Lecture 10

Integer Addition
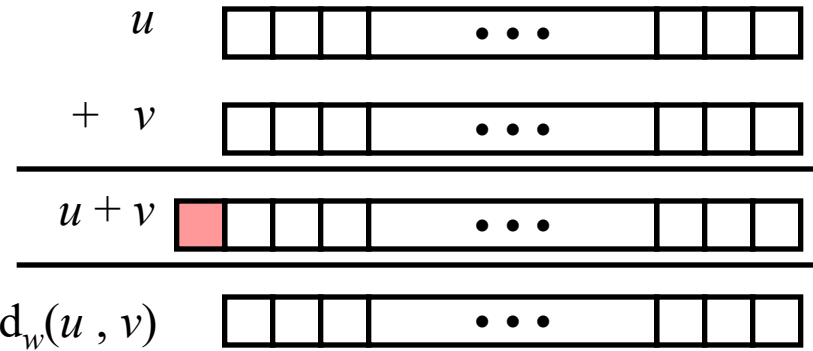
CPSC 275
Introduction to Computer Systems

Unsigned Addition

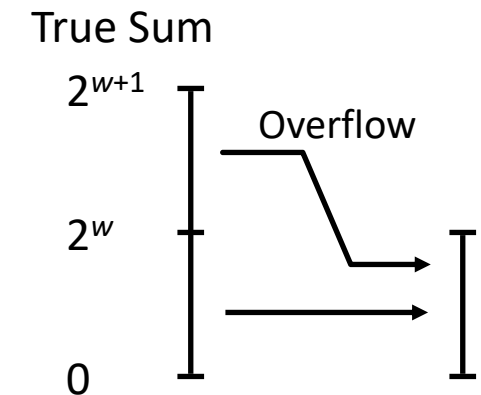Operands: w bits

u

$+\ v$

True Sum: $w+1$ bits

$u + v$

Discard Carry: w bits

$\mathrm{UAdd}_w(u\ ,\ v)$

- True sum requires w+1 bits
- Drop off MSB
- Treat remaining bits as unsigned integer
- Consider two cases.

True Sum

2^{w+1}

Overflow

2^w

$$UAdd_w(u,v) \;=\; \begin{cases} u+v & u+v < 2^w \\ u+v-2^w & u+v \geq 2^w \end{cases}$$

0

Two's Complement Addition
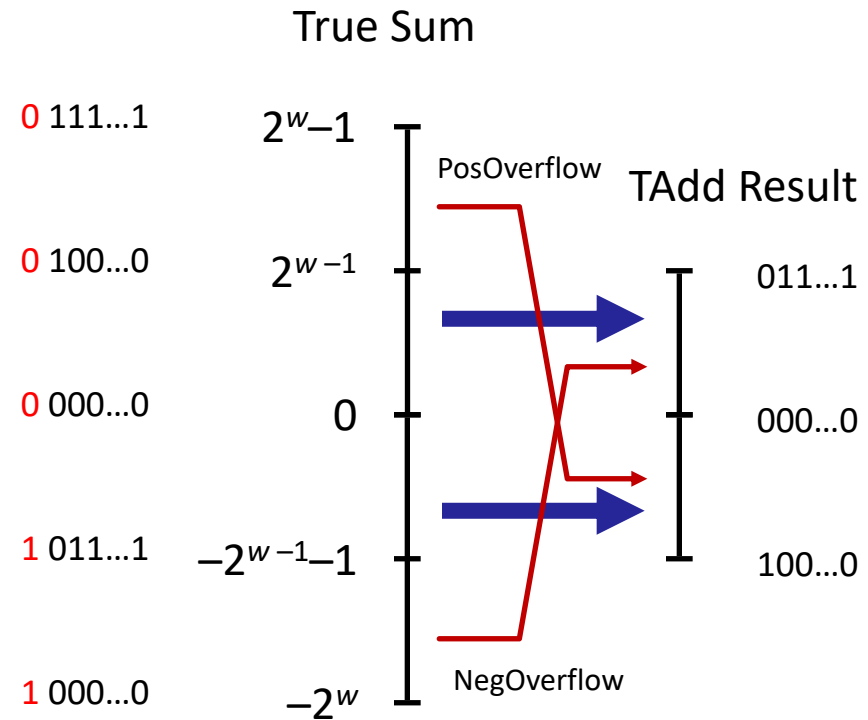
Operands: w bits

True Sum: $w+1$ bits

Discard Carry: w bits

- TAdd and UAdd have identical bit-level behavior!

TAdd Overflow

- True sum requires $w+1$ bits
- Drop off MSB
- Treat remaining bits as 2's comp. integer
- Consider four cases.

True Sum

0 111...1	2^w-1
0 100...0	2^{w-1}
0 000...0	0
1 011...1	$-2^{w-1}-1$
1 000...0	-2^w

PosOverflow

TAdd Result

011...1

000...0

100...0

NegOverflow

Example

Assume a 4-bit unsigned binary representation.

0110 + 0111 = ?

  6   + 7

Example

Assume a 4-bit unsigned binary representation.

0110 + 0111 = 1101

6 + 7 = 13

Example

Assume a 4-bit unsigned binary representation.

1101 + 0101 = ?
 13 + 5

Example

Assume a 4-bit unsigned binary representation.

1101 + 0101 = 10010 (overflow)

13 + 5   =   2 (?)

Example

Assume a 4-bit signed binary representation.

1110 + 0011 = ?
 -2   +   3

Example

Assume a 4-bit signed binary representation.

1110 + 0011 = 0001

-2 + 3 = 1

Example

Assume a 4-bit signed binary representation.

0110 + 0011 = ?

    6 +   3

Example

Assume a 4-bit signed binary representation.

0110 + 0011 = 1001 (overflow)

   6   +   3 =   -7 (?)

Example

Assume a 4-bit signed binary representation.

1001 + 1101 = ?
-7   +   -3

Example

Assume a 4-bit signed binary representation.

1001 + 1101 = 10110 (overflow)
 -7   +   -3   =   6 (?)