Announcement

- C Workshop
  - 7:00 – 7:50 p.m. today (last one!)
  - MECC 127
  - Required (need to attend only one)
  - Topic: String operations in C

Lecture 6

Integer Representations II

CPSC 275
Introduction to Computer Systems

Encoding Integers

x: w-bit binary representation of an integer

Unsigned

$$\sum_{i=0}^{w-1} x_i \cdot 2^i$$

Two's Complement

$$-x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

Encoding Integers, cont'd

```
x =  15213: 00111011 01101101
y = -15213: 11000100 10010011
```

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		**15213**		**-15213**

Numeric Ranges

- Unsigned Values

  UMin

  000…0 = 0

  UMax

  111…1 = $2^w - 1$

- Two's Complement Values

  TMin

  100…0 = -2^{w-1}

  TMax

  011…1 = $2^{w-1} - 1$

- Other Values

  –1 in 2's complement?

  111…1

Values for $w = 16$

	Decimal	Hex	Binary
UMax	**65535**	**FF FF**	11111111 11111111
TMax	**32767**	7F FF	01111111 11111111
TMin	**-32768**	80 00	10000000 00000000
-1	**-1**	FF FF	11111111 11111111
0	**0**	00 00	00000000 00000000

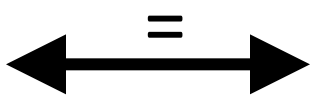
Unsigned & Signed Numeric Values

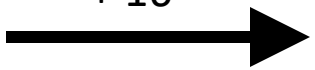
x	unsigned	signed
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	–8
1001	9	–7
1010	10	–6
1011	11	–5
1100	12	–4
1101	13	–3
1110	14	–2
1111	15	–1

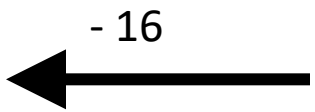
- Same encoding for non-negative integers
- Each integer has a unique bit encoding.

Mapping between unsigned and signed integers

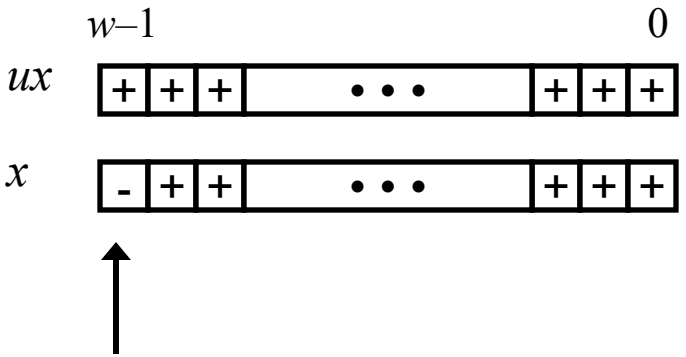
Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15







Relation between Signed & Unsigned



Large negative weight
becomes
Large positive weight

$$ux = \begin{cases} x & x \geq 0 \\ x + 2^w & x < 0 \end{cases}$$

Signed → Unsigned Conversion