QEA Project 1: Boat Design
Technical Mini-Report
SPARISTOPHER

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1 Relevant Definitions

The choice of origin affects the position vectors of the center of mass and center of buoyancy. The origin (in the reference frame of the boat) is shown in figure 1:

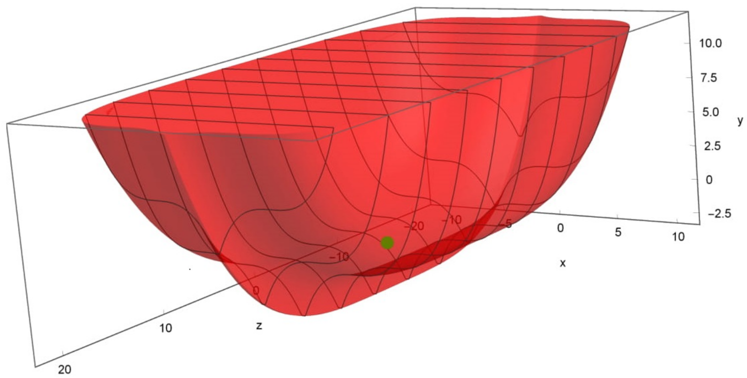

Figure 1: This figure shows the frame of reference and the choice of origin for the boat analysis. The green dot at (0, 0, 0) labels the origin on the 3D plot of the boat.

All the units in this document are in **g, cm, s**.

The term **mass (M)**, in physics, is defined as the quantity of matter contained in a body, regardless of its volume or of any forces acting on it. M, as used in this document, accounts for the mass of the hardboard that was used in constructing the boat, as well as the mass of the mast.

A **ballast** is usually a piece of heavy/dense material placed low in the boat to lower the center of mass and hence, to improve stability.

$$Mass_{Ballast}$$

is the **mass of the ballast**, and has been used independently wherever required. This is essentially the mass of the lead shots that we placed in the boat as a ballast.

The **acceleration due to gravity (g)** is the acceleration of any object moving under the sole influence of gravity. It was assumed to be:

$$g = 980cm/s^2$$

In the domain of fluid mechanics, **displacement** occurs when an object is immersed in a fluid. The volume of the fluid displaced can then be measured. An object that sinks displaces an amount of fluid equal to the object's volume.

The **Volume of water displaced** by the boat is used as 'V' throughout the document, and the density of the boat is used as 'rho' throughout the document.

$$V = M/\rho$$

The **center of mass (COM)** of a distribution of mass in space is the unique point where the weighted relative position of the distributed mass sums to zero, or the point where if a force is applied it moves in the direction of the force without rotating. The distribution of mass is balanced around the center of mass and the average of the weighted position coordinates of the distributed mass defines its coordinates. The COM of each part can be calculated using the following equation, where μ is the density of the solid that we are using:

$$\iiint_V \mu.(x, y, z)\, dx\, dy\, dz$$

divided by the integral of μ over the volume of the boat

$$\iiint_V \mu\, dx\, dy\, dz$$

The **center of buoyancy (COB)** of an object is the centroid of the volume of fluid displaced by the boat as it floats in the water. It can be determined in 3 dimensions, with mu is the density of water (D) as follows:

$$\iiint_V \mu.(x, y, z)\, dx\, dy\, dz$$

divided by the integral of mu over the volume of the boat

$$\iiint_V \mu\, dx\, dy\, dz$$

Draft is the depth of water that the boat needs in order to float flat on the surface of the water. The draft is measured as the distance from the waterline to the lowest point of the boat - the tip of the hull. The **draft of the boat** is used as 'd' throughout the document.

An **eye bolt** is a bolt with a circular opening at one end.

The **waterline** is the line where the hull of a ship meets the surface of the water. The general **equation of the waterline** where tan(theta) is the slope of the waterline with respect to the boat, and d is the draft of the boat is as follows:

$$y = tan(theta).x + d$$

The waterline has been manipulated to form **different cases of heel angles** for the analysis of the boat, as shown in Figure 2.

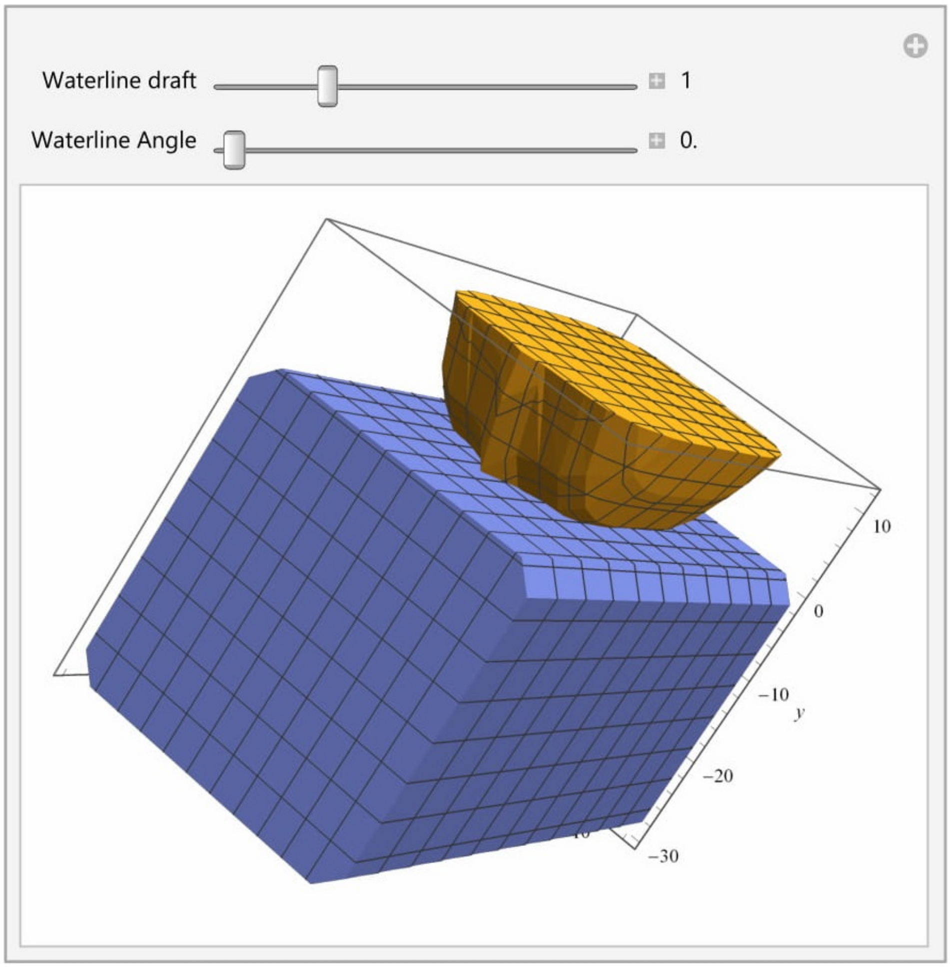

Figure 2: This figure shows the manipulate functionality for the draft of the boat, and the angle of waterline. The region under water is not visible. The water is in the blue color while the boat is in the orange color.

2 General Methodology

In order to determine the right design for the boat, and placement of the ballast on the boat, we went through a series of steps:

1. **Background Research** was necessary for the author to get acquainted with the terms specific to boats and naval architecture. The most important outcome from this process was learning how to visualize the waterline and draft.

2. **Back-of-the-Envelope Calculations and sketches** were carried out on paper to get a basic sense of what the author imagined would be a good fit to the design constraints. The basic idea about the surface integrals was laid out and the 3D sketches were formed.

3. **A Detailed Analysis** of the designs was carried out in Mathematica, as well as SolidWorks. This included the mathematical calculations of the physical quantities and the simulation of boat design constraints. This has been portrayed in the picture below. The waterline is being studied with respect to the boat - varying the tilt of the boat with respect to the waterline made the visualization easier.

3 Design Constraints

Boat Fabrication Requirements were as follows:

1. The boat must be constructed on a single 24" x 18" piece of hardboard.

2. The boat must be created from an equation-driven CAD model.

3. The boat must accept a cargo with a minimum mass of 700 g and a maximum mass of 1000 g. You should have your cargo preapproved before securing it within your boat.

4. The boat must be a keel-less mono-hull displacement design (no outriggers or catamarans).

5. The boat must include the mast. The mast will be a 0.5 meter long, 3/8" diameter aluminum dowel. The entire length of the mast must extend vertically above the bottom of your boat's hull.

6. The eyelet should be attached to the hull at the desired drag point to enable speed testing.

4 Performance Requirements

Performance Requirements for the boat were as follows:

1. The boat must float when fully loaded.

2. The deck of the boat should be parallel to the surface of the water (off by ¡5 degrees) when the boat is fully loaded.

3. The angle of vanishing stability (AVS) for the fully loaded boat should be between 120 degrees and 140 degrees.

4. The boat should should have a maximum computed righting moment of at least 0.2 N-m.

5. The hull should be as fast as possible. This will be determined using a drag test: a constant force of 0.1 Newtons will be applied via a horizontal string at the screw eyelet.

5 The Boat Design

After some design considerations based on the shapes and functionality of existing ships (At one point, the US Aircraft Carrier and US Navy John Paul Jones were also considered as potential inspirations), the design of the hull was chosen to be a **cubic curve that is reflected about the y-axis** to converge at the bottom tip of the hull. The two cubic curves were bounded by a straight line in the x-z plane. The boat was further defined as a **bi-quadratic curve in the z-direction** to make it more streamlined in water.

5.1 Boat Equations

Equations that define the shape of our boat:
The part of the cross-section of the hull in the (-x)(+y) plane is defined by the following y bounds:

$$-(x/3+1.5)^3+(z/13)^4 \leq y \leq 20$$

where x is the horizontal distance from the center plane, for negative values of x, and y is the vertical distance from the bottom to the hull.

The part of the cross-section of the hull in the (+x)(+y) plane is defined by the following y bounds:

$$(x/3-1.5)^3+(z/13)^4 \leq y \leq 20$$

where x is the horizontal distance from the center plane, for positive values of x, and y is the vertical distance from the bottom to the hull.

5.2 Design Considerations

The goal of these design considerations was to achieve the corresponding advantages:

(i) The bottom row tips of the hull give the boat **a streamlined shape for increased speed output**.

(ii) The shape concentrates more mass towards the bottom, that **pulls the center of mass downwards**. It gave us more room to play with the positioning of the ballast on the boat.

5.3 Ballast and its properties:

The mass of the ballast is 1000g.

We placed it at (0, 1.72755, 0) to get the Angle of Vanishing Stability Curve that is presented below in this document. Figure 3 below shows the placement of the ballast.

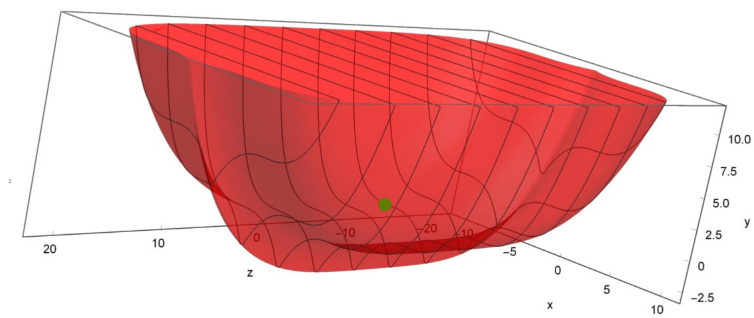

Figure 3: This figure shows the placement of the ballast on the boat. The green dot at (0, 1.72755, 0) labels the placement on the 3D plot of the boat.

5.4 Boat Equation Parameters

We varied the following parameters as we refined the boat design:

(i) **The height of the boat** - the limit of y for the boat functions. The adjustment was made according to the total volume of the boat versus how much mass it contained to displace enough water for it to float flat on the surface. In Physics terms, we analyzed the effective density of the boat. We considered several cases - the extremes being:

a. The mass was very high, but the volume was very low: we concluded that this type of boat is not ideal for us because of the high density of the boat.

For a boat of mass (M), and volume of water displaced (V), density (rho) can be given as:

$$\rho = M/V$$

Greater Mass for lesser volume turns out a high density.

b. The mass was very low, but the volume was very high: this type of boat would never float flat on the surface of the water due to the fact that most of the boat would be outside of the water - the Righting Moment would always be a negative torque.

For a boat of mass (M), and volume of water displaced (V), density (rho) can be given as:

$$\rho = M/V$$

Lesser Mass for greater volume turns out a low density.

(ii) **The length of the boat** - the limit of z for the boat functions. The adjustment was made in relation to the height of the boat. We wanted to make the boat as streamlined as we could for a good speed output.

The mass of the boat was determined **piece-wise in SolidWorks**. Broadly speaking, the pieces included the slices of the boat hull, the mast, and the deck. We require the mass of the boat to determine the mass of water displaced by the boat, and the force of buoyancy. In our case, the total mass of the boat,

$$M = 315g$$

The center of mass (COM) was determined after incorporating: the boat, the mast, as well as, the ballast, in a SolidWorks model as shown in figure 4.The COM is presented in each of the 3 dimensions (COM-X, COM-Y, COM-Z). The center of mass is required to find the torque vector - the righting moment arm. In our case, the COM in 3 dimensions was as follows:

$$COM_X = 0$$

$$COM_Y = 3.25$$

$$COM_Z = 0$$

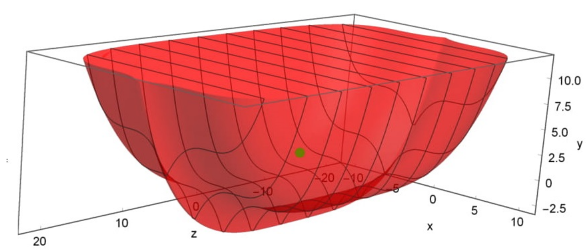

Figure 4: This figure shows the position of the COM on the boat. The green dot at (0, 3.25, 0) labels the COM on the 3D plot of the boat.

The SolidWorks boat assembly is shown in figure 5:

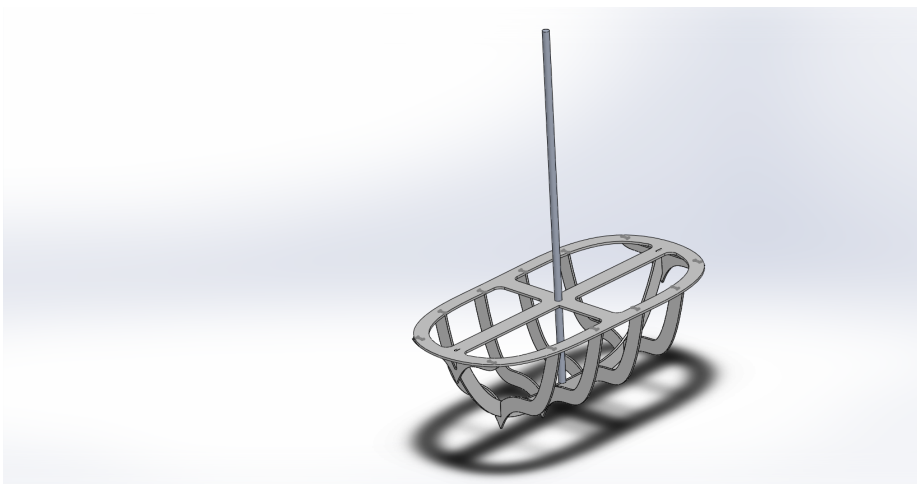

Figure 5: This figure shows the SolidWorks assembly of the boat. This includes the slices of the hull, the mast, as well as, the deck of the boat.

6 The Case of 0 Degree Heel Angle - The Boat Floats on The Water

At a **heel angle of 0 degrees**, the forces acting on the boat in static equilibrium:

A. The **force of gravity exerted on the boat is equal and opposite to the force of buoyancy** exerted due to the water displaced by the boat. Since the force vectors lie along the y-axis,

$$Moment = [COB - COM] \times F_{Buoyancy}$$

B. The **righting moment is zero** as [COB - COM] vector lies on the same axis as the force vectors. Hence, **the boat floats on the water with no net torque** on the heel. This is shown in figure 6

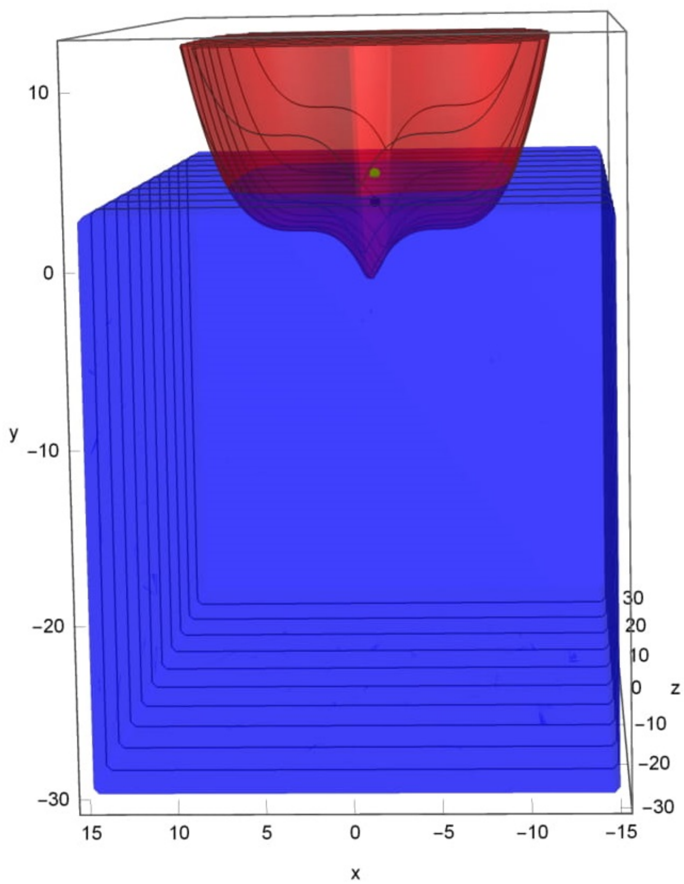

Figure 6: This figure shows the 0 degree heel angle case for the boat. The green label is the COM, the black label is the COB, the boat is red, and the water is blue.

7 Generating the Stability Curve

7.1 Summary of the derivation

The general description of the process is as follows:

(i) To derive the stability curve, we derived the values for **righting moment versus the heel angle** for our boat.

(ii) We calculated the righting moment using the **force of buoyancy** acting on the boat.

(iii) The **righting moment** is a cross product of the [COB-COM] and the Force of Buoyancy.

(iv) We calculated the **COM of the boat** in SolidWorks.

(v) For the calculation of the **COB**, we used mass of water displaced by the boat.

7.2 Analytical Solution

The gravitational force F(g) on the boat was determined using Newtonian Mechanics:

$$F_g = M \times g$$

$$F_g = 1288700 g.cm/s^2$$

Assuming static equilibrium,

The buoyant force F(B) on the boat was calculated as follows:

$$F_B = -F_g$$

$$F_B = -1288700 g.cm/s^2$$

The volume of water displaced by the boat in the static equilibrium condition can be used to calculate the center of buoyancy - it is the center of mass of the water displaced by the boat. It can be determined using the following equation:

$$V.D.g = F(g)$$

$$V = M/D$$

$$V = 1315 cm^3$$

The center of buoyancy (COB) can be determined using the volume of water displaced as found using the equation defined in section of the document. It is necessary for the calculation of the righting moment. That will enable us to plot the AVS curve and analyze the behaviour of the boat for various draft angles.

An expression for the **buoyancy vector** (in the coordinate system attached to the boat) as a function of heel angle (theta) can be given as follows:

$$M \times g \times [-Sin(\theta), Cos(\theta), 0]$$

As an example, for the heel angle of 30 degrees with the waterline, the center of buoyancy that we derived was as follows:

$$COB_X = 4.32359$$

$$COB_Y = 2.3439$$

$$COB_Z = 0$$

The righting moment (tau) was determined using vector.COB and vector.COM, about the Center of Mass of the boat as follows:

$$\tau = (COB - COM) \times F_B$$

The **stability curve** that we derived following the steps described in this section is shown in Figure 7.

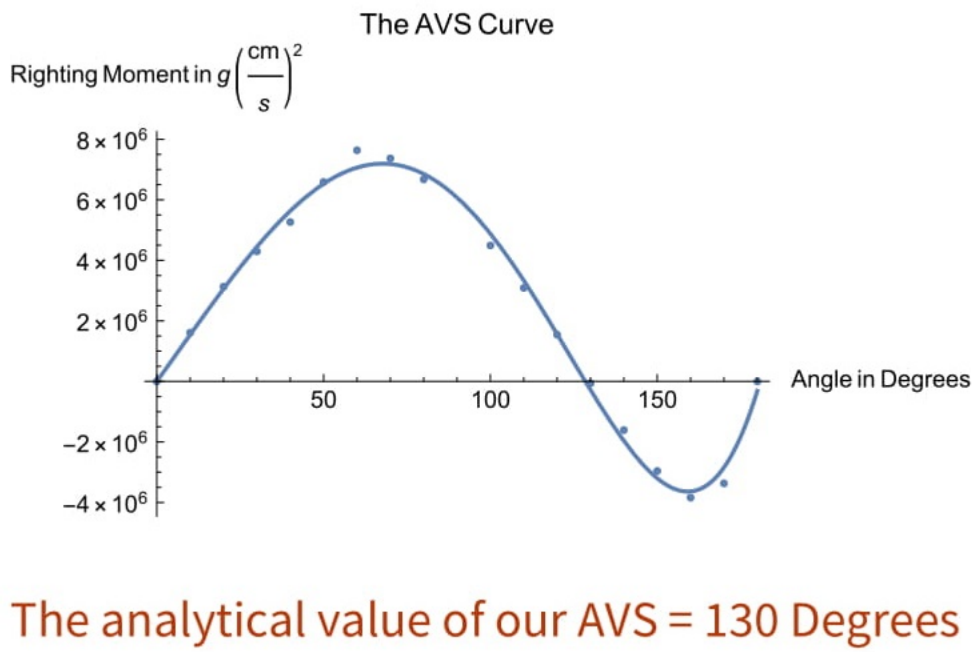

Figure 7: This figure shows the stability curve that we have derived. The analytical value of the AVS is the point where the graph crosses the x-axis, which is at 130 degrees in this case.

8 Diagrams for Various Heel Angles - Analysis

While theta is less than 130 degrees, the COB shifts towards (+y)(+x) direction. As an example, I have included Cases I, II, and III:

8.1 Case I - 30 Degrees - Figure 8

$$d = 2.01655$$

$$COB = [4.27488, 2.51152, 0]$$

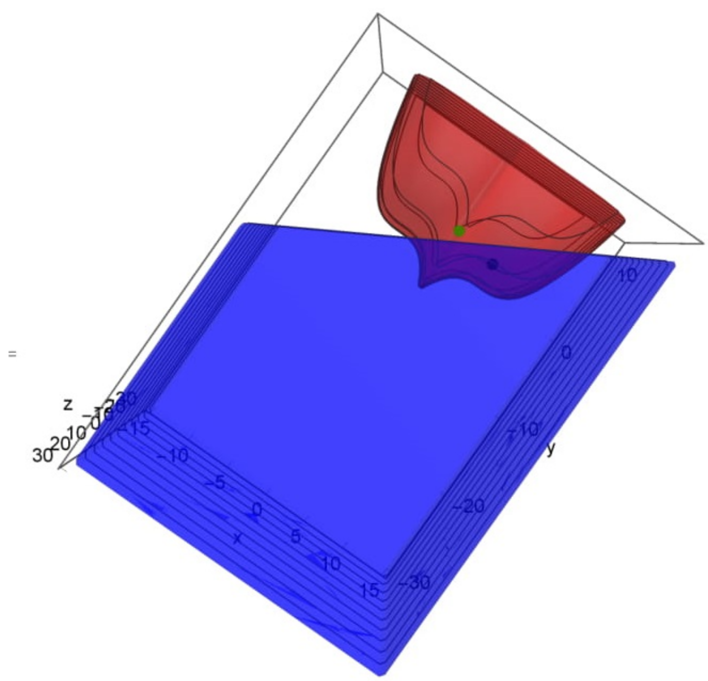

Figure 8: This figure shows the 30 degrees heel angle case for the boat. The green label is the COM, the black label is the COB, the boat is red, and the water is blue.

8.2 Case II - 60 Degrees - Figure 9

$$d = -3.91481$$

$$COB = [7.46538, 5.78314, 0]$$

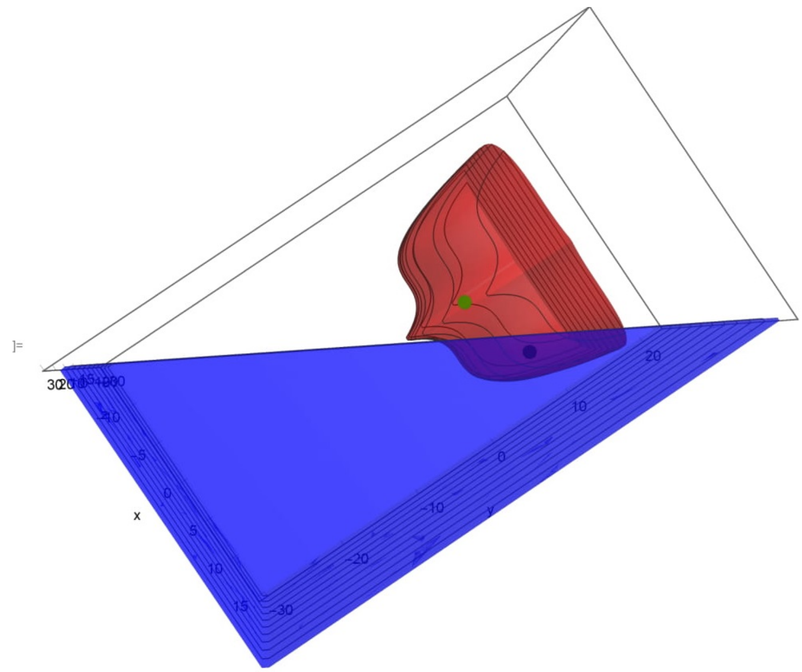

Figure 9: This figure shows the 60 degree heel angle case for the boat. The green label is the COM, the black label is the COB, the boat is red, and the water is blue.

8.3 Case III - 120 Degrees - Figure 10

$$d = 17.7406$$

$$COB = [7.66226, 9.05742, 0]$$

While theta is equal to the calculated AVS (130 degrees), the COB passes from under the COM (the direction of torque exerted by the hydro-static pressure about the COM switches). Past this angle, the righting moment tends to flip the boat. As an example, I have included Case IV:

8.4 Case IV - 130 Degrees - Figure 11

$$d = 14.7071$$

$$COB = [7.41353, 9.41234, 0]$$

While theta is greater than 130 degrees, the COB shifts towards (+y)(-x) direction. As an example, I have included Cases V, and VI:

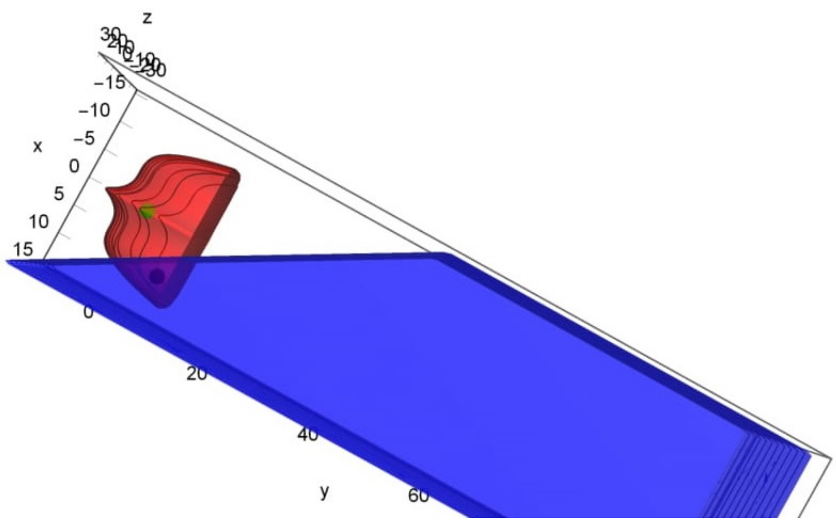

Figure 10: This figure shows the 120 degree heel angle case for the boat. The green label is the COM, the black label is the COB, the boat is red, and the water is blue.

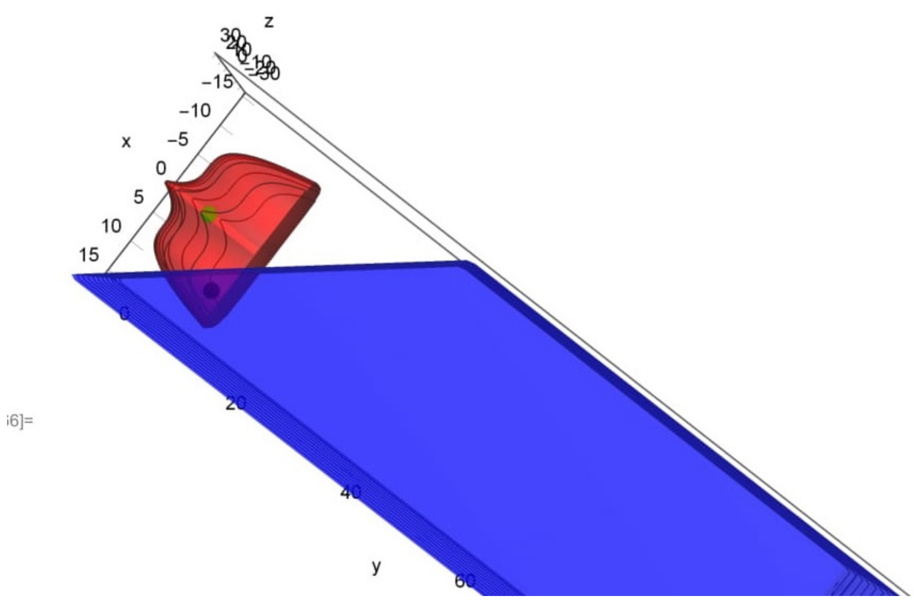

Figure 11: This figure shows the 130 degree heel angle case for the boat. The green label is the COM, the black label is the COB, the boat is red, and the water is blue.

8.5 Case V - 150 Degrees - Figure 12

$$d = 11.6427$$

$$COB = [6.59459, 10.0814, 0]$$

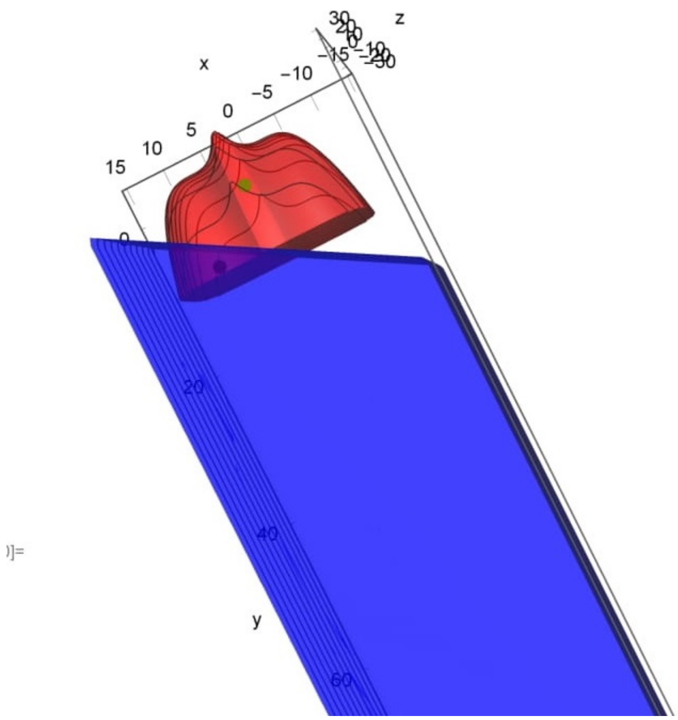

Figure 12: This figure shows the 150 degree heel angle case for the boat. The green label is the COM, the black label is the COB, the boat is red, and the water is blue.

8.6 Case VI - 180 Degrees - Figure 13

$$d = 10.4711$$

$$COB = [0, 11.243, 0]$$

Analysis

Since the **righting moment is positive** for the cases 0 - 130 degrees, which means that the boat will be in a stable equilibrium for this range of angles. In other words, if released from any heel angle in this range, it will come to rest in the floating position.

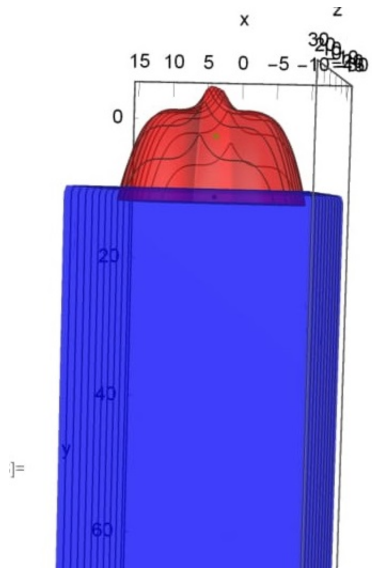

Figure 13: This figure shows the 180 degree heel angle case for the boat. The green label is the COM, the black label is the COB, the boat is red, and the water is blue.

Since the **righting moment is negative** for the cases 130 - 180 degrees, the boat will be in an unstable equilibrium for this range of angles. In other words, if released from any heel angle in this range, it will come to rest in a flipped (upside down) floating position.

9 Conclusion

The boat will float because there is no righting moment acting on it at the heel angle of zero degree as per the stability curve that we have derived. Since there is no net torque on the boat, it will float without any rotation. The forces of gravity and buoyancy are balanced - a case of static equilibrium. Our model predicts an **angle of vanishing stability of 130 degrees**. It is essentially the point where the stability curve meets the x-axis for the second time after the case of origin. We have made the boat as streamlined as we could, and we predict that it would be moderately fast (average).