Evaluation of Postfix and Prefix Expressions

Part II: Evaluation of Expressions

Expression (a) — Postfix

Postfix Expression:

$$4\ 5\ +\ 10\ \times\ 6\ 2\ \div\ -$$

Step-by-Step Evaluation:

Step 1: Push 4. *Stack:* [4]

Step 2: Push 5. *Stack:* [4, 5]

Step 3: Operator +: Pop 5 and 4; compute $4 + 5 = 9$; push 9. *Stack:* [9]

Step 4: Push 10. *Stack:* [9, 10]

Step 5: Operator ×: Pop 10 and 9; compute $9 \times 10 = 90$; push 90. *Stack:* [90]

Step 6: Push 6. *Stack:* [90, 6]

Step 7: Push 2. *Stack:* [90, 6, 2]

Step 8: Operator ÷: Pop 2 and 6; compute $6 \div 2 = 3$; push 3. *Stack:* [90, 3]

Step 9: Operator −: Pop 3 and 90; compute $90 - 3 = 87$; push 87. *Stack:* [87]

Final Result: $\boxed{87}$

Expression (b) — Postfix

Postfix Expression:

$$3\ 4\ \times\ (5\ 6\ -)\ 2\ 3\ +\ \times$$

Step-by-Step Evaluation:

Step 1: Push 3. *Stack:* [3]

Step 2: Push 4. *Stack:* [3, 4]

Step 3: Operator $\times$**:** Pop 4 and 3; compute $3 \times 4 = 12$; push 12. *Stack:* [12]

Step 4: Subexpression (5 6 $-$)**:**

- **Push** 5.
- **Push** 6.
- **Operator** $-$**:** Pop 6 and 5; compute $5 - 6 = -1$; push -1.

Stack now: [12, -1]

Step 5: Push 2. *Stack:* [12, -1, 2]

Step 6: Push 3. *Stack:* [12, -1, 2, 3]

Step 7: Operator $+$**:** Pop 3 and 2; compute $2 + 3 = 5$; push 5. *Stack:* [12, -1, 5]

Step 8: Operator $\times$**:** Pop 5 and -1; compute $(-1) \times 5 = -5$; push -5. *Stack:* [12, -5]

Step 9: Final Operator $\times$**:** Pop -5 and 12; compute $12 \times (-5) = -60$; push -60. *Stack:* [-60]

Final Result: $\boxed{-60}$

Expression (c) — Prefix

Prefix Expression:

$$- \ 29 \ + \ 5 \ * \ 4 \ 6$$

Step-by-Step Evaluation:

Step 1: The first token is the subtraction operator $-$, which requires two operands.

Step 2: First operand: 29.

Step 3: Second operand: Evaluate the subexpression starting with $+$:

- The operator $+$ requires two operands: the first is 5 and the second is the subexpression $* \ 4 \ 6$.
- **Evaluate** $* \ 4 \ 6$: Multiply 4 and 6 to get 24.
- Now compute $5 + 24 = 29$.

Step 4: Finally, compute the subtraction: $29 - 29 = 0$.

Final Result: $\boxed{0}$

Expression (d) — Prefix

Prefix Expression:

$$\times\ 5\ \div\ 6\ +\ 6\ \times\ 3\ 2$$

Step-by-Step Evaluation:

Step 1: The first token is the multiplication operator $\times$. It requires two operands:

- **First operand:** 5.
- **Second operand:** The subexpression starting with $\div$.

Step 2: Evaluate the subexpression $\div\ 6\ +\ 6\ \times\ 3\ 2$:

- The operator $\div$ requires two operands:
  - **First operand:** 6.
  - **Second operand:** Evaluate the subexpression starting with $+$.
- **Evaluate** the subexpression $+\ 6\ \times\ 3\ 2$:
  - The operator $+$ requires two operands: the first is 6 and the second is the subexpression $\times\ 3\ 2$.
  - **Evaluate** $\times\ 3\ 2$: Multiply 3 and 2 to get 6.
  - Now compute $6 + 6 = 12$.
- Now, with the subexpression evaluated, compute the division: $6 \div 12 = 0.5$.

Step 3: Finally, compute the initial multiplication: $5 \times 0.5 = 2.5$.

Final Result: $\boxed{2.5}$