1 Capacitors and Inductors

1.1 Capacitors

A capacitor stores energy in the electric field between its plates.
Key Relationships:

$$C = \frac{Q}{V}, \quad i(t) = C\frac{dv(t)}{dt}, \quad v(t) - v(t_0) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau$$

$$w(t) = \frac{1}{2}Cv^2(t), \quad P(t) = i(t)v(t)$$

Behavior:

- Acts as an open circuit in steady-state DC.
- Voltage cannot change instantaneously ($i \to \infty$ for rapid voltage changes).
- Frequency selective: responds differently to fast and slow voltage changes.

Capacitor Combinations:

- Series: $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$
- Parallel: $C_{\text{eq}} = C_1 + C_2 + \cdots$

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1.2 Inductors

An inductor stores energy in its magnetic field.
Key Relationships:

$$v(t) = L\frac{di(t)}{dt}, \quad i(t) - i(t_0) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau$$

$$w(t) = \frac{1}{2}Li^2(t), \quad P(t) = v(t)i(t)$$

Behavior:

- Acts as a short circuit in steady-state DC.
- Current cannot change instantaneously ($v \to \infty$ for rapid current changes).
- Frequency selective: opposes changes in current.

Inductor Combinations:

- Series: $L_{\text{eq}} = L_1 + L_2 + \cdots$
- Parallel: $\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots$

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2 RC and RL Circuits

2.1 RC Circuits

Capacitor Properties:

$$i(t) = C\frac{dv(t)}{dt}, \quad w(t) = \frac{1}{2}Cv^2(t)$$

- Capacitor acts as an open circuit in steady-state DC.
Transient Analysis:

$$v(t) = V_f + (V_i - V_f)e^{-t/\tau}, \quad \tau = RC$$

$$i(t) = \frac{V_s}{R}e^{-t/RC}$$

2.2 RL Circuits

Inductor Properties:

$$v(t) = L\frac{di(t)}{dt}, \quad w(t) = \frac{1}{2}Li^2(t)$$

- Inductor acts as a short circuit in steady-state DC.
Transient Analysis:

$$i(t) = I_f + (I_i - I_f)e^{-t/\tau}, \quad \tau = \frac{L}{R}$$

$$v(t) = L\frac{di(t)}{dt}$$

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3 Kirchhoff's Laws

3.1 Kirchhoff's Current Law (KCL)

The sum of currents entering and leaving a node is zero:

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

3.2 Kirchhoff's Voltage Law (KVL)

The sum of voltage drops in a closed loop equals the sum of voltage rises:

$$\sum V_{\text{drops}} = \sum V_{\text{rises}}$$

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4 Nodal Analysis

4.1 Steps for Nodal Analysis

1. Identify essential nodes.
2. Choose a reference node (ground).
3. Apply KCL at each node using node voltages.
4. Solve the system of equations for unknown voltages.

Example for RC Circuit:

$$\frac{v_1 - v_2}{R} + C\frac{dv_2}{dt} = 0$$

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5 Mesh Analysis

5.1 Steps for Mesh Analysis

1. Identify meshes (loops without other loops inside).
2. Assign mesh currents.
3. Apply KVL in each mesh.
4. Solve for mesh currents.

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6 Transient Responses

6.1 General Solution

The total response is the sum of the natural and forced responses:

$$y(t) = y_h(t) + y_p(t)$$

- $y_h(t)$: Homogeneous solution, exponential decay:

$$y_h(t) = Ke^{-t/\tau}$$

- $y_p(t)$: Particular solution, steady-state response.

6.2 Time Constant

- RC Circuits: $\tau = R_{eq}C$ - RL Circuits: $\tau = \frac{L}{R_{eq}}$
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7 Example: Step Response

7.1 RC Circuit

Voltage across capacitor:

$$v_C(t) = V(1 - e^{-t/RC})$$

Current through resistor:

$$i_R(t) = \frac{V}{R}e^{-t/RC}$$

7.2 RL Circuit

Current through inductor:

$$i_L(t) = I(1 - e^{-tR/L})$$

Voltage across resistor:

$$v_R(t) = IR(1 - e^{-tR/L})$$