1. Basic Circuit Elements

1.1 Resistors

Resistor resists the flow of charge. The resistance R is a function of length, area, and resistivity:

$$R = \frac{\rho \ell}{A} \quad \text{or} \quad R = \frac{\ell}{\sigma A}$$

- ρ: Resistivity
- ℓ: Length
- A: Area

Ohm's Law: Voltage, current, and resistance are related:

$$V = IR \quad \text{or} \quad I = \frac{V}{R}$$

1.2 Power Dissipation and Passive Sign Convention

Power Dissipation:

$$P = IV = I^2 R = \frac{V^2}{R}$$

- In the passive sign convention, if the current enters the positive terminal of an element, the element is absorbing power:

$$P = VI$$

- If the current enters the negative terminal, the element is delivering power:

$$P = -VI$$

1.3 Conductance

Conductance is the reciprocal of resistance:

$$G = \frac{1}{R} \quad \text{in Siemens (S)}$$

1.4 Ideal Conductors

$$R = 0, \quad \sigma \to \infty$$

No voltage drop across an ideal conductor.
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2. Kirchhoff's Laws

2.1 Kirchhoff's Current Law (KCL)

The sum of currents entering and leaving a node is zero:

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

Example: If $2\,A$, $3\,A$, and $5\,A$ enter a node:

$$i_x = 10\,A$$

2.2 Kirchhoff's Voltage Law (KVL)

The sum of voltage drops in a closed loop equals the sum of voltage rises:

$$\sum V_{\text{drops}} = \sum V_{\text{rises}}$$

Example: In a loop with voltage drops of $10\,V$, $6\,V$, and $4\,V$, and supply $V_s = 24\,V$:

$$V_x = 4\,V$$

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3. Resistors in Series and Parallel

3.1 Series Resistors

Resistors in series carry the same current:

$$R_{\text{eq}} = R_1 + R_2 + \cdots + R_n$$

Voltage Division:

$$V_k = V_s \frac{R_k}{R_{\text{eq}}}$$

Example: For two resistors $R_1 = 10\,\Omega$, $R_2 = 5\,\Omega$, and source $V_s = 30\,V$:

$$V_1 = 20\,V, \quad V_2 = 10\,V$$

3.2 Parallel Resistors

Resistors in parallel share the same voltage:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

Current Division:

$$i_k = I_s \frac{R_{\text{eq}}}{R_k}$$

Example: For two resistors $R_1 = 6\,\Omega$, $R_2 = 3\,\Omega$, and $I_s = 6\,A$:

$$R_{\text{eq}} = 2\,\Omega, \quad i_1 = 2\,A, \quad i_2 = 4\,A$$

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4. Nodal Analysis (Node Voltage Method)

4.1 Steps for Nodal Analysis

1. Identify essential nodes.
2. Choose a reference node (ground).
3. Write KCL equations at each essential node using node voltages.
4. Solve the system of equations for unknown node voltages.

4.2 Example of KCL at Node

At node V_1:

$$\frac{V_1 - V_{\text{source}}}{R_1} + \frac{V_1 - V_2}{R_2} = 0$$

Solve the system of equations to find node voltages.
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5. Mesh Analysis (Mesh Current Method)

5.1 Steps for Mesh Analysis

1. Identify meshes (loops without other loops inside).
2. Assign mesh currents.
3. Apply KVL in each mesh to write voltage equations.
4. Solve the system of equations for mesh currents.

5.2 Example of KVL in Mesh

For a mesh with resistors R_1, R_2, and voltage source V_s:

$$i_a(R_1 + R_2) - i_b R_2 = V_s$$

Solve for i_a and i_b in the system of equations.
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6. Thevenin and Norton Equivalent Circuits

6.1 Thevenin's Theorem

Any linear circuit can be reduced to a single voltage source V_{Th} in series with R_{Th}.

$$V_L = V_{Th} \frac{R_L}{R_{Th} + R_L}$$

6.2 Norton's Theorem

Any linear circuit can be reduced to a single current source I_N in parallel with R_N.

$$I_N = \frac{V_{\text{Th}}}{R_{\text{Th}}}, \quad R_N = R_{\text{Th}}$$

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7. Source Transformations

7.1 Voltage to Current Source Transformation

A voltage source V_s in series with R can be transformed into a current source:

$$I_s = \frac{V_s}{R}, \quad \text{in parallel with } R$$

7.2 Current to Voltage Source Transformation

A current source I_s in parallel with R can be transformed into a voltage source:

$$V_s = I_s R, \quad \text{in series with } R$$

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8. Delta-Y (Δ-Y) Conversion

8.1 Delta to Y Conversion

For a delta network with resistors R_a, R_b, and R_c, the equivalent Y-resistances are:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \quad R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Example: For a delta network with $R_a = 10\,\Omega$, $R_b = 20\,\Omega$, and $R_c = 30\,\Omega$:

$$R_1 = 12\,\Omega, \quad R_2 = 5\,\Omega, \quad R_3 = 3.33\,\Omega$$

8.2 Y to Delta Conversion

For a Y-network with resistors R_1, R_2, and R_3, the equivalent delta-resistances are:

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}, \quad R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

Example: For a Y-network with $R_1 = 3\,\Omega$, $R_2 = 4\,\Omega$, and $R_3 = 5\,\Omega$:

$$R_a = 9.33\,\Omega, \quad R_b = 7.67\,\Omega, \quad R_c = 6\,\Omega$$