1. Basic Circuit Elements

1.1 Resistors

Resistor resists the flow of charge. The resistance R is a function of length, area, and resistivity:

$$R = \frac{\rho \ell}{A} \quad \text{or} \quad R = \frac{\ell}{\sigma A}$$

- ρ: Resistivity
- ℓ: Length
- A: Area

Ohm's Law: Voltage, current, and resistance are related:

$$V = IR \quad \text{or} \quad I = \frac{V}{R}$$

Example: If $V = 10\,V$ and $R = 5\,\Omega$, then $I = \frac{10\,V}{5\,\Omega} = 2\,A$.

Power Dissipation:

$$P = IV = I^2 R = \frac{V^2}{R}$$

1.2 Conductance

Conductance is the reciprocal of resistance:

$$G = \frac{1}{R} \quad \text{in Siemens (S)}$$

1.3 Ideal Conductors

$$R = 0, \quad \sigma \to \infty$$

No voltage drop across an ideal conductor.

2. Circuit Laws

2.1 Kirchhoff's Current Law (KCL)

The sum of currents entering a node equals the sum of currents leaving the node:

$$\sum_{k=1}^{n} i_k = 0$$

Example: At a node, $2\,A$, $3\,A$, and $5\,A$ enter. If i_x leaves, then:

$$2 + 3 + 5 = i_x \quad \Rightarrow \quad i_x = 10\,A$$

2.2 Kirchhoff's Voltage Law (KVL)

The sum of voltage drops in a loop equals the sum of voltage rises:

$$\sum_{k=1}^{n} v_k = 0$$

Example: In a loop with three voltage drops $10\,V$, $6\,V$, and $4\,V$ and a supply voltage $V_s = 24\,V$:

$$10 + 6 + 4 + V_x = 24 \quad \Rightarrow \quad V_x = 4\,V$$

3. Resistors in Series and Parallel

3.1 Series Resistors

Resistors in series share the same current:

$$R_{\text{eq}} = R_1 + R_2 + \cdots + R_n$$

Voltage Division: The voltage across R_k is:

$$V_k = V_s \frac{R_k}{R_{\text{eq}}}$$

Example: For two resistors in series, $R_1 = 10\,\Omega$ and $R_2 = 5\,\Omega$, and the source voltage $V_s = 30\,V$:

$$V_1 = 30\frac{10}{15} = 20\,V, \quad V_2 = 30\frac{5}{15} = 10\,V$$

3.2 Parallel Resistors

Resistors in parallel share the same voltage:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

Current Division:

$$i_k = I_s \frac{R_{\text{eq}}}{R_k}$$

Example: For two resistors in parallel, $R_1 = 6\,\Omega$ and $R_2 = 3\,\Omega$, and a total current $I_s = 6\,A$:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2} \quad \Rightarrow \quad R_{\text{eq}} = 2\,\Omega$$

$$i_1 = 6\,A \times \frac{2}{6} = 2\,A, \quad i_2 = 6\,A \times \frac{2}{3} = 4\,A$$

3.3 Series-Parallel Combination Example

Find the equivalent resistance of the following circuit:

$$R_1 = 4\,\Omega, \quad R_2 = 6\,\Omega \quad \text{(in parallel)}, \quad R_3 = 2\,\Omega \quad \text{(in series)}$$

$$\frac{1}{R_{\text{eq1}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4} + \frac{1}{6} = \frac{5}{12} \quad \Rightarrow R_{\text{eq1}} = 2.4\,\Omega$$

$$R_{\text{total}} = R_{\text{eq1}} + R_3 = 2.4\,\Omega + 2\,\Omega = 4.4\,\Omega$$

4. Sources

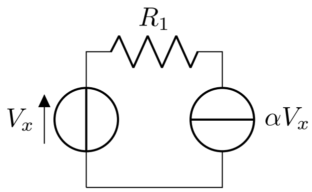
4.1 Independent Sources

Ideal Voltage Source: Constant voltage irrespective of current.
Ideal Current Source: Constant current irrespective of voltage.

4.2 Dependent Sources

Dependent Voltage Source: Voltage depends on a current or voltage elsewhere.
Dependent Current Source: Current depends on a current or voltage elsewhere.

4.3 Example



5. Equivalent Resistance and Δ-Y Conversion

5.1 Simplification of Resistive Networks

To simplify circuits:

- Combine series and parallel resistors.
- Reduce complex networks step-by-step.

5.2 Delta-Wye (Δ-Y) Conversion

For resistors in a delta configuration, the equivalent Y-resistance is calculated as:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \quad R_2 = \frac{R_a R_c}{R_a + R_b + R_c}, \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Example: For a delta network with $R_a = 10\,\Omega$, $R_b = 20\,\Omega$, and $R_c = 30\,\Omega$:

$$R_1 = \frac{20 \times 30}{10 + 20 + 30} = 12\,\Omega, \quad R_2 = \frac{10 \times 30}{60} = 5\,\Omega, \quad R_3 = \frac{10 \times 20}{60} = 3.33\,\Omega$$

6. Passive Sign Convention

In the passive sign convention:

- If current enters through the positive terminal of an element, the element is absorbing power:

$$P = VI$$

- If current enters through the negative terminal, the element is delivering power:

$$P = -VI$$

6.1 Example: Power Calculation

Example: A $12\,V$ source supplies a current of $2\,A$ to a resistor. Power delivered by the source:

$$P = -VI = -(12\,V)(2\,A) = -24\,W$$

This means the source is delivering $24\,W$ of power.

7. Practice Problems

Problem 1: Ohm's Law

A $10\,\Omega$ resistor has a current of $1.5\,A$. What is the voltage across it?

Solution:

$$V = IR = 1.5\,A \times 10\,\Omega = 15\,V$$

Problem 2: Equivalent Resistance

Find the equivalent resistance of $4\,\Omega$ and $6\,\Omega$ in parallel.

Solution:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{4} + \frac{1}{6} = \frac{5}{12} \quad \Rightarrow \quad R_{\text{eq}} = 2.4\,\Omega$$