rz} is a function of r.
From the symmetry argument, $\tau_{rz}(r)$ satisfies:

$$\tau_{rz} = -m\left|\frac{du_z}{dr}\right|^{n-1}\frac{du_z}{dr},$$

where m and n are material-specific parameters for the power-law fluid.
The velocity profile is derived as:

$$u_z(r) = \left(\frac{3n+1}{n+1}\right)\left[1 - \left(\frac{r}{R}\right)^{(n+1)/n}\right]\bar{u}_z,$$

where $\bar{u}_z$ is the mean velocity defined as:

$$\bar{u}_z = \frac{2}{R^2}\int_0^R u_z r\, dr.$$

Finally, the volumetric flow rate is expressed as:

$$Q = \pi R^2 \bar{u}_z = \left(\frac{n\pi}{3n+1}\right)\left[\frac{R^{n+1}}{2m}\frac{dp}{dz}\right]^{1/n}.$$

PRACTICE PROBLEMS