Problem 1

Consider a 3-bit analog to digital convertor A/D with a full-scale range of $\pm 10V$. An input signal in the form $x(t) = 5\sin(2000\pi t)$ is acquired by the A/D using a sampling rate $F_s = 8kHz$.

a. Determine the resolution of the A/D.

b. Determine the maximum quantization error for the given input signal.

c. Compute and sketch the quantization levels of the given A/D.

d. Write an expression for the sampled discrete time signal $x[n] = x(nT_s)$ where T_s is the sampling interval, $T_s = \frac{1}{F_s}$. Determine the discrete time frequency, Ω.

e. What is the minimum sampling frequency that can be used to sample $x(t)$?

f. Now assume the given input signal is acquired by the A/D using a sampling frequency, $F_s = 8kHz$. Sketch the samples and quantized signal over one cycle of the input signal, assuming each sample is held constant at the corresponding quantization level, during the sampling interval.

Problem 2

Consider a certain sensor that produces a signal of interest having peak values of $\pm 0.2V$. The highest frequency component of the signal is $4kHz$. The signal is acquired by an analog to digital convertor A/D with a full-scale range of $\pm 5V$. The quantization error should not exceed 0.1% of the maximum signal amplitude.

a. What is the minimum number of bits required by the A/D.

b. What is the minimum sampling frequency?

c. If the input signal is amplified first to extend over the entire full-scale range of the A/D, what should the gain factor of the amplifier be? If the quantization error should not exceed 0.1% of the maximum signal amplitude, what is the minimum number of bits required by the A/D in this case?

Problem 3

Consider a waveform, $x(t) = 2cos(1000\pi t) + 3cos(2000\pi t)$ is to be uniformly sampled for digital transmission. The analog to digital convertor, A/D is a 10-bit system and covers a signal range of $\pm 10V$.

a. What is the minimum allowable sampling rate that will ensure perfect signal reproduction?

b. What is the number of quantization levels for the A/D?

c. What is the maximum quantization error?

d. If a sampling frequency of $4kHz$ is used and we want to reproduce 10 min of this waveform, how many sample values need to be stored?

e. Will any of your answers in parts a,b,c change if the waveform is $x(t) = 8cos(5000\pi t) + 3cos(2000\pi t)$

Problem 4

An audio signal is recorded by a microphone which produced the following analog signal,

$$x(t) = 2cos(2000\pi t + \pi/3) + 3cos(6000\pi t) + 2\cos(3000\pi t)$$

The analog signal is then sampled at a sampling rate of $F_s = 4000Hz(samples/second)$ and stored as a discrete time signal, $x[n] = x(nT_s)$.

The discrete time signal, $x[n]$, is then played back using a speaker connected to the computer. The same sampling rate was used to reconstruct the signal. The reconstructed continuous time signal is denoted by $x_r(t)$.

a. What frequencies can be heard in the original audio signal $x(t)$?

b. Write an expression for the discrete time signal, $x[n]$. The frequencies of the discrete time signal, Ω, should be converted to the smallest equivalent value in the range $-\pi \leq \Omega \leq \pi$. Use symmetry to obtain positive discrete time frequencies, for example $\cos(-\Omega_o n + \theta) = \cos(\Omega_o n - \theta)$ and $\sin(-\Omega_o n + \theta) = -\sin(\Omega_o n - \theta)$.

c. What frequencies will be heard in the reconstructed signal $x_r(t)$? Do these frequencies match the frequencies in the original signal $x(t)$?