Observation of the Classical and Quantum Hall Effects

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Abstract

In this experiment, the workings of the classical and integral quantum Hall effects were investigated. The classical Hall effect were used to determine whether electrons or holes were the primary charge carriers in each of two samples of doped silicon. A sample of Gallium Arsenide at high magnetic field strength and low temperature was afterward used to observe the quantum Hall effect. Values for the von Klitzing constant R_K and the fine-structure constant α were calculated using collected Hall resistance measurements.

1 Introduction

The classical Hall effect describes the generation of a voltage in a direction transverse to that of the current flowing through a conductor in the presence of a perpendicular magnetic field. As the magnetic field strength increases, so does the deflection of charge carriers, and the potential difference in the direction transverse to the current rises [1].

For a system exhibiting the quantum Hall effect, changes in the Hall resistance are quantized. As the magnetic field strength increases, the Hall resistance plateaus at values that are even integer fractions of the von Klitzing constant R_K.

2 Classical Hall effect

In this experiment, given two semiconductor samples labeled A and B, the goal was to determine whether electrons or holes were being influenced by the Hall effect to generate a potential difference transverse to the current going through the semiconductor. Because the force on a particle with charge q moving at speed v through a magnetic field of strength B is

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}, \tag{1}$$

we choose perpendicular pairs of leads connected to opposite edges of the sample, and place the sample facing the magnetic field generated by an electromagnet. Because electrons have negative charge and holes are equivalent to positively charged particles, the voltage observed in the direction transverse to the current will tell us which charge carriers are being deflected.

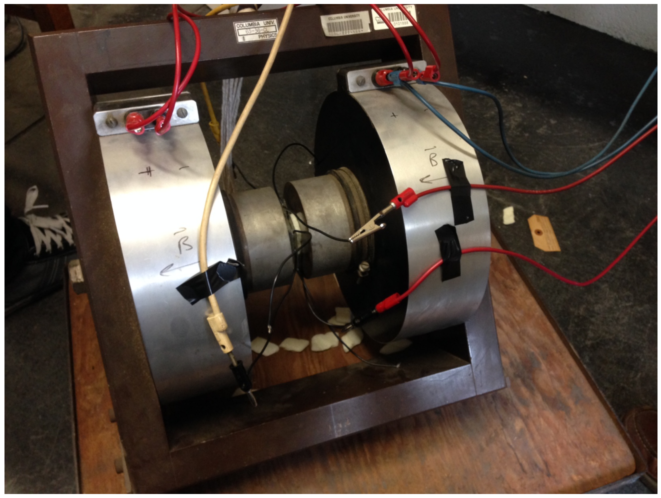

Figure I: One of the samples suspended between the poles of the electromagnet, both connected to power supplies and multimeters.

The magnetic field strength was increased, and the transverse voltage was recorded. Amusingly, a Hall effect gaussmeter was used to determine the minimum and maximum values of the magnetic field strength, and the average of these two values was used for plotting, with the minimum and maximum serving as error bars. The uncertainty in voltage measurements from the multimeter was also recorded.

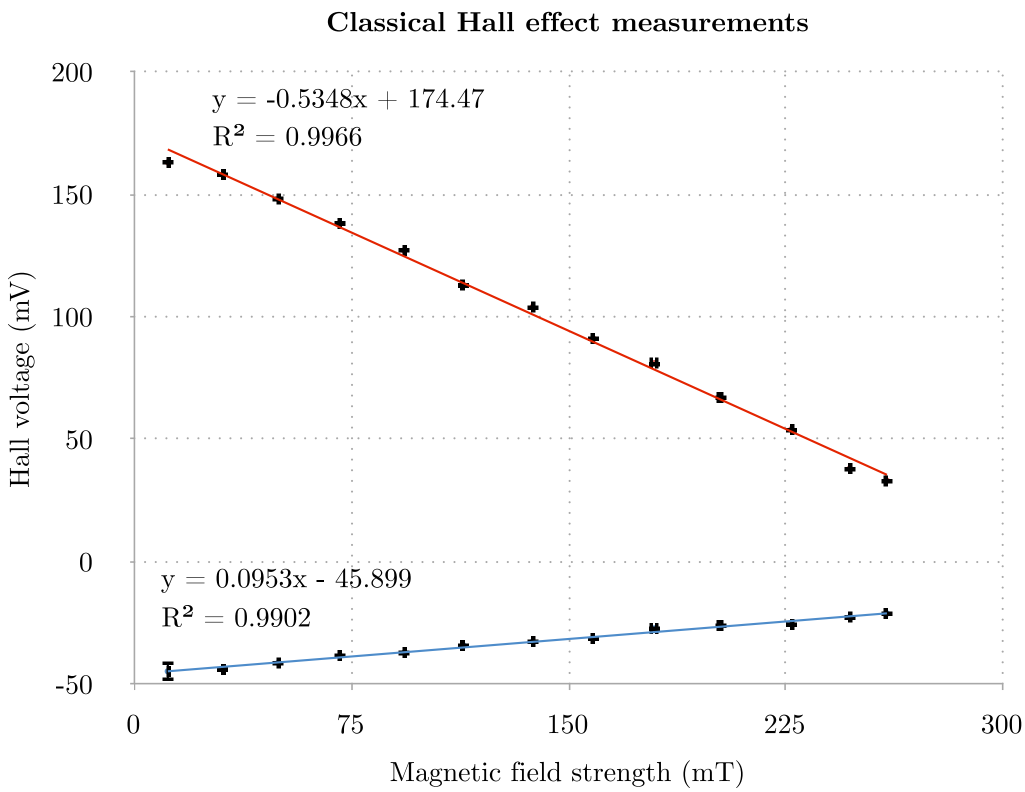

Figure II: Plot of Hall voltage recorded vs. magnetic field strength, with uncertainty in both measurements indicated by error bars. Sample A is shown in red, and sample B in blue.

For sample A, a current of 0.03 A flowing through the sample was measured, with a voltage drop of 16.89±0.05 mV. For sample B, the current was 0.09, and the voltage drop 16.54±0.05 mV.

We observe that the Hall voltage is changing in opposite ways for each sample as the magnetic field strength is increased. Because electrons and holes are of opposite charge, the deflection resulting from equation (1) is in the same direction in both cases. But for the same reason, the change in Hall voltage induced from increasing deflection is opposite. The negative slope of the red line implies that the sign of its charge carriers is negative, and the positive slope of the blue line indicates that its charge carriers have a positive sign.

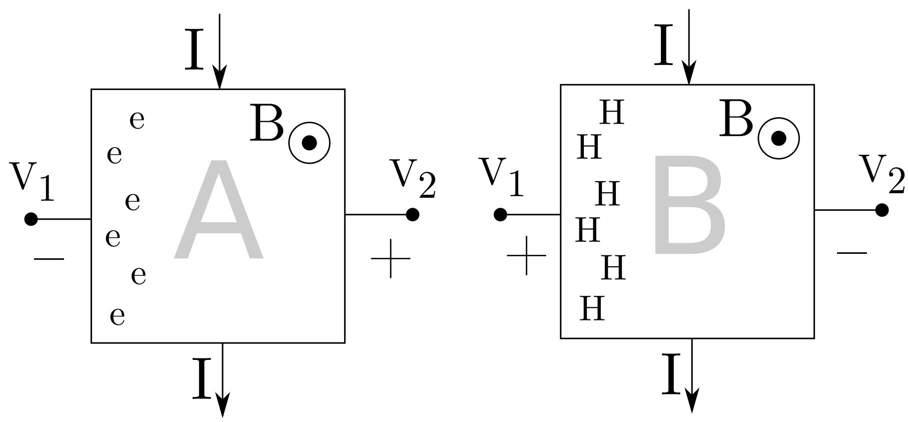

FIGURE III: Diagram of the effects of the charge carriers' drift on the measured Hall voltage.

The qualitative analysis of figure II is substantiated when we take into account another effect: the offset voltage. The voltage observed between opposite leads in the direction perpendicular to the flow of current through the sample is the sum of of the Hall voltage and an offset voltage:

$$V_{\perp} = V_H + V_{\text{offset}}(I) \quad [1]. \tag{2}$$

V_{offset} arises from the fact that the leads are not arranged in exactly the perpendicular direction, and the resistance of the sample in the direction of current flow comes into play. It is a function of the current I because $V = IR$, with a constant

$$R = \frac{\rho l}{A} \tag{3}$$

where ρ is the resistivity of the sample, l is the distance between the leads in the direction parallel to the current flowing through it, and A is its cross-sectional area (in this case, the sample's width multiplied by thickness).

Because we expect that there would be no induced Hall voltage in the absence of a magnetic field, we can estimate V_{offset} for each of the samples thanks to equation (2): it corresponds to the y-intercept of each line in figure II. For sample A, $V_{\text{offset}} = 174.47$ mV (corresponding to $R = 0.17447/0.03 = 5.82\ \Omega$), and for sample B, $V_{\text{offset}} = -45.899$ mV (so $R = 0.045899/0.09 = 0.510\ \Omega$). Had we measured the length of the leads' misalignment, as well as the width and thickness of each sample, we could estimate their resistivities.

3 Quantum Hall effect

3.1 Setup

To observe the quantum Hall effect, a sample of Gallium Arsenide (GaAs) heterojunction semiconductor was cooled to the boiling point of liquid helium (around 4 K) and subjected to a strong magnetic field.

FIGURE IV: The sample used [3].

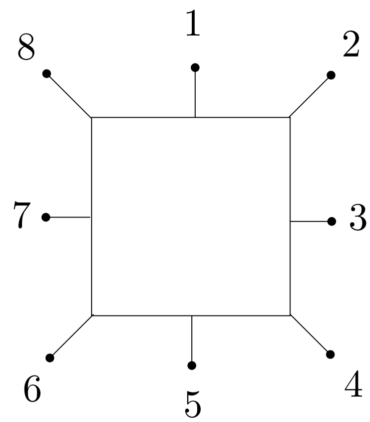

FIGURE V: Labeled sample connection leads.

An 8 Tesla superconducting solenoid connected to a 6260B Hewlett-Packard power supply was used to generate the magnetic field. A ramp generator was not available, and the current supplied to the solenoid was increased slowly by hand.

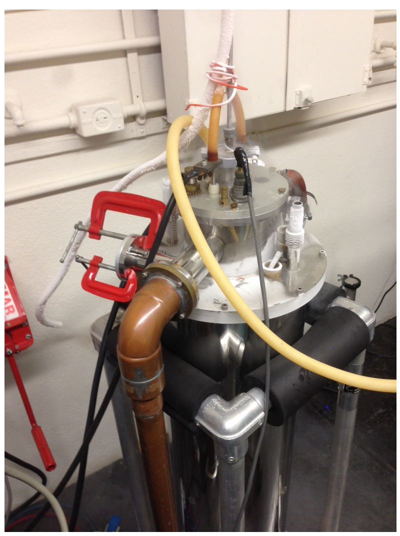

FIGURE VI: The cryostat consisting of a 10L helium dewar containing the magnet.

The voltage drop across the precision resistor was measured using a voltmeter (with internal resistance much larger than R_P so as not to affect the current) and a 67.8 V battery. Its resistance was determined to be $R_P = 12408\ \Omega$, and because this does not vary across a range of temperatures and currents, we can use the measurement to calculate the Hall voltage across the sample. The precision resistor, and then the sample, were arranged in series with 754 kΩ, 6 MΩ, and 68 MΩ resistors.

3.2 Corrections and analysis

The current in the circuit is not measured in real time, but calculated from the voltage drop of the precision resistor. The variation of the Hall resistance with magnetic field strength causes the current to change during measurements, which leads to inaccuracies in R_{Hall}. There is a first-order correction to be made to the recorded measurements:

$$R'_{Hall} = R_{Hall}\left(\frac{R_x + R'_{Hall} - \Delta R}{R_x + R_p}\right) \tag{4}$$

where $\Delta R = R'_{Hall} - R_{Hall}$ [2]. A simple Python program was written to perform the correction of equation (4) on all data points and determine which points on the curve are part of a plateau:

```
def correct(Rh):
    Rhp = Rh*(Rh+Rx)/(Rx+Rp)
    delRh = Rhp - Rh
    Rhp = Rh*(Rx+Rhp)/(Rx+Rp) - Rh*delRh/(Rx+Rp)
    return Rhp

def plateaus(b_field, resistance):
    overall_slope = (resistance[len(resist)-1]-resistance[0])
                    /(b_field[len(b_field)-1]-b_field[0])
    plateaus_y = []
    plateaus_x = []
    for i in range(len(resistance)-1):
        local_slope = (resistance[i+1]-resistance[i-1])/(b_field[i+1]-b_field[i-1])
        if local_slope/overall_slope < .5:   # arbitrary
            plateaus_y.append(resistance[i])
            plateaus_x.append(b_field[i])
    return (plateaus_x, plateaus_y)
```

In practice, the average of the correction differences for all 68 MΩ resistor tests was 0.29939 Ω, which is quite small compared to plateau values on the order of kΩ but slightly relevant to their uncertainties, as shown below.

3.3 68 MΩ resistor tests

The current flowing through the precision resistor was calculated as follows:

$$I = \frac{V_{Rp}}{R_p} = \frac{V_{battery}}{R_x + R_p} = \frac{67.8}{12408 + 68 \cdot 10^6} = 9.9688 \cdot 10^{-7}\ \text{A} \tag{5}$$

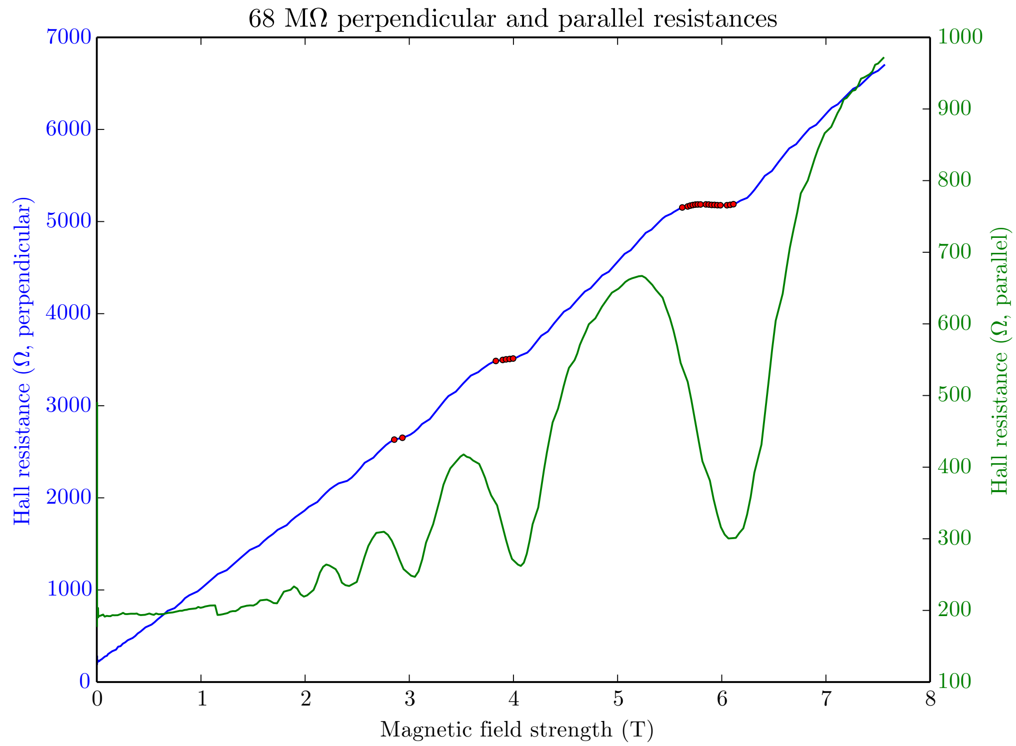

FIGURE VII: Measurements from perpendicular and parallel lead pairs on the GaAs sample, with plateau points shown in red.

For each plateau, the mean and standard error of the points determined to belong to it were calculated. Three sets of measurements were taken with the 68 MΩ resistor, and the mean and its standard error were calculated for each plateau over all three measurements.

	Pass 1	Pass 2	Pass 3	Mean
$i = 10$	2642.3 ± 10.068 Ω	2565.4 ± 6.0759 Ω	2594.8 ± 7.6382 Ω	2600.8 ± 22.407 Ω
$i = 8$	3501.6 ± 4.5691 Ω	3421.2 ± 2.4071 Ω	3413.9 ± 4.9675 Ω	3445.6 ± 28.101 Ω
$i = 6$	5178.7 ± 2.2767 Ω	5093.8 ± 2.2201 Ω	5088.6 ± 2.1015 Ω	5120.5 ± 29.212 Ω

Table 1: Average plateau resistance values and their uncertainities for the 68 MΩ resistor tests.

Systematic error in our measurements is most evident for the plateau corresponding to the integer Hall coefficient $i = 6$. In this case, the mean resistance of all points on the plateau in our first measurement was calculated to be 5178.7 Ω with a relatively low standard error of the mean of 2.2767 Ω corresponding to random error. Two additional passes were made using the same configuration, and the mean Hall resistance for $i = 6$ was determined to be 5120.5 ± 29.212 Ω. The accepted value would be $25812.8/6 = 4302.1$ Ω [4]. The systematic error is then around 16%.

For the plateau in the middle corresponding to $i = 8$, the first pass resulted in a mean of 3501.6 ± 4.5691 Ω, and the mean of the values from the three passes was 3445.6 ± 28.101 Ω. With an accepted value of $25812.8/8 = 3226.6$ Ω [4], the systematic error for $i = 8$ is around 6.4%.

Finally, for the plateau corresponding to $i = 10$, the first pass resulted in a mean of 2642.3 ± 10.068 Ω, and the mean of the values from the three passes was 2600.8 ± 22.407 Ω. With an accepted value of $25812.8/10 = 2581.3$ Ω [4], the systematic error for $i = 10$ is around .75%.

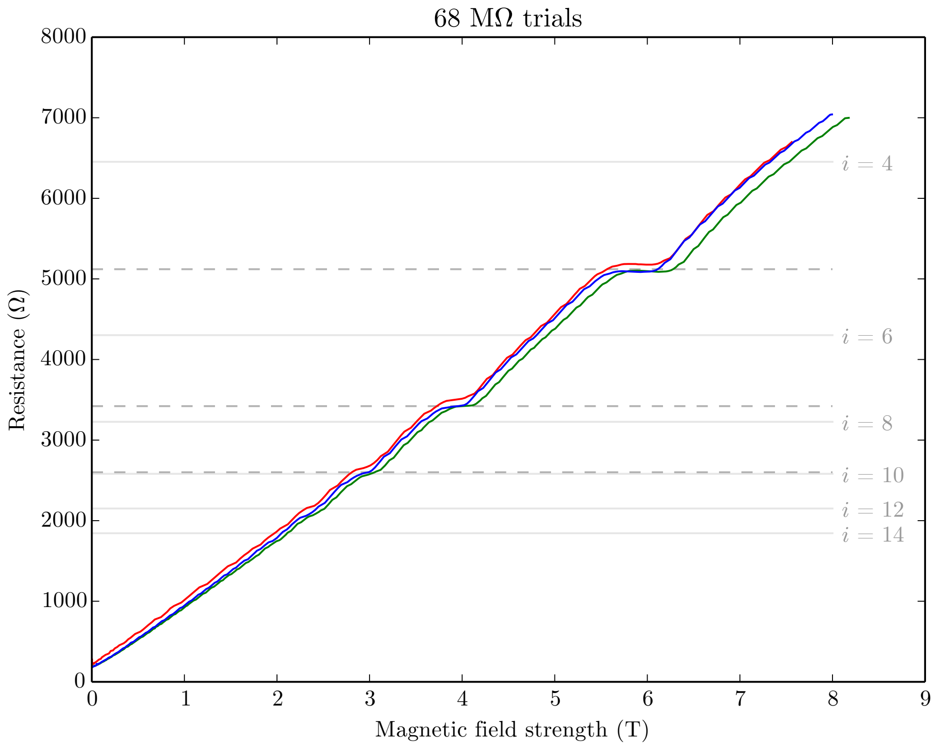

FIGURE VIII: In red, green, and blue are superimposed the first, second, and third passes through the range of magnetic field strength using the 68 MΩ resistor. The gray lines show accepted even integer values of the Hall resistance, and the dotted ones show the calculated means across the three measurements.

The measurements for the 68 MΩ resistor were the most pronounced, and we can use them to calculate a mean value for the von Klitzing constant:

$$R_K = \frac{10R_{H10} + 8R_{H8} + 6R_{H6}}{3} = \frac{2600.8 \cdot 10 + 3445.6 \cdot 8 + 5120.5 \cdot 6}{3} = 28099 \pm 31.112 \ \Omega \qquad (6)$$

in which error was propagated using the same formula. Alternatively, we might use the Hall resistance obtained from the $i = 10$ plateau, which has the least systematic error.

3.4 Other resistor tests

Other tests were carried out with the 6MΩ and 754 kΩ resistors in series with the sample.

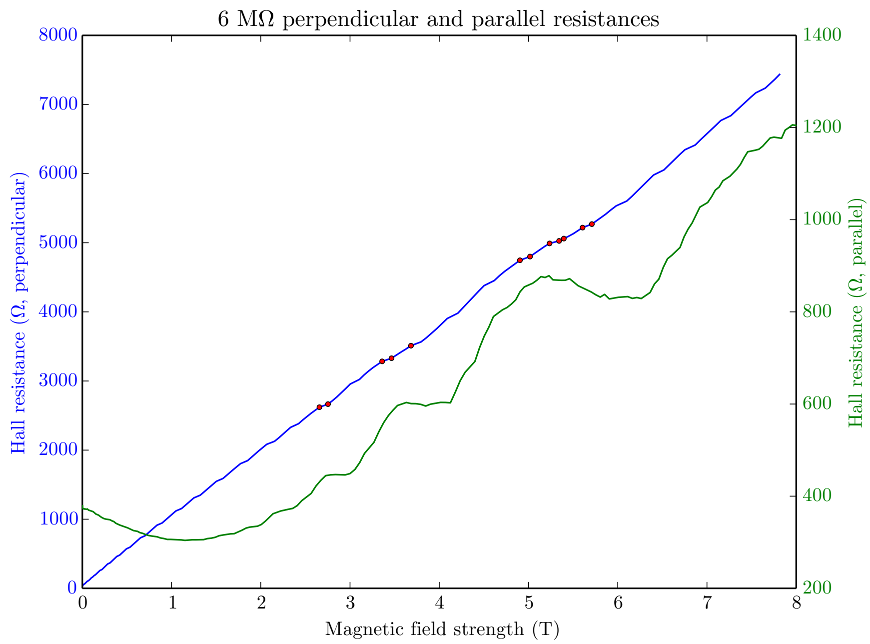

FIGURE IX: Measurements with the 6 MΩ resistor in series with the sample.

Plateaus for these samples are much less distinctive because of the lower signal to noise ratio. Performing the same calculation as in equation (5) yields higher current values, which induce higher temperatures that obscure the manifestation of the Quantum hall effect in the sample.

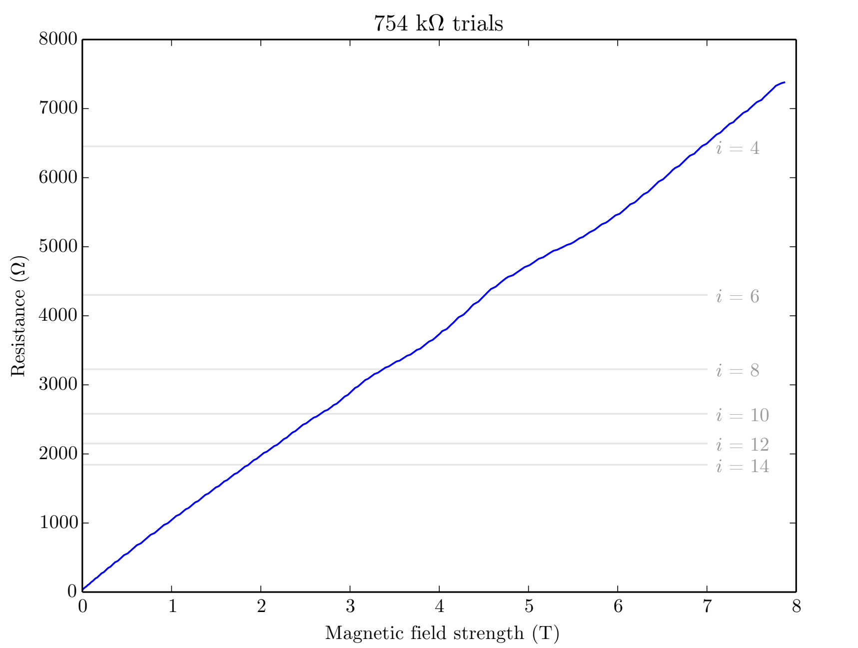

FIGURE X: Measurements with the 754 kΩ resistor in series with the sample.

3.5 Fine-structure constant

The von Klitzing constant is related to the fine-structure constant as follows:

$$R_K = \frac{h}{e^2} = \frac{\mu_0 c}{2\alpha} \longrightarrow \alpha = \frac{\mu_0 c}{2R_K} \tag{7}$$

Thanks to the 1983 redefinition of the meter in terms of the speed of light, the only uncertainty added to this equation is that in the value of R_K, if measured in Ohms [4].

Using our value of $R_K = 28099 \pm 31.112\ \Omega$ from the 35, we obtain

$$\alpha = \frac{\mu_0 c}{2R_K} = \frac{4\pi \cdot 10^{-7} \cdot 2.9979 \cdot 10^8}{2 \cdot 28099} = 0.0067036 \tag{8}$$

$$\frac{\Delta\alpha}{0.0067036} = \frac{31.112}{28099} \longrightarrow \alpha = 0.0067036 \pm 0.000007422 \tag{9}$$

This corresponds to a random error of 0.11%. The calculated value of the fine structure constant differs from the accepted value of 0.0072974 by 8.1% [4].

3.6 Why are only even Landau levels observed?

The Landau levels undergo Zeeman splitting, which creates two new levels with an energy gap 70 times smaller than that between adjacent Landau levels without the effects of spin. The lower new level corresponds to an odd number of levels filled. In this case, the energy gap to the next level (an even number) is very small and easy to attain. The plateaus on the graph of Hall resistance vs. magnetic field strength appear only for even integer fractions of R_K because the gap to the next level takes a much greater difference in energy to overcome [6].

4 Conclusion

The classical Hall effect and integral quantum Hall effect were both investigated. Data for the classical Hall effect supported qualitative reasoning on the signs of the charge carriers for two unknown samples A and B. In sample A, the primary charge carriers were electrons, while in sample B, these were holes.

The investigation of the quantum Hall effect, with measurements of the Hall resistance values for even integer fractions of 6, 8, and 10 of the von Klitzing constant R_K, made possible the calculation of $R_K = 28099 \pm 31.112\ \Omega$ along with the fine-structure constant $\alpha = 0.0067036 \pm 0.000007422$.

References

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