Sampling and Quantization Problems

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1 Problem 1: A/D Converter with Full-Scale Range of $\pm$10V

(a) Resolution of the A/D

The resolution of the A/D converter is given by:

$$\text{Resolution} = \frac{\text{Full-scale range}}{\text{Number of levels}}$$

For a 3-bit A/D converter, the number of levels is $2^3 = 8$. Thus:

$$\text{Resolution} = \frac{20}{8} = 2.50\,\text{V}$$

(b) Maximum Quantization Error

The maximum quantization error is half of the resolution:

$$\text{Maximum Error} = \frac{\text{Resolution}}{2} = \frac{2.50}{2} = 1.25\,\text{V}$$

(c) Quantization Levels

The quantization levels for a 3-bit A/D converter with a range of $\pm$10 V are:

$$-8.75, -6.25, -3.75, -1.25, 1.25, 3.75, 6.25, 8.75\,\text{V}$$

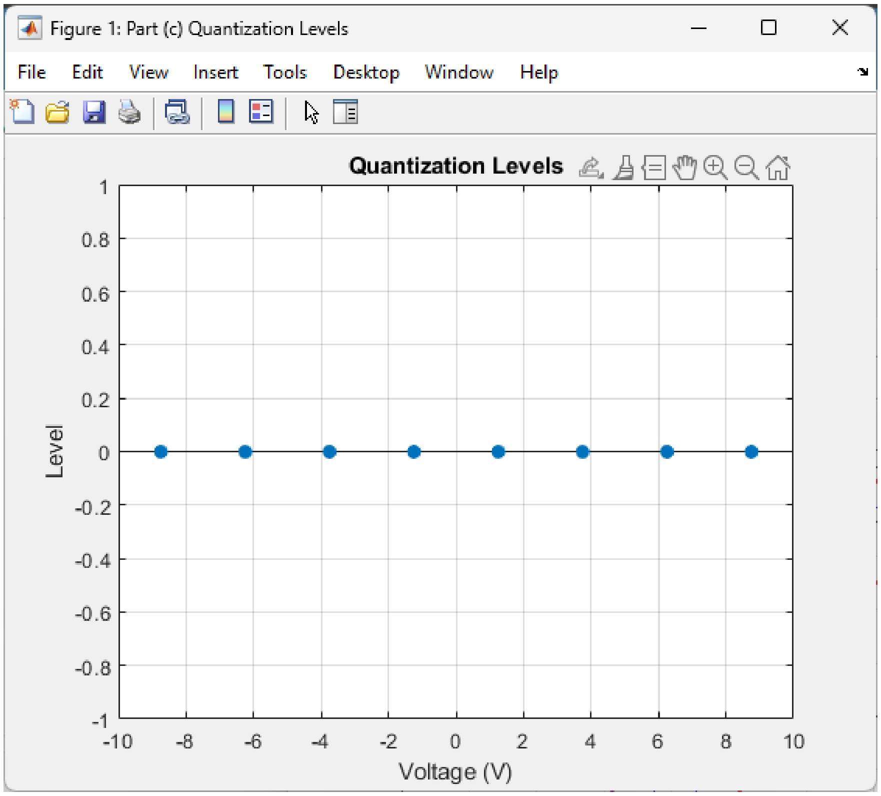

Figure 1: Quantization Levels for 3-bit A/D Converter.

(d) Expression for Discrete-Time Signal

The discrete-time signal is given by:

$$x[n] = x(nT_s) = 5\sin(2000\pi nT_s)$$

The sampling interval T_s is:

$$T_s = \frac{1}{F_s} = \frac{1}{8000} = 0.000125\,\mathrm{s}$$

The discrete-time frequency Ω is:

$$\Omega = 2\pi f T_s = 0.25\pi \text{ radians/sample}$$

(e) Minimum Sampling Frequency

The Nyquist theorem states:

$$F_s \geq 2f_{\max}$$

For $f_{\max} = 1000\,\mathrm{Hz}$, the minimum sampling frequency is:

$$F_s = 2000\,\mathrm{Hz}$$

(f) Quantized Signal

The samples and quantized signal are shown in Figure 2.

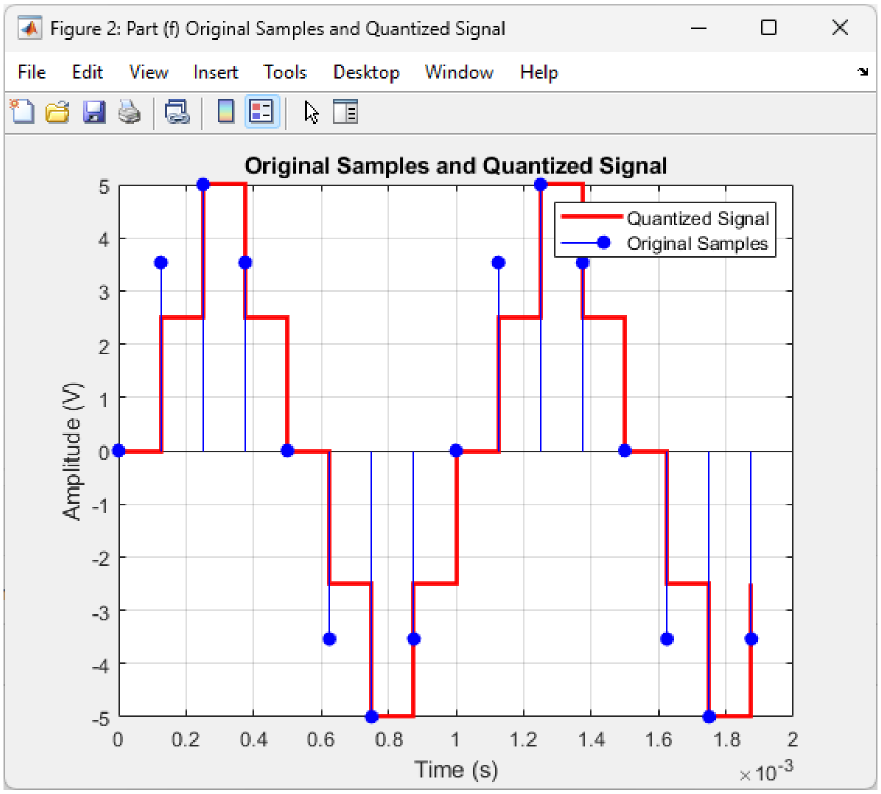

Figure 2: Original Samples and Quantized Signal.

2 Problem 2: Signal with Peak Values of ±0.2V

(a) Minimum Number of Bits Without Amplification

Quantization error should not exceed 0.1% of the maximum signal amplitude ($A = 0.2$):

$$\text{Error} = 0.001 \times A = 0.0002\,\text{V}$$

The resolution must be less than or equal to 0.0002 V:

$$\text{Resolution} = \frac{\text{Full-scale range}}{2^N} \implies 2^N \geq \frac{5}{0.0002} = 25000$$

Thus, $N \geq 15$.

(b) Minimum Sampling Frequency

The Nyquist theorem gives:

$$F_s \geq 2f_{\max} = 2 \times 4000 = 8000\,\text{Hz}$$

(c) Amplified Signal

The gain factor required to extend the signal range from ±0.2V to ±5V is:

$$\text{Gain} = \frac{5}{0.2} = 25$$

With amplification, the resolution is recalculated:

$$2^N \geq \frac{5}{0.0002} \implies N \geq 10$$

3 Problem 3: Composite Waveform

(a) Minimum Sampling Rate

The highest frequency component is 1000 Hz. Thus:

$$F_s \geq 2 \times 1000 = 2000\,\text{Hz}$$

(b) Quantization Levels

The number of quantization levels for a 10-bit system is:

$$2^{10} = 1024$$

(c) Maximum Quantization Error

The resolution is:

$$\text{Resolution} = \frac{20}{1024} = 0.01953\,\text{V}$$

The maximum quantization error is half the resolution:

$$\text{Error} = \frac{0.01953}{2} = 0.00977\,\text{V}$$

(d) Storage Requirement

For a 4 kHz sampling rate over 10 minutes:

$$\text{Samples} = F_s \times \text{time} = 4000 \times (10 \times 60) = 2400000$$

(e) New Signal

For $f_{\text{max}} = 2500\,\text{Hz}$, the new sampling rate is:

$$F_s = 5000\,\text{Hz}$$

The quantization levels and maximum error remain unchanged.

4 Problem 4: Audio Signal Sampling

(a) Original Frequencies

The original frequencies present in $x(t)$ are:

$$1000\,\text{Hz}, 1500\,\text{Hz}, 3000\,\text{Hz}$$

(b) Discrete-Time Frequencies

The discrete-time frequencies are:

$$\Omega = \{1.57, 2.36, -1.57\}\,\text{radians/sample}$$

(c) Reconstructed Frequencies

The reconstructed frequencies are:

$$1000\,\text{Hz}, 1500\,\text{Hz}$$

These match the audible frequencies in the original signal.