Mathematical Determination of Righting Moment vs. Heel Angle SPRIS

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1 The Boat Design

After some design considerations based on the existing navy ships (At one point, the US Aircraft Carrier and US Navy John Paul Jones were also considered as potential inspirations), the design of the hull was chosen to be a cubic curve that is reflected about the y-axis to converge at the end of the hull. The two cubic curves were bounded by a straight line in the x-z plane. The boat was then given a bi-quadratic shape in the z-direction to make it more streamlined.

Part one of the cross-section of the hull is defined by the following y bounds:

$$-(x/3 + 1.5)^3 + (z/13)^4 <= y <= 20$$

where x is the horizontal distance from the center plane, for negative values of x, and y is the vertical distance from the bottom to the hull.

Part two of the cross-section of the hull is defined by the following y bounds:

$$(x/3 - 1.5)^3 + (z/13)^4 <= y <= 20$$

where x is the horizontal distance from the center plane, for positive values of x, and y is the vertical distance from the bottom to the hull.

The mass and the density of the boat is determined using SolidWorks. The density of water is assumed to be uniform, and equal to 1g per cubic cm.

2 Relevant Definitions

The origin (in the reference frame of the boat) is considered to be at the bottom, center of the boat.

The term 'mass(M)' as used in this document accounts for the mass of the hardboard that was used in constructing the boat. Mass-Ballast is the mass of the ballast, and has been used independently wherever required.

The acceleration due to gravity (g) was assumed to be:

$$g = 980cm/s^2$$

The Volume of water displaced by the boat is used as 'V' throughout the document.

The Mass of water displaced by the boat is used as 'MW' throughout the document.

The draft of the boat is used as 'd' throughout the document.

The general equation of the waterline where tan(theta) is the slope of the waterline with respect to the boat, and d is the draft of the boat is as follows:

$$y = tan(theta).x + d$$

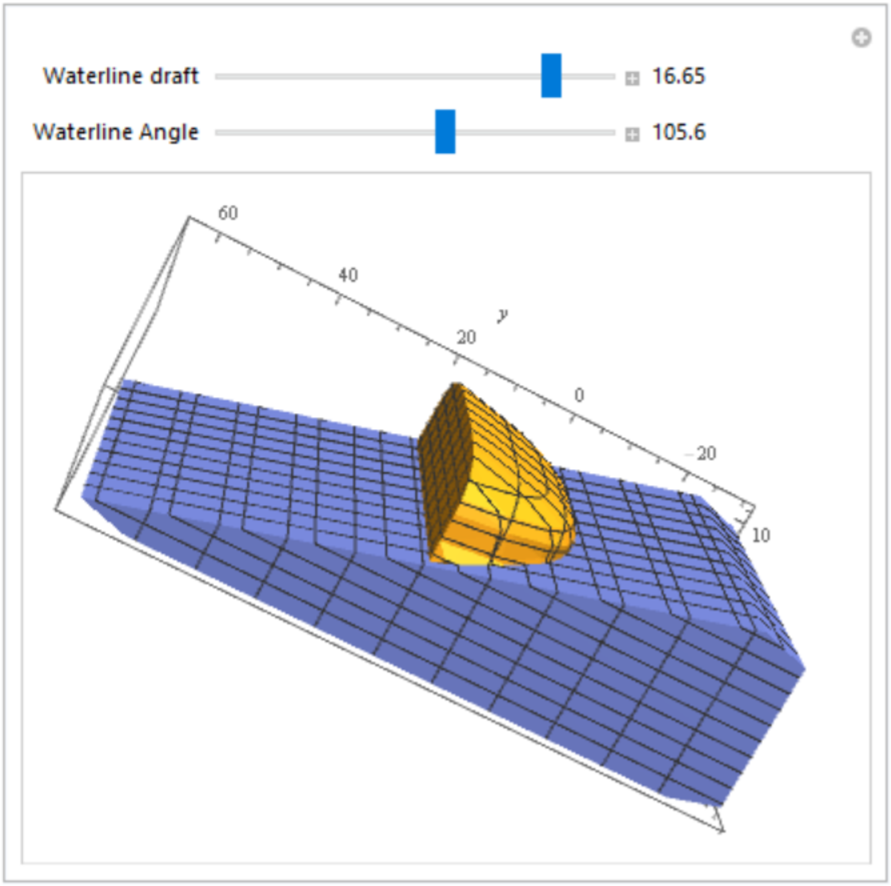

Figure 1: The waterline and the boat

3 Physical Quantities

As stated in the boat design section, the mass has been determined piece-wise in SolidWorks. Broadly speaking, the pieces included the slices of the boat, the mast, the ballast, and the deck. We require the mass of the boat to determine the mass of water displaced by the boat, and the force of buoyancy. Total Mass of the boat and the ballast (M),

$$M = 402.94 grams$$

The Center of Mass (COM) was determined after incorporating both: the boat, the mast, as well as the ballast, in SolidWorks, in each of the 3 dimensions (represented as COM-X, COM-Y, COM-Z). The center of mass is required to find the torque vector - the righting moment arm.

$$COM_X = 0$$

$$COM_Y = 18.217$$

$$COM_Z = 0$$

COM of each part can be calculated using the following equation, where mu is the density of the solid that we are using:

$$\iiint_V \mu.(x,y,z)\,dx\,dy\,dz$$

$$\iiint_V \mu\,dx\,dy\,dz$$

The gravitational force F(g) on the boat was determined using Newtonian Mechanics:

$$F_g = M.g$$

$$F_g = 395284.14g.com/s^2$$

Assuming static equilibrium,

The buoyant force F(B) on the boat was calculated as follows:

$$F_B = -.F_g$$

$$F_B = -395284.14g.com/s^2$$

The volume of water displaced by the boat in the static equilibrium condition can be used to calculate the center of buoyancy - it is the center of mass of the water displaced by the boat. It can be determined using the following equation:

$$V.D.g = F(g)$$

$$V = M/D$$

$$V = 402.94cm^3$$

The Center of Buoyancy (COB) can be determined using the volume of water displaced as found using the previous equation. It is necessary for the calculation of the righting moment. That will enable us to plot the AVS curve and analyze the behaviour of the boat for various draft angles. It was determined in 3 dimensions, with mu is the density of water (D) as follows:

$$\iiint_V \mu.(x,y,z)\,dx\,dy\,dz$$

$$\iiint_V \mu\,dx\,dy\,dz$$

For the tilt of 30 degrees with the waterline, the Center of Buoyancy was as follows:

$$COB_X = 4.32359$$

$$COB_Y = 2.3439$$

$$COB_Z = 0$$

The righting moment (T) was determined using vector.COB and vector.COM, about the Center of Mass of the boat as follows:

$$T = (COB - COM).F_B$$

4 Methodology

In order to determine the right design for the boat, and placement of the ballast on the boat, I went through a series of steps:

1. Background Research was necessary for the author to get acquainted with the terms specific to boats and naval architecture. The most important outcome from this process was learning how to visualize the waterline and draft.

2. Back-of-the-Envelope Calculations and sketches were carried out on paper to get a basic sense of what the author imagined would be a good fit to the design constraints. The basic idea about the surface integrals was laid out and the 3D sketches were formed.

3. A Detailed Analysis of the designs was carried out in Mathematica, as well as SolidWorks. This included the mathematical calculations of the physical quantities and the simulation of boat design constraints. This has been portrayed in the picture below. The waterline is being studied with respect to the boat - varying the tilt of the boat with respect to the waterline made the visualization easier:

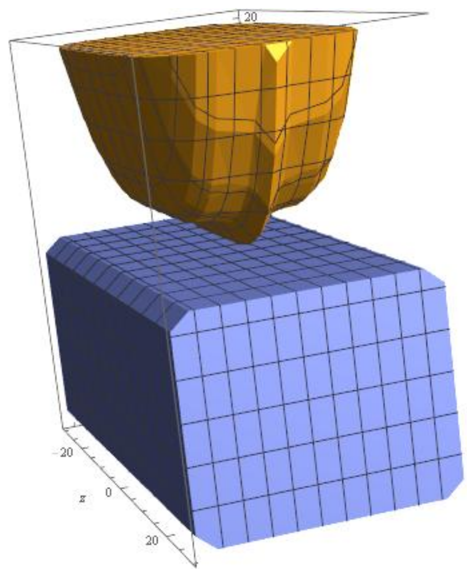

Figure 2: Simulation of the Floating Boat in Mathematica

5 Design Constraints

Boat Fabrication Requirements were as follows:

1. You may only use one 24" x 18" piece of hardboard.

2. The boat must be created from an equation-driven CAD model.

3. The boat must accept a cargo with a minimum mass of 700 g and a maximum mass of 1000 g. You should have your cargo preapproved before securing it within your boat.

4. The boat must be a keel-less mono-hull displacement design (no outriggers or catamarans).

5. The boat must include the mast. The mast will be a 0.5 meter long, 3/8" diameter aluminum dowel. The entire length of the mast must extend vertically above the bottom of your boat's hull.

6. The eyelet should be attached to the hull at the desired drag point to enable speed testing.

6 Performance Requirements

Performance Requirements for the boat were as follows:

1. The boat must float when fully loaded.

2. The deck of the boat should be parallel to the surface of the water (off by ¡5 degrees) when the boat is fully loaded.

3. The angle of vanishing stability (AVS) for the fully loaded boat should be between 120 degrees and 140 degrees.

4. The boat should should have a maximum computed righting moment of at least 0.2 N-m.

5. The hull should be as fast as possible. This will be determined using a drag test: a constant force of 0.1 Newtons will be applied via a horizontal string at the screw eyelet.

7 Result

The following is a resultant Torque (Righting Moment Arm) versus Heel Angle plot in Mathematica. As per the design requirements, the angle of AVS must lie within the range of 120 to 140 degrees.

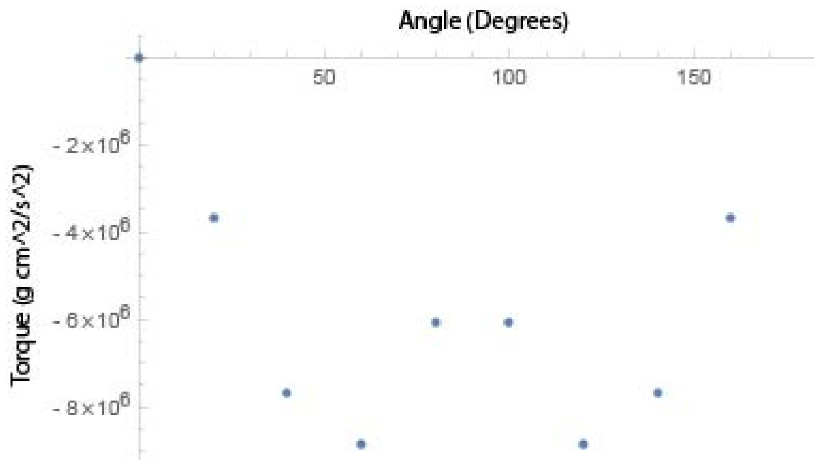

Figure 3: The plot of Righting Moment versus Heel Angle