Computer Modelling and New Technologies, 2013, Vol.17, No. 4, 136-141
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EIGENFACES AND IDENTIFICATION OF COLOR FACE IMAGES

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To preserve the relation of three-primary colours of colour images, this paper represents a colour image by a quaternion matrix. Colour eigenfaces are defined by eigenvectors of the covariance matrix of a set of faces. Several main colour eigenfaces form the eigenface subspace, which is used to calculate the distance between new image and known colour faces. That helps us to judge whether an input colour image is a face and further more whether it is a known one.

Keywords: colour face, quaternion matrix, colour eigenface, face subspace

1. Introduction

Three basic colours for colon images are red, green, and blue (RGB for short). Quaternion has been introduced in accordance with the relation of RGB to represent a set of colon face images and compute the eigenvalues and eigenvectors, which are used to span the face subspace. The identification of colour face images can be operated in the assistance of such face subspace.

Irish mathematician, physicist, and astronomer -William Rowan Hamilton in 1843 invented quaternion [1], which is his greatest achievement. When he was considering expanding complex to higher dimension (a complex can be seen as one point of the plane), he found four-dimensional space could create quaternion. In recent years, quaternion has been applied to many advanced fields [2, 3, 4]. The position and rotation of three-dimensional objects can be expressed by it when we draw pictures by computer and do some related graphic analysis. At the same time, we can find its appearance in cybernetic, signal processing, attitude control, physics and orbital mechanics and it is also used to express position and rotation. Explicitly, quaternion is the non-commutative expansion of complex. It represents a four-dimensional space if the set of quaternions is considered as multi-dimensional real space relative to a two-dimensional space of complex. That is why quaternion plays an important role in the facet of colour image processing [5, 6].

A quaternion q including four components-one real part and three imaginary parts is represented as:

$$q = q_r + q_i i + q_j j + q_k k$$

where q_r, q_i, q_j and q_k are four real numbers, i, j, k are three imaginary units and they are related to each other as follows:

$$i^2 = j^2 = k^2 = ijk = -1, \tag{1}$$

$$ij = -ji = k, jk = -kj = i, ki = -ik = j. \tag{2}$$

A quaternion is called pure quaternion when the real part is set as zero. One pure quaternion can represent one pixel of the colour image [5] by the expression: $Ri + Gj + Bk$, in which *R*, *G*, and *B* each stands for the value of RGB.

For a quaternion matrix $X_{(q)} = X_r + X_i i + X_j j + X_k k \times Q^{m\times n}, X_n \times R^{m\times n}(n = r, i, j, k)$, then we define its real representation [7, 8]:

$$X_{e(r)} = \begin{matrix} X_r & X_i & X_j & X_k \\ -X_i & X_r & -X_k & X_j \\ -X_j & X_k & X_r & -X_i \\ -X_k & -X_j & X_i & X_r \end{matrix} \times R^{4m\times 4n} .$$

We have $X_{(q)}, Y_{(q)} \in Q^{m\times n}$ and make the following conclusion by upper definition:

$$\left(X_{(q)} + Y_{(q)}\right)_{e(r)} = X_{e(r)} + Y_{e(r)},\ \left(X_{(q)}Y_{(q)}\right)_{e(r)} = X_{e(r)}Y_{e(r)},\ \left(X_{(q)}^H\right)_{e(r)} = \left(X_{e(r)}\right)^T,$$

where $X_{(q)}^H$ means the conjugate and transpose of $X_{(q)}$.

In this paper, we use quaternion matrices to deal with colour faces, that is, a quaternion matrix can express a colour image. By making use of its equivalent real matrix, it is possible to calculate the colour eigenfaces. Then, we choose several eigenvectors to span the colour face subspace. In practice, we input one new colour image and compute its distance from the subspace and the face classes respectively to figure out whether it belongs to the known individual or not.

2. Eigenfaces and Face Identification

A computational model of face recognition [9, 10, 11] will have huge influence both in theory and in practice. In this paper, we develop a rapid, reasonable and simple one suitable for a constrained environment.

Colour eigenfaces can be considered as a group of standardized human faces. Each colour face image can be represented by the linear combination of eigenfaces. The recognition of new colour face image is conducted in the subspace spanned by colour eigenfaces. We represent a $N \times N$ colour face image by an $N^2 \times 1$ order pure quaternion column vector. Let the training set of colour face images be the column vectors $F1, F2, F3, \cdots, F_M$ (as it shows in Figure 1, $M = 16, N = 128$). Then,

$$\Psi = \frac{1}{M}\sum_{n=1}^{M} F_n$$

is defined as the average face vector. Each face differs from Ψ by $\Phi_i = F_i - \Psi$. The orthogonal vectors u_k are chosen such that

$$\lambda_k = \frac{1}{M}\sum_{n=1}^{M}\left(u_k^T \Phi_n\right)^2$$

is the k -th maximal eigenvalue. The vectors u_k and scalars λ_k are respectively eigenvectors and eigenvalues of the covariance matrix

$$C = \frac{1}{M}\sum_{n=1}^{M}\Phi_n\Phi_n^T = AA^T ,$$

where $A = [\Phi_1\Phi_2 \ldots \Phi_M]$, $A \in Q^{N^2\times M}$. Due to $C \in Q^{N^2\times N^2}$, there are N^2 pairs of eigenvalues and eigenvectors. If N is huge, working out the eigenfaces of such a big matrix is tough. Generally, M is far smaller than N^2. So, can we figure out the eigenvectors and eigenvalues of a small $M \times M$ order matrix to directly obtain those of C ? We find that:

$$A^T A v_i = \mu_i v_i, \quad (3)$$

$$AA^T A v_i = \mu_i A v_i. \quad (4)$$

From Eq. (3) and Eq. (4), we know that if we want the eigenvectors of $C = AA^T$, we can firstly compute the eigenvector v_i of $A^T A$. Because $A^T A$ is an $M \times M$ order quaternion matrix and M is much smaller than N^2, we can work out v_i easier.

In fact, we just need to use M' eigenfaces corresponding to M' biggest eigenvalues to span the subspace Z. Set $M'=8$ then we can get the subspace. Figure 4 shows the eight main eigenfaces. We assume the new face image Γ (in Figure 3, including the face image and animal's face image), represented by column vector F.

Let the colour image projected onto the subspace based on the following formula:

$$\omega_k = u_k^T (F - \Psi), k = 1,\ldots,M'.$$

Vector $\Omega^T = [\omega_1, \omega_2, \ldots, \omega_{M'}]$ denotes the weights of new input face image that describe the distribution of eigenfaces spanned the subspace. Ω_k is the average weights obtained from a face class, which can be used to check out which face class the new face image belongs to or which is the best suitable face class. We have three images of each individual here, for example, in Figure 2. To find the right face class, we define

$$\varepsilon_k = \|(\Omega - \Omega_k)\| / \|\Omega\|.$$

The simple way to find out the exact face class is to compare the minimal ε_k. If ε_k is less than a certain value, we can tell that the new face belongs to the k-th face class, otherwise it's the face of an unknown individual. To conclusion, ε_k means the distance between new face image and the face classes.

Set $wi = \Phi_i^{'} Z$, $weight = (F - \Psi)' Z$ and $WM = \sum_{i=1}^{M} norm(wi)$ so that $\delta = abs((WM - norm(weight)) / norm(weight))$. If δ is less than a certain value, we can judge that the new face image belongs to a human being. To conclusion, δ means the distance between new face image and the face subspace.

Figure 1. Some colour faces in the training set from [12]

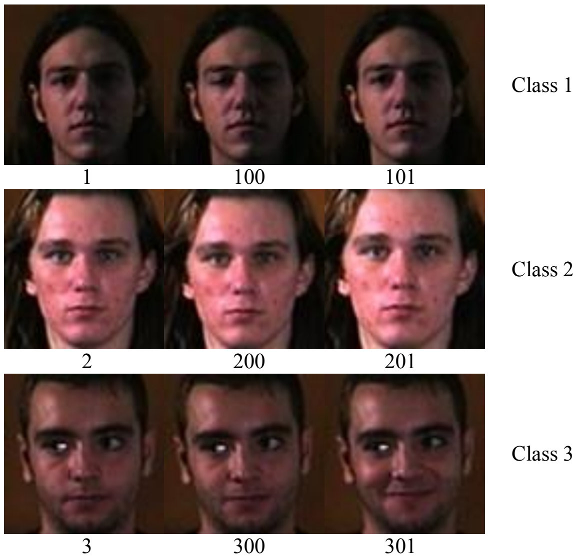

Figure 2. Three face classes

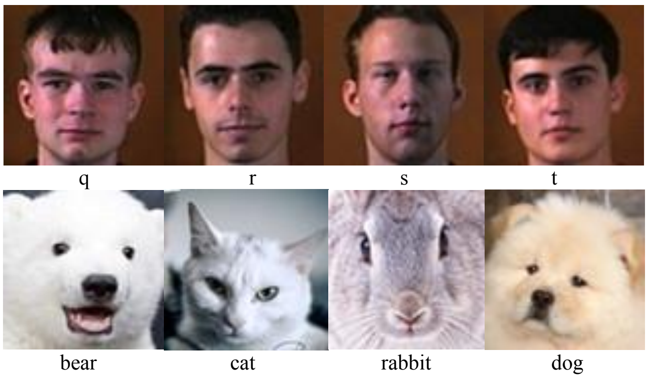

Figure 3. The new face images of both human and animals

3. Result Analysis

3.1. Computation of Eigenfaces

Figure 4. Eight main eigenfaces calculated from the input image

From the above figure, we find that the last eigenface with the biggest eigenvalue containing the most information of the training set.

3.2. Judgment

TABLE 1. Data of δ and ε_k of the faces belonging to the input individual

figure	**1**	**2**	**3**	**4**
δ	0.2154	0.1140	0.6098	0.8640
ε_k	.0356	0.1135	0.0568	0.2413
figure	**5**	**6**	**7**	**8**
δ	0.0262	0.4051	0.2124	0.5567
ε_k	0.1329	0.0223	0.0408	0.0276
figure	**9**	**10**	**11**	**12**
δ	0.0416	0.5881	0.3742	0.5730
ε_k	0.0940	0.1496	0.0299	0.1020
figure	**13**	**14**	**15**	**16**
δ	0.3168	0.6512	0.9345	0.2762
ε_k	0.1242	0.1294	0.0633	0.0865
figure	**17**	**18**	**19**	**20**
δ	0.1860	0.0707	0.4349	0.3255
ε_k	0.0598	0.1348	0.1491	0.2903
figure	**21**	**22**	**23**	**24**
δ	0.3042	0.6519	0.1522	0.5441
ε_k	0.3238	0.2328	0.2422	0.0308
figure	**25**	**26**	**27**	**28**
δ	0.1093	0.3649	0.3356	0.6175
ε_k	0.2142	0.0634	0.1604	0.1130
figure	**29**	**30**	**31**	**32**
δ	0.4300	0.0750	0.6270	0.2557
ε_k	0.0460	0.1116	0.0917	0.0553

TABLE 2. Data of δ and ε_k of the new face of both human and animals

figure	**q**	**r**	**s**	**t**
δ	0.6160	0.4588	0.3181	0.6801
ε_k	0.5205	0.3728	0.5125	0.6576
figure	**bear**	**cat**	**rabbit**	**dog**
δ	0.8180	0.7383	0.7883	0.7926
ε_k	0.6394	0.5915	0.5958	0.5765

4. Conclusion

Situation 1: Generally, if $\delta < 0.7$ and $\varepsilon_k < 0.3$, that is to say, the new face is near to one of the face classes and the subspace, we can judge that the new face belongs to certain individual and we can recognize him or her.

Situation 2: Generally, if $\delta < 0.7$ and $\varepsilon_k < 0.3$, that is to say, the new face is near to the subspace but far away from any face class, we can judge that the new face doesn't belong to any individual and it's a new human face.

Situation 3: Generally, if $\delta < 0.7$ and $\varepsilon_k < 0.3$, that is to say, the new face is far away from the subspace and face classes, we can judge that the new face doesn't belong to any individual and it's also not a human face.

Because of light and shooting angle, the distance δ between a human face and the subspace may be bigger than 0.7 and the distance between an animal face and the subspace may be less than 0.7. These cases are rare. People can also make a conclusion depending on the second parameter ε_k. For example, the fourth known face have a big δ, but its ε_k is smaller than 0.3. We can say that it is a known face.

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Received on the 21st of June 2013