QEA Project 1: Boat Design

Quantitative Engineering Analysis Project 1

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Boat Design - Sparistopher

In this section, we define (in this order)
(i) The curves which comprise the shape of the hull of our boat,
(ii) The waterline with respect to the boat,
(iii) The region of the boat that is under the water, and
(iv) A function for analysis of the hull design - the variations of the waterline in terms of the draft and the waterline angle.

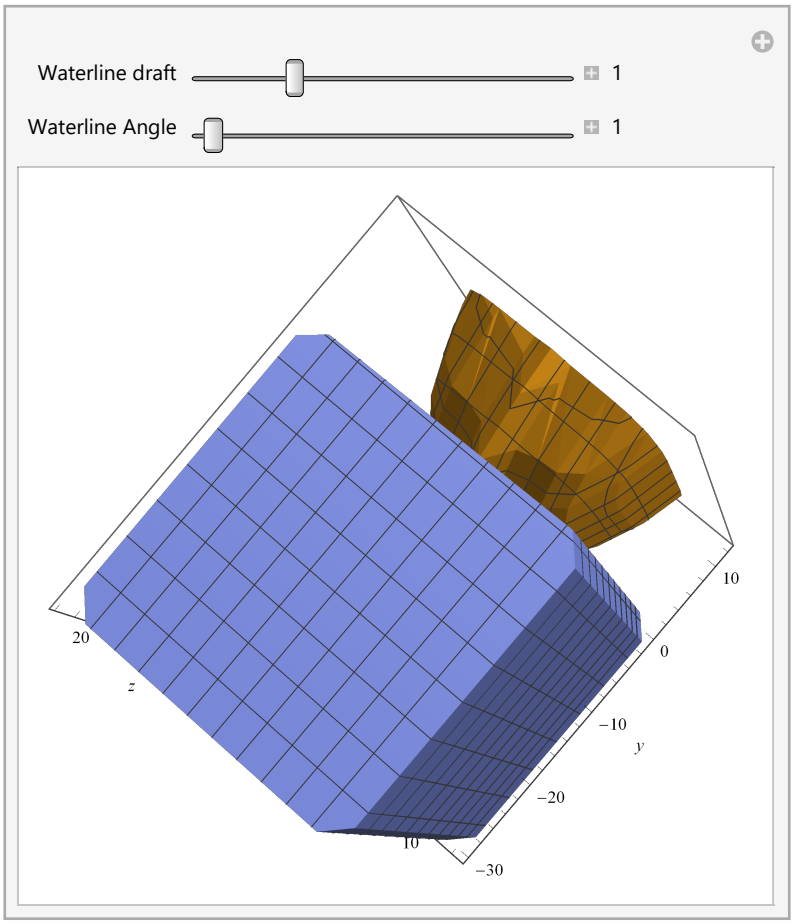
In[1]:=
```
boat1 = ImplicitRegion[
    -14 ≤ x ≤ 14 && -(x/3 + 1.5)^3 + Abs[((z)/11)^4)] ≤ y ≤ 12 && -28 ≤ z ≤ 28, {x, y, z}];
boat2 = ImplicitRegion[-14 ≤ x ≤ 14 && (x/3 - 1.5)^3 + Abs[((z)/11)^4)] ≤ y ≤ 12 &&
     -28 ≤ z ≤ 28, {x, y, z}];
boat = RegionIntersection[boat1, boat2]
water = ImplicitRegion[
  y < Tan[θ Degree] * x + d && -15 < x < 15 && -30 < y < 100 && -30 <= z ≤ 30, {x, y, z}]
under = RegionIntersection[boat, water]
Manipulate[RegionPlot3D[{under /. {d → dd, θ → θθ}, water /. {d → dd, θ → θθ}, boat},
  AspectRatio → Automatic, PlotTheme → "Scientific", AxesLabel → {x, y, z}],
 {{dd, 1, "Waterline draft"}, -5, 20, Appearance → "Labeled"},
 {{θθ, 1, "Waterline Angle"}, 0, 180, Appearance → "Labeled"}]
```

Out[3]=
$$\text{ImplicitRegion}\left[-28 \le z \le 28 \&\& -14 \le x \le 14 \&\& 0.037037\,(-4.5+x)^3 + \frac{z^4}{14\,641} \le y \le 12 \&\& -0.037037\,(4.5+x)^3 + \frac{z^4}{14\,641} \le y \le 12,\ \{x, y, z\}\right]$$

Out[4]=
$$\text{ImplicitRegion}\left[y < d + x\,\text{Tan}[^\circ\,\theta] \&\& -15 < x < 15 \&\& -30 < y < 100 \&\& -30 \le z \le 30,\ \{x, y, z\}\right]$$

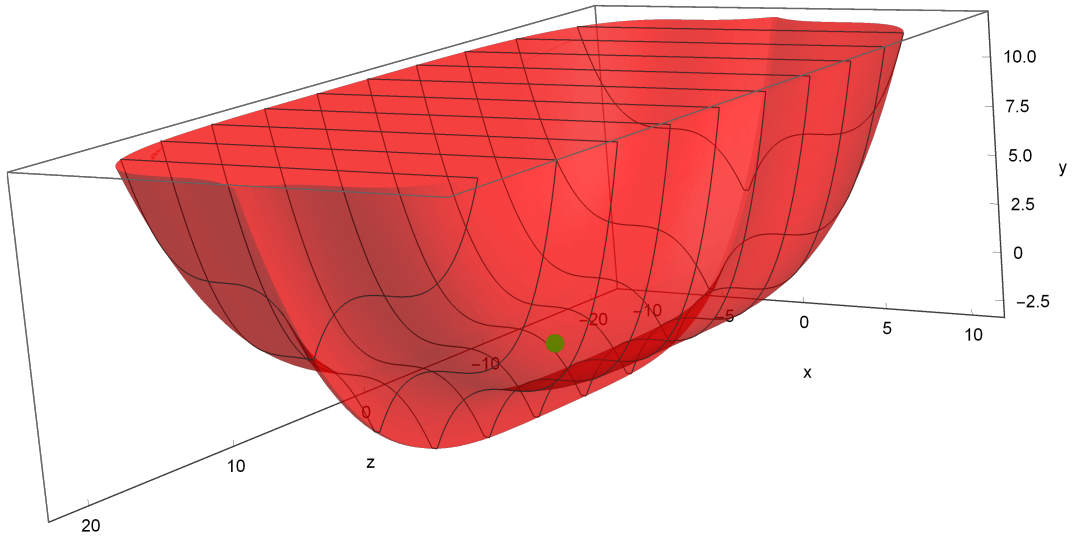
Out[5]=
$$\text{ImplicitRegion}\left[-28 \le z \le 28 \&\& -14 \le x \le 14 \&\& 0.037037\,(-4.5+x)^3 + \frac{z^4}{14\,641} \le y \le 12 \&\& -0.037037\,(4.5+x)^3 + \frac{z^4}{14\,641} \le y \le 12 \&\& y < d + x\,\text{Tan}[^\circ\,\theta] \&\& -15 < x < 15 \&\& -30 < y < 100 \&\& -30 \le z \le 30,\ \{x, y, z\}\right]$$

Out[6]=



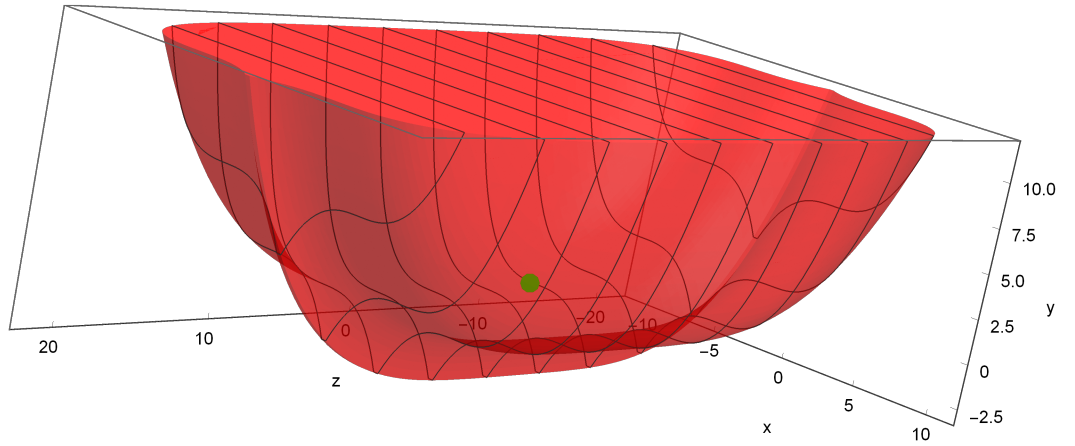
In[7]:=
```
(* Test Plotter *)
Show[
 RegionPlot3D[boat, PlotTheme → "Web", AxesLabel → {"x", "y", "z"},
  PlotPoints → 100, PlotStyle → Directive[Red, Opacity[0.5]]],
 ListPointPlot3D[{{0, 0, 0}}, PlotStyle → Directive[Green, Opacity[1]]]]
```

Out[7]=



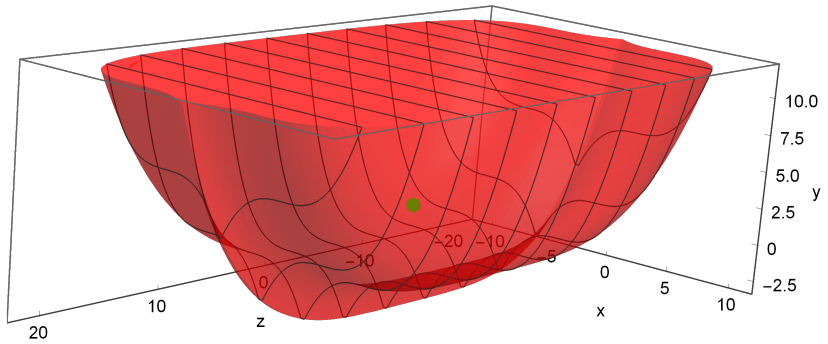
In[8]:=
```
(* Test Plotter *)
Show[
 RegionPlot3D[boat, PlotTheme → "Web", AxesLabel → {"x", "y", "z"},
  PlotPoints → 100, PlotStyle → Directive[Red, Opacity[0.5]]],
 ListPointPlot3D[{{0, 1.72755, 0}}, PlotStyle → Directive[Green, Opacity[1]]]]
```

Out[8]=



In[9]:=
```
(* Test Plotter *)
Show[
 RegionPlot3D[boat, PlotTheme → "Web", AxesLabel → {"x", "y", "z"},
  PlotPoints → 100, PlotStyle → Directive[Red, Opacity[0.5]]],
 ListPointPlot3D[{{0, 3.25, 0}}, PlotStyle → Directive[Green, Opacity[1]]]]
```

Out[9]=



In this section, we plot the Center of Mass and Center of Buoyancy on a 3D plot of our boat for several heel angles:

0 Degree

Green Label - COM

Black Label - COB

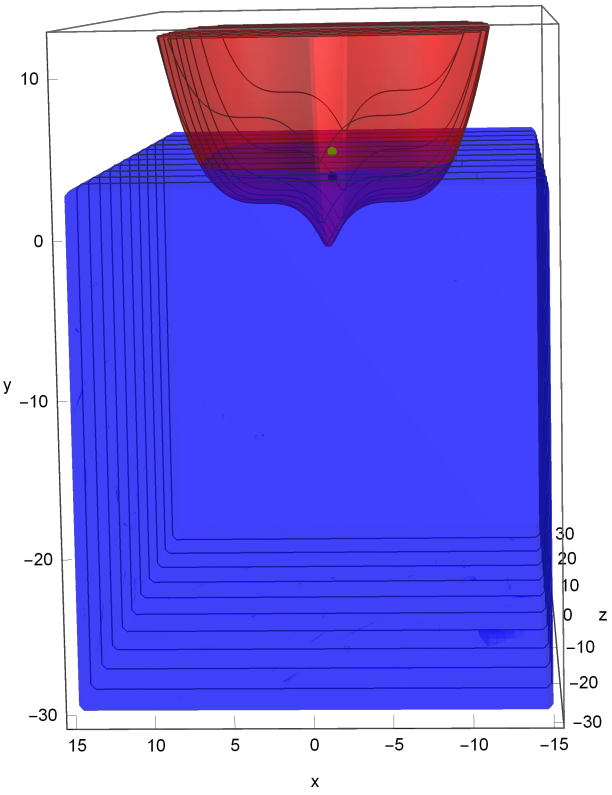
In[ ]:=
```
mass = 1315;
com = {0, 3.25, 0};
water = ImplicitRegion[
   y < Tan[0 Degree] * x + d && -15 < x < 15 && -30 < y < 100 && -30 <= z ≤ 30, {x, y, z}];
under = RegionIntersection[boat, water];
disp = Integrate[1, {x, y, z} ∈ under /. {θ → 0}];
draft = N[d /. FindRoot[disp == mass, {d, -20, 20}][[1]]]
cob = RegionCentroid[under /. {d → draft, θ → 0}]
```

Out[ ]= 3.04709

Out[ ]= $\{-5.97325 \times 10^{-17}, 1.41403, -2.55604 \times 10^{-7}\}$

In[•]:=
```
Show[
 RegionPlot3D[boat, PlotTheme → "Web", AxesLabel → {"x", "y", "z"},
  PlotPoints → 100, PlotStyle → Directive[Red, Opacity[0.5]]],
 RegionPlot3D[water /. {d → draft, θ → 0}, PlotTheme → "Web", AxesLabel → {"x", "y", "z"},
  PlotPoints → 100, PlotStyle → Directive[Blue, Opacity[0.5]]],
 ListPointPlot3D[{{com}}, PlotStyle → Directive[Green, Opacity[1]]],
 ListPointPlot3D[{{cob}}, PlotStyle → Directive[Black, Opacity[1]]]]
```

Out[•]=



30 Degree

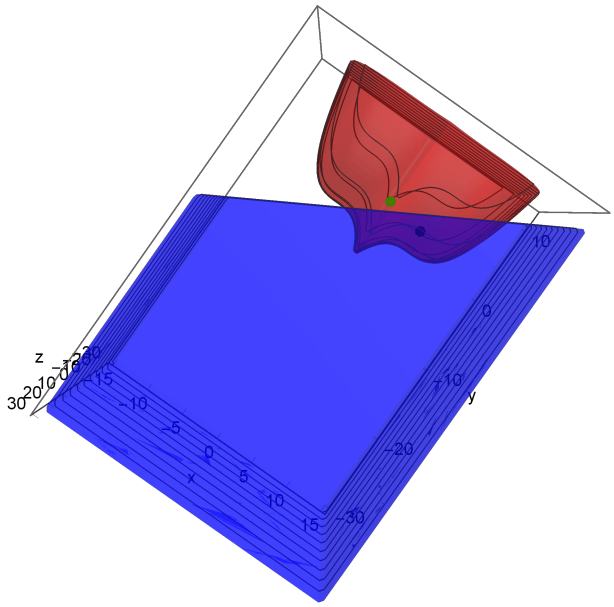
In[•]:=
```
mass = 1315;
com = {0, 3.25, 0};
water = ImplicitRegion[
   y < Tan[30 Degree] * x + d && -15 < x < 15 && -30 < y < 100 && -30 <= z ≤ 30, {x, y, z}];
under = RegionIntersection[boat, water];
disp = Integrate[1, {x, y, z} ∈ under /. {θ → 30}];
draft = N[d /. FindRoot[disp == mass, {d, -20, 20}][[1]]]
cob = RegionCentroid[under /. {d → draft, θ → 30}]
```

Out[•]= 2.01655

Out[•]= $\{4.27488, 2.51152, -5.48583 \times 10^{-17}\}$

In[ ]:=
```
Show[
 RegionPlot3D[boat, PlotTheme → "Web", AxesLabel → {"x", "y", "z"},
  PlotPoints → 100, PlotStyle → Directive[Red, Opacity[0.5]]],
 RegionPlot3D[water /. {d → draft, θ → 30}, PlotTheme → "Web", AxesLabel → {"x", "y", "z"},
  PlotPoints → 100, PlotStyle → Directive[Blue, Opacity[0.5]]],
 ListPointPlot3D[{{com}}, PlotStyle → Directive[Green, Opacity[1]]],
 ListPointPlot3D[{{cob}}, PlotStyle → Directive[Black, Opacity[1]]]]
```

Out[ ]=



60 Degree

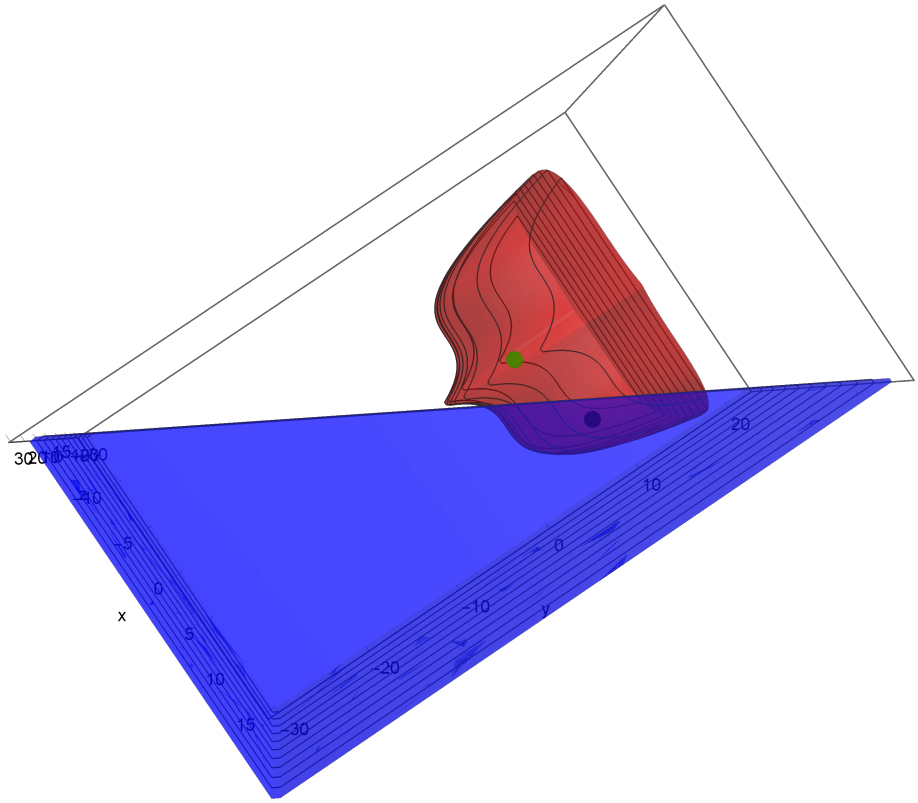
In[ ]:=
```
mass = 1315;
com = {0, 3.25, 0};
water = ImplicitRegion[
   y < Tan[60 Degree] * x + d && -15 < x < 15 && -30 < y < 100 && -30 <= z ≤ 30, {x, y, z}];
under = RegionIntersection[boat, water];
disp = Integrate[1, {x, y, z} ∈ under /. {θ → 60}];
draft = N[d /. FindRoot[disp == mass, {d, -20, 20}][[1]]]
cob = RegionCentroid[under /. {d → draft, θ → 60}]
```

Out[ ]= −3.91481

Out[ ]= $\{7.46538, 5.78314, -4.25666 \times 10^{-16}\}$

In[ ]:=
```
Show[
 RegionPlot3D[boat, PlotTheme → "Web", AxesLabel → {"x", "y", "z"},
  PlotPoints → 100, PlotStyle → Directive[Red, Opacity[0.5]]],
 RegionPlot3D[water /. {d → draft, θ → 60}, PlotTheme → "Web", AxesLabel → {"x", "y", "z"},
  PlotPoints → 100, PlotStyle → Directive[Blue, Opacity[0.5]]],
 ListPointPlot3D[{{com}}, PlotStyle → Directive[Green, Opacity[1]]],
 ListPointPlot3D[{{cob}}, PlotStyle → Directive[Black, Opacity[1]]]]
```

Out[ ]=



120 Degree

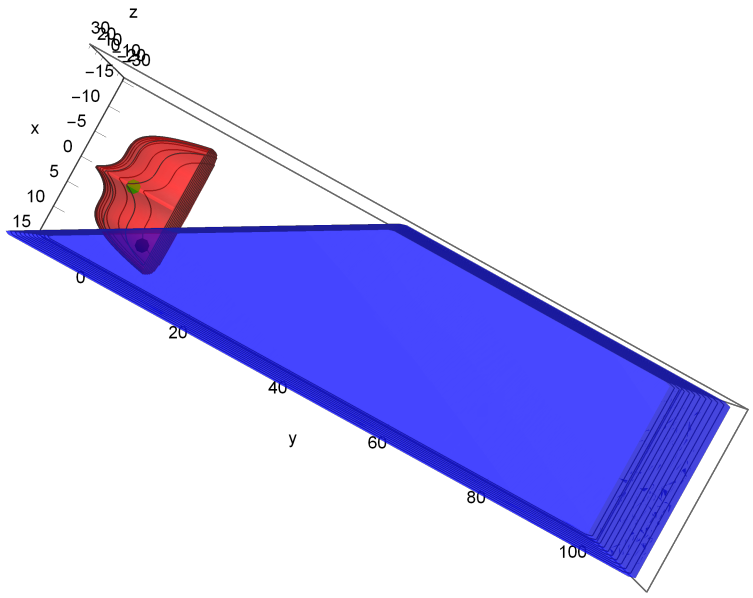
In[ ]:=
```
mass = 1315;
com = {0, 3.25, 0};
water = ImplicitRegion[
   y > Tan[120 Degree] * x + d && -15 < x < 15 && -30 < y < 100 && -30 <= z ≤ 30, {x, y, z}];
under = RegionIntersection[boat, water];
disp = Integrate[1, {x, y, z} ∈ under /. {θ → 120}];
draft = N[d /. FindRoot[disp == mass, {d, -20, 20}][[1]]]
cob = RegionCentroid[under /. {d → draft, θ → 120}]
```

Out[ ]= 17.7406

Out[ ]= $\{7.66226, 9.05742, -4.78493 \times 10^{-17}\}$

In[ ]:=
```
Show[
 RegionPlot3D[boat, PlotTheme → "Web", AxesLabel → {"x", "y", "z"},
  PlotPoints → 100, PlotStyle → Directive[Red, Opacity[0.5]]],
 RegionPlot3D[water /. {d → draft, θ → 120}, PlotTheme → "Web", AxesLabel → {"x", "y", "z"},
  PlotPoints → 100, PlotStyle → Directive[Blue, Opacity[0.5]]],
 ListPointPlot3D[{{com}}, PlotStyle → Directive[Green, Opacity[1]]],
 ListPointPlot3D[{{cob}}, PlotStyle → Directive[Black, Opacity[1]]]]
```

Out[ ]=



130 Degree

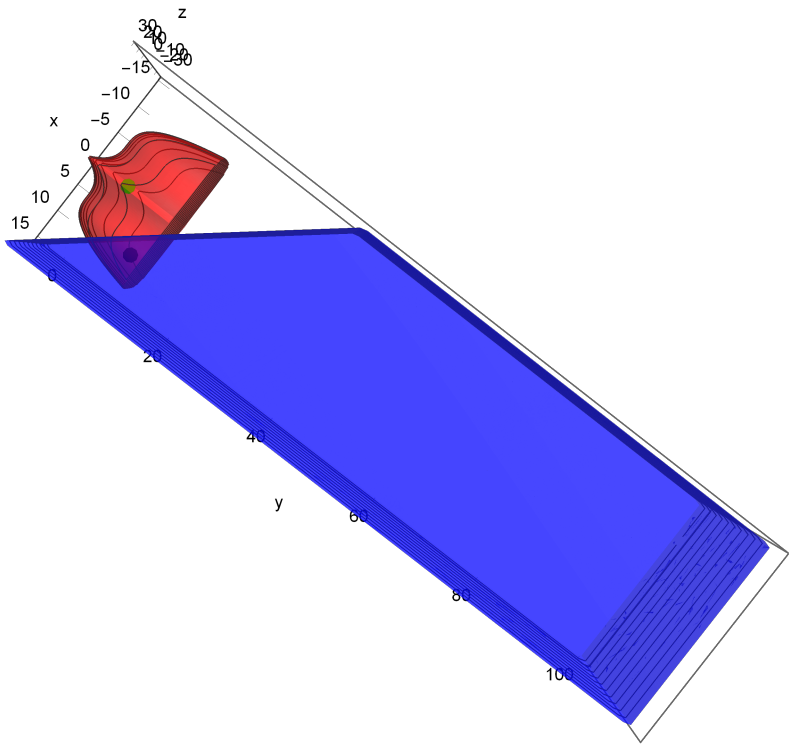
In[ ]:=
```
mass = 1315;
com = {0, 3.25, 0};
water = ImplicitRegion[
   y > Tan[130 Degree] * x + d && -15 < x < 15 && -30 < y < 100 && -30 <= z ≤ 30, {x, y, z}];
under = RegionIntersection[boat, water];
disp = Integrate[1, {x, y, z} ∈ under /. {θ → 130}];
draft = N[d /. FindRoot[disp == mass, {d, -20, 20}][[1]]]
cob = RegionCentroid[under /. {d → draft, θ → 130}]
```

Out[ ]= 14.7071

Out[ ]= $\{7.41353, 9.41234, -1.13808 \times 10^{-16}\}$

In[ ]:=
```
Show[
 RegionPlot3D[boat, PlotTheme → "Web", AxesLabel → {"x", "y", "z"},
  PlotPoints → 100, PlotStyle → Directive[Red, Opacity[0.5]]],
 RegionPlot3D[water /. {d → draft, θ → 130}, PlotTheme → "Web", AxesLabel → {"x", "y", "z"},
  PlotPoints → 100, PlotStyle → Directive[Blue, Opacity[0.5]]],
 ListPointPlot3D[{{com}}, PlotStyle → Directive[Green, Opacity[1]]],
 ListPointPlot3D[{{cob}}, PlotStyle → Directive[Black, Opacity[1]]]]
```

Out[ ]=



150 Degree

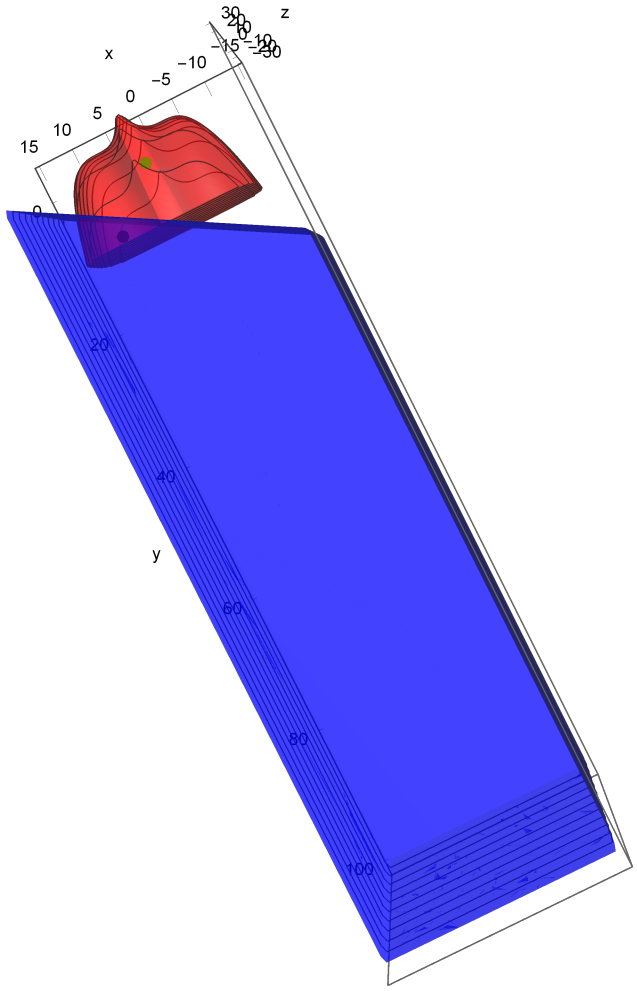
In[ ]:=
```
mass = 1315;
com = {0, 3.25, 0};
water = ImplicitRegion[
   y > Tan[150 Degree] * x + d && -15 < x < 15 && -30 < y < 100 && -30 <= z ≤ 30, {x, y, z}];
under = RegionIntersection[boat, water];
disp = Integrate[1, {x, y, z} ∈ under /. {θ → 150}];
draft = N[d /. FindRoot[disp == mass, {d, -20, 20}][[1]]]
cob = RegionCentroid[under /. {d → draft, θ → 150}]
```

Out[ ]= 11.6427

Out[ ]= $\{6.59459, 10.0814, -9.93713 \times 10^{-17}\}$

In[ ]:=
```
Show[
 RegionPlot3D[boat, PlotTheme → "Web", AxesLabel → {"x", "y", "z"},
  PlotPoints → 100, PlotStyle → Directive[Red, Opacity[0.5]]],
 RegionPlot3D[water /. {d → draft, θ → 150}, PlotTheme → "Web", AxesLabel → {"x", "y", "z"},
  PlotPoints → 100, PlotStyle → Directive[Blue, Opacity[0.5]]],
 ListPointPlot3D[{{com}}, PlotStyle → Directive[Green, Opacity[1]]],
 ListPointPlot3D[{{cob}}, PlotStyle → Directive[Black, Opacity[1]]]]
```

Out[ ]=



180 Degree

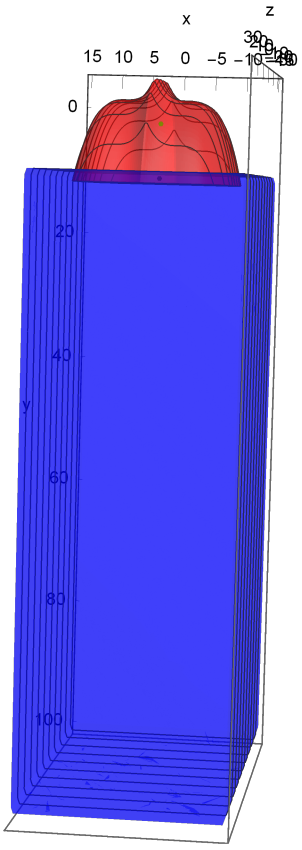
In[ ]:=
```
mass = 1315;
com = {0, 3.25, 0};
water = ImplicitRegion[
   y > Tan[180 Degree] * x + d && -15 < x < 15 && -30 < y < 100 && -30 <= z ≤ 30, {x, y, z}];
under = RegionIntersection[boat, water];
disp = Integrate[1, {x, y, z} ∈ under /. {θ → 180}];
draft = N[d /. FindRoot[disp == mass, {d, -20, 20}][[1]]]
cob = RegionCentroid[under /. {d → draft, θ → 180}]
```

Out[ ]= 10.4711

Out[ ]= $\{-1.56094 \times 10^{-17}, 11.243, 3.04664 \times 10^{-16}\}$

In[ ]:=
```
Show[
 RegionPlot3D[boat, PlotTheme → "Web", AxesLabel → {"x", "y", "z"},
  PlotPoints → 100, PlotStyle → Directive[Red, Opacity[0.5]]],
 RegionPlot3D[water /. {d → draft, θ → 180}, PlotTheme → "Web", AxesLabel → {"x", "y", "z"},
  PlotPoints → 100, PlotStyle → Directive[Blue, Opacity[0.5]]],
 ListPointPlot3D[{{com}}, PlotStyle → Directive[Green, Opacity[1]]],
 ListPointPlot3D[{{cob}}, PlotStyle → Directive[Black, Opacity[1]]]]
```

Out[ ]=



In this section, we define the variables that we obtained from the SolidWorks

Model of our boat:
(i) Mass of the Boat
(ii) COM of the Boat
(iii) Mass of the Ballast
(iv) The COM of the boat - ballast assembly (Calculated in Mathematica)

```
In[•]:= newcomcode[comballast_] := Module[{},
          massboat = 315;
          comboat = {0, 11.21, 0};
          massballast = 1000;
          newcom = ((massboat * comboat) + (massballast * comballast)) / (massboat + massballast);
          newcom
         ]

In[•]:= newcomcode[{0, 0.7485, 0}]

Out[•]= {0, 3.25449, 0}
```

In this section, we define the functions that we use to obtain the righting moment that is acting on the boat. Waterline Angle < 90 Degrees

```
In[•]:= Submergedless90[θθ_] := Module[{},
          mass = 1315;
          com = {0, 3.25, 0};
          water = ImplicitRegion[
            y < Tan[θθ Degree] * x + d && -15 < x < 15 && -30 < y < 100 && -30 <= z ≤ 30, {x, y, z}];
          under = RegionIntersection[boat, water];
          disp = Integrate[1, {x, y, z} ∈ under /. {θ → θθ}];
          draft = N[d /. FindRoot[disp == mass, {d, -20, 20}][[1]]];
          cob = RegionCentroid[under /. {d → draft, θ → θθ}];
          buoyancy = mass * 980 * {-Sin[θθ Degree], Cos[θθ Degree], 0};
          torque = Cross[cob - com, buoyancy][[3]];
          torque
         ]
```

In this section, we define the functions that we use to obtain the righting moment that is acting on the boat. Waterline Angle > 90 Degrees

```
In[•]:= Submergedmore90[θθ_] := Module[{},
  mass = 1315;
  com = {0, 3.25, 0};
  water = ImplicitRegion[
    y > Tan[θθ Degree] * x + d && -15 < x < 15 && -30 < y < 100 && -30 <= z ≤ 30, {x, y, z}];
  under = RegionIntersection[boat, water];
  disp = Integrate[1, {x, y, z} ∈ under /. {θ → θθ}];
  draft = N[d /. FindRoot[disp == mass, {d, -20, 20}][[1]]];
  cob = RegionCentroid[under /. {d → draft, θ → θθ}];
  buoyancy = mass * 980 * {-Sin[θθ Degree], Cos[θθ Degree], 0};
  torque = Cross[cob - com, buoyancy][[3]];
  torque
 ]
```

```
In[•]:= before90 =
 Table[{theta, Submergedless90[theta]}, {theta, {0, 10, 20, 30, 40, 50, 60, 70, 80}}]
```

$$\text{Out[•]=}\ \{\{0, -7.69773\times10^{-11}\}, \{10, 1.6118\times10^{6}\}, \{20, 3.13069\times10^{6}\},\ \{30, 4.29512\times10^{6}\}, \{40, 5.26508\times10^{6}\}, \{50, 6.59151\times10^{6}\},\ \{60, 7.63742\times10^{6}\}, \{70, 7.36941\times10^{6}\}, \{80, 6.67464\times10^{6}\}\}$$

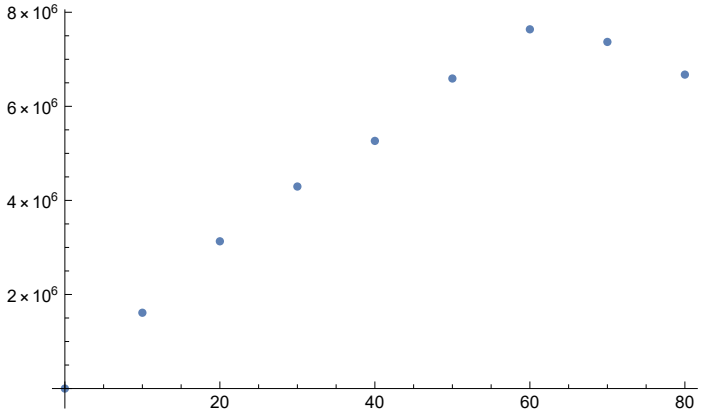
```
In[•]:= after90 = Table[{theta, Submergedmore90[theta]},
   {theta, {100, 110, 120, 130, 140, 150, 160, 170, 180}}]
```

$$\text{Out[•]=}\ \{\{100, 4.49229\times10^{6}\}, \{110, 3.08922\times10^{6}\}, \{120, 1.54417\times10^{6}\},\ \{130, -57597.4\}, \{140, -1.60825\times10^{6}\}, \{150, -2.95807\times10^{6}\},\ \{160, -3.84148\times10^{6}\}, \{170, -3.37014\times10^{6}\}, \{180, 2.01158\times10^{-11}\}\}$$

In this section, we find and plot the results.

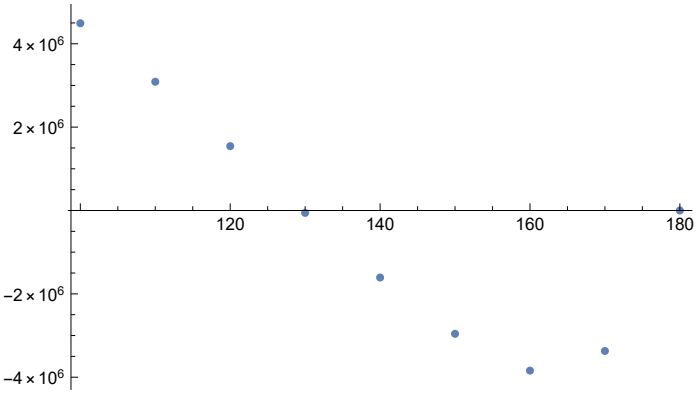
```
In[•]:= ListPlot[before90]
ListPlot[{{{0, -7.697731341238518`*^-11}, {10, 1.6117969999596088`*^6},
   {20, 3.130685400111096`*^6}, {30, 4.295119286440562`*^6},
   {40, 5.26508138860967`*^6}, {50, 6.591513650189281`*^6}, {60, 7.637417976334997`*^6},
   {70, 7.369406305421029`*^6}, {80, 6.674636087897369`*^6}}}]
```

Out[•]=



In[•]:=
```
ListPlot[{{100, 4.492285882425818`*^6},
  {110, 3.0892233977019624`*^6}, {120, 1.5441742377184266`*^6},
  {130, -57597.35070126683`}, {140, -1.6082452983155984`*^6},
  {150, -2.958066348619032`*^6}, {160, -3.8414826675554756`*^6},
  {170, -3.370135743164603`*^6}, {180, 2.0115779159578246`*^-11}}]
```

Out[•]=

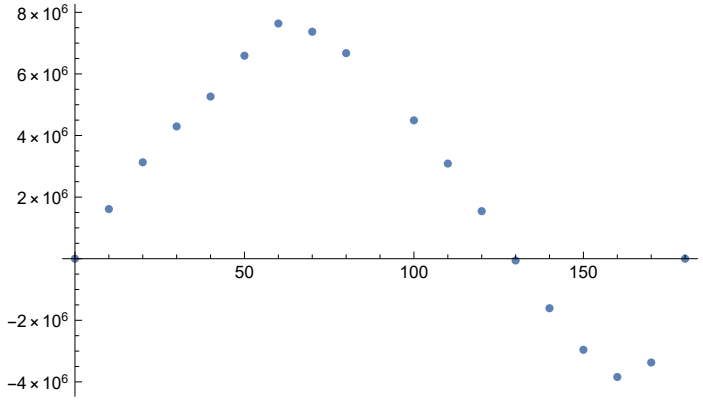


In[•]:=
```
AVS = {{0, -7.697731341238518`*^-11}, {10, 1.6117969999596088`*^6},
  {20, 3.130685400111096`*^6}, {30, 4.295119286440562`*^6}, {40, 5.26508138860967`*^6},
  {50, 6.591513650189281`*^6}, {60, 7.637417976334997`*^6}, {70, 7.369406305421029`*^6},
  {80, 6.674636087897369`*^6}, {100, 4.492285882425818`*^6},
  {110, 3.0892233977019624`*^6}, {120, 1.5441742377184266`*^6},
  {130, -57597.35070126683`}, {140, -1.6082452983155984`*^6},
  {150, -2.958066348619032`*^6}, {160, -3.8414826675554756`*^6},
  {170, -3.370135743164603`*^6}, {180, 2.0115779159578246`*^-11}}
```

Out[•]=
$$\{\{0, -7.69773\times10^{-11}\}, \{10, 1.6118\times10^{6}\}, \{20, 3.13069\times10^{6}\},$$
$$\{30, 4.29512\times10^{6}\}, \{40, 5.26508\times10^{6}\}, \{50, 6.59151\times10^{6}\}, \{60, 7.63742\times10^{6}\},$$
$$\{70, 7.36941\times10^{6}\}, \{80, 6.67464\times10^{6}\}, \{100, 4.49229\times10^{6}\}, \{110, 3.08922\times10^{6}\},$$
$$\{120, 1.54417\times10^{6}\}, \{130, -57\,597.4\}, \{140, -1.60825\times10^{6}\}, \{150, -2.95807\times10^{6}\},$$
$$\{160, -3.84148\times10^{6}\}, \{170, -3.37014\times10^{6}\}, \{180, 2.01158\times10^{-11}\}\}$$

In[•]:=
```
ListPlot[{{0, -7.697731341238518`*^-11}, {10, 1.6117969999596088`*^6},
  {20, 3.130685400111096`*^6}, {30, 4.295119286440562`*^6}, {40, 5.26508138860967`*^6},
  {50, 6.591513650189281`*^6}, {60, 7.637417976334997`*^6}, {70, 7.369406305421029`*^6},
  {80, 6.674636087897369`*^6}, {100, 4.492285882425818`*^6},
  {110, 3.0892233977019624`*^6}, {120, 1.5441742377184266`*^6},
  {130, -57597.35070126683`}, {140, -1.6082452983155984`*^6},
  {150, -2.958066348619032`*^6}, {160, -3.8414826675554756`*^6},
  {170, -3.370135743164603`*^6}, {180, 2.0115779159578246`*^-11}}]
```

Out[•]=



In[•]:=
```
curve = A * x^5 + B * x^4 + C * x^3 + D * x^2 + F * x
params = {{A}, {B}, {C}, {D}, {F}}
bestparams = FindFit[AVS, curve, params, x]
bestcurve = curve /. bestparams
Show[ListPlot[AVS], Plot[bestcurve, {x, 0, 180}]]
```

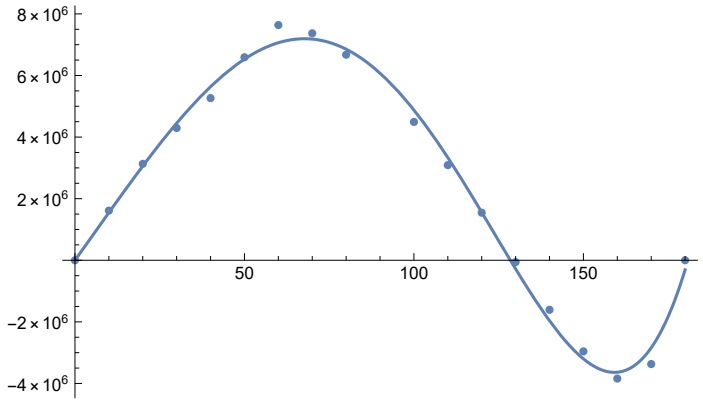
Out[•]= $F x + D x^2 + C x^3 + B x^4 + A x^5$

Out[•]= $\{\{A\}, \{B\}, \{C\}, \{D\}, \{F\}\}$

Out[•]= $\{A \to 0.000475537, B \to -0.0485358, C \to -12.8235, D \to 245.067, F \to 153503.\}$

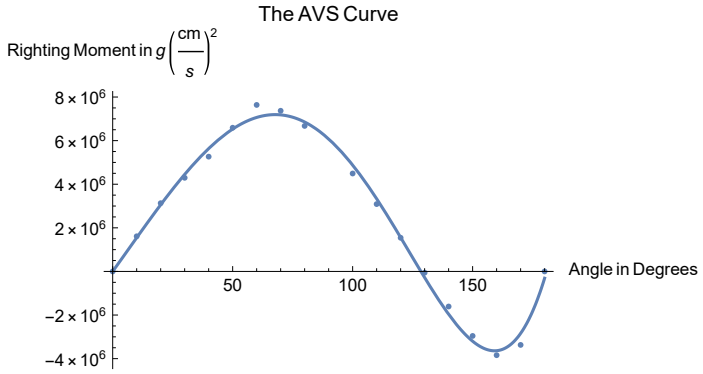
Out[•]= $153503. x + 245.067 x^2 - 12.8235 x^3 - 0.0485358 x^4 + 0.000475537 x^5$

Out[•]=



In[ ]:=
```
Show[%36, AxesLabel → {HoldForm[Angle in Degrees], HoldForm[Righting Moment in g (cm/s)^2]},
 PlotLabel → HoldForm[The AVS Curve], LabelStyle → {GrayLevel[0]}]
```

Out[ ]=



The analytical value of our AVS = 130 Degrees