1 Basic Circuit Elements

1.1 Resistors

Resistor resists the flow of charge. The resistance R is a function of length, area, and resistivity:

$$R = \frac{\rho \ell}{A} \quad \text{or} \quad R = \frac{\ell}{\sigma A}$$

- ρ: Resistivity
- ℓ: Length
- A: Area

Ohm's Law: Voltage, current, and resistance are related:

$$V = IR \quad \text{or} \quad I = \frac{V}{R}$$

1.2 Power Dissipation and Passive Sign Convention

Power Dissipation:

$$P = IV = I^2 R = \frac{V^2}{R}$$

- In passive sign convention, if current enters positive terminal, element absorbs power:

$$P = VI$$

- If current enters negative terminal, element delivers power:

$$P = -VI$$

1.3 Conductance

Conductance is the reciprocal of resistance:

$$G = \frac{1}{R} \quad \text{in Siemens (S)}$$

1.4 Ideal Conductors

$$R = 0, \quad \sigma \to \infty$$

No voltage drop across an ideal conductor.
—

2 Kirchhoff's Laws

2.1 Kirchhoff's Current Law (KCL)

The sum of currents entering and leaving a node is zero:

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

2.2 Kirchhoff's Voltage Law (KVL)

The sum of voltage drops in a closed loop equals the sum of voltage rises:

$$\sum V_{\text{drops}} = \sum V_{\text{rises}}$$

—

3 Resistors in Series and Parallel

3.1 Series Resistors

Resistors in series carry the same current:

$$R_{\text{eq}} = R_1 + R_2 + \cdots + R_n$$

Voltage Division:

$$V_k = V_s \frac{R_k}{R_{\text{eq}}}$$

3.2 Parallel Resistors

Resistors in parallel share the same voltage:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

Current Division:

$$i_k = I_s \frac{R_{\text{eq}}}{R_k}$$

—

4 Nodal Analysis (Node Voltage Method)

4.1 Steps for Nodal Analysis

1. Identify essential nodes.
2. Choose a reference node (ground).
3. Write KCL equations at each essential node using node voltages.
4. Solve the system of equations for unknown node voltages.

—

5 Mesh Analysis (Mesh Current Method)

5.1 Steps for Mesh Analysis

1. Identify meshes (loops without other loops inside).
2. Assign mesh currents.
3. Apply KVL in each mesh to write voltage equations.
4. Solve the system of equations for mesh currents.

—

6 Thevenin and Norton Equivalent Circuits

6.1 Thevenin's Theorem

Any linear circuit can be reduced to a single voltage source V_{Th} in series with R_{Th}.

$$V_L = V_{Th} \frac{R_L}{R_{Th} + R_L}$$

6.2 Norton's Theorem

Any linear circuit can be reduced to a single current source I_N in parallel with R_N.

$$I_N = \frac{V_{\text{Th}}}{R_{\text{Th}}}, \quad R_N = R_{\text{Th}}$$

—

7 Source Transformations

7.1 Voltage to Current Source Transformation

A voltage source V_s in series with R can be transformed into a current source:

$$I_s = \frac{V_s}{R}, \quad \text{in parallel with } R$$

7.2 Current to Voltage Source Transformation

A current source I_s in parallel with R can be transformed into a voltage source:

$$V_s = I_s R, \quad \text{in series with } R$$

—

8 Delta-Y (Δ-Y) Conversion

8.1 Delta to Y Conversion

For a delta network with resistors R_a, R_b, and R_c, the equivalent Y-resistances are:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \quad R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

8.2 Y to Delta Conversion

For a Y-network with resistors R_1, R_2, and R_3, the equivalent delta-resistances are:

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}, \quad R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

—

9 Operational Amplifiers (Op-Amps)

9.1 Introduction to Op-Amps

Operational Amplifier (Op-Amp) is a high-gain IC used in various mathematical and amplification operations:

- Ideal characteristics: infinite input impedance, zero output impedance, infinite open-loop gain, and bandwidth.
- Common applications: integrator, differentiator, summing amplifier, buffer amplifier, difference amplifier.

9.2 Ideal Op-Amp Model

- Infinite input impedance $R_{in} = \infty$.
- Zero output impedance $R_{out} = 0$.
- Infinite open-loop gain $A_{vo} \to \infty$.
- Differential input voltage $V_d = V^+ - V^-$, ideally $V_d \approx 0$.

9.3 Basic Op-Amp Configurations

9.3.1 Inverting Amplifier

Gain $A_v = -\frac{R_f}{R_{in}}$

$$V_{out} = -V_{in} \cdot \frac{R_f}{R_{in}}$$

9.3.2 Non-Inverting Amplifier

Gain $A_v = 1 + \frac{R_f}{R_1}$

$$V_{out} = V_{in} \cdot \left(1 + \frac{R_f}{R_1}\right)$$

9.3.3 Voltage Follower (Buffer)

Gain $A_v = 1$, used for isolation

$$V_{out} = V_{in}$$

9.4 Real-World Op-Amp Limitations

9.4.1 Bandwidth and Slew Rate

Bandwidth: Gain-Bandwidth Product (GBP): $A_v \times f_{BW} =$ constant **Slew Rate:** Max rate of change of V_{out}:

$$\text{Slew Rate} = \frac{\Delta V_{out}}{\Delta t}$$

9.4.2 Input Offset Voltage and Bias Current

Input Offset Voltage V_{OS}: Small voltage difference to zero V_{out}. **Input Bias Current:** Average current into input terminals.

9.5 Op-Amp Analysis Techniques

- Assume ideal conditions ($V^+ = V^-$, $I^+ = I^- = 0$).
- Apply KCL at nodes, considering virtual ground in inverting amplifiers.

9.6 Feedback in Op-Amps

Types of Feedback:

- Negative Feedback: Stabilizes gain.
- Positive Feedback: Used in oscillators.

Closed-Loop Gain:

$$A_{CL} = \frac{A_{vo}}{1 + A_{vo} \cdot \beta}$$