Ay190 – Worksheet 7
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Date: January 29, 2014

1 Problem 1: Are you thinking about food?

To calculate π, I drew deliciously random (x, y) points inside 1×1 square with a circle inscribed (picture raisins sprinkled in a certain baked round dessert. Or, better yet, chocolate chips!). I then looked at the fraction of points that were inside the circle. Since the points were chosen uniformly, this fraction should only depend on the ratio of the circle's area to the square's area:

$$P = \frac{A_{circ}}{A_{sqr}} = \frac{\pi r^2}{l^2} = \frac{\pi}{4} \tag{1}$$

Thus, multiplying the fraction I find by 4 gives a measurement of π. My results are shown in Figure 1. Since choosing the points is a random counting process, the spread in my results should decrease with the number of points chosen like $1/\sqrt{N}$ (meanwhile, this would make a great dessert spread). This can be seen in Figure 1, where my result converges to π at this rate. For $N = 10,000$ points drawn, I find $\pi \approx 3.1400$. Now, if only it was someone's birthday so I could bake them a π. Or two people's. I wonder what the odds of that would be...

2 Problem 2: The Barta Conundrum.

To find the minimum number of people needed for there to be a greater than 50% chance that two of them share a birthday, I did a Monte Carlo simulation that first generates random days of the year for N people to be born on. This is done 10,000 times, and the number of times a pair of people share a birthday is counted for each N. This is divided by 10,000 to determine the probability of two people having the same birthday. The results are shown in Figure 2. I find that 23 people are required for the probability to be above 50%. For $N = 23$, I find $P = 0.5099$.

To calculate what the answer should be, use the probability that no two people share a birthday. This is given by

$$P' = 1 \times \frac{364}{365} \times \frac{363}{365} \times ... \times \frac{365 - (N-1)}{365} \tag{2}$$

$$P' = \frac{365 \times 364 \times 363 \times ... \times (365 - (N-1))}{365^N} \tag{3}$$

$$P' = \frac{365!}{365^N (365 - N)!} \tag{4}$$

For $N = 22$, this evaluates to $P' = 0.5243$, so $P = 0.4757$. For $N = 23$, $P' = 0.4927$, so $P = 0.5073$. So 23 is the correct answer.

Interestingly enough, in a group of 7 people, $P = 0.05$. Thus, since $P <= 0.05$, I conclude that Marta and Becky are the same person.

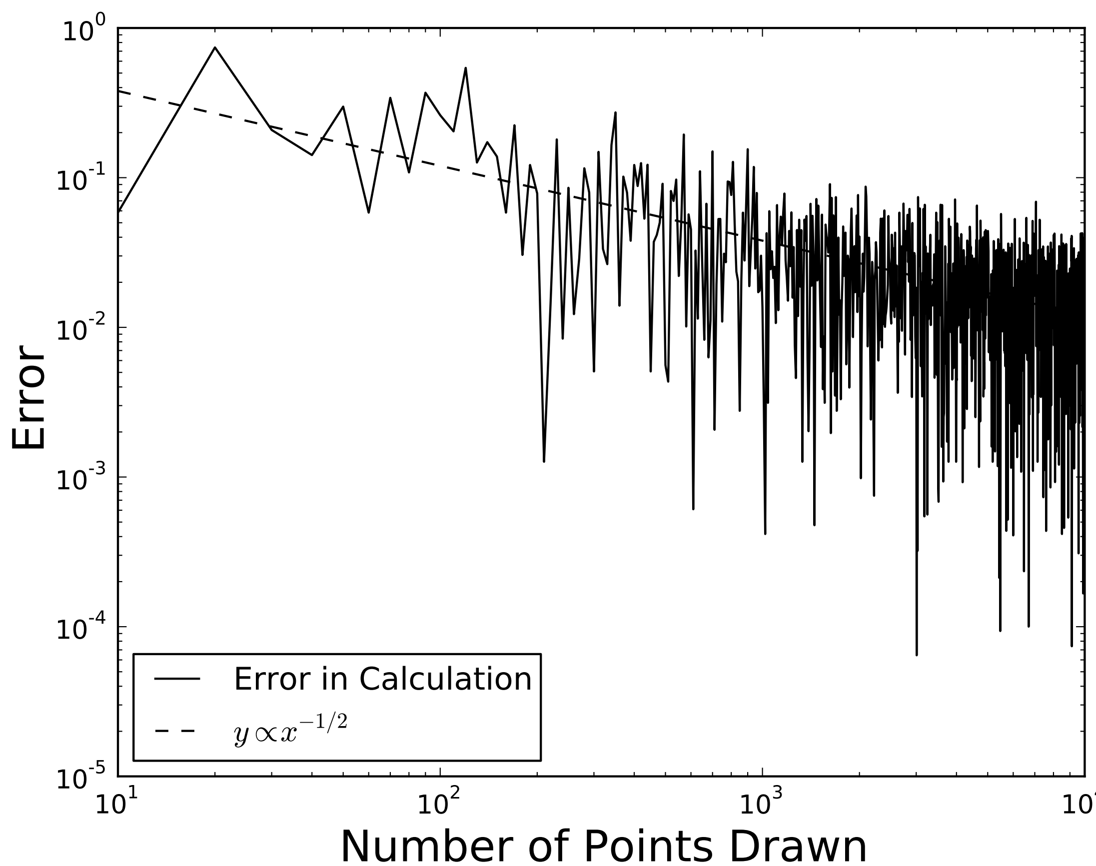

Figure 1: Error from π of my calculation, which falls off like $\frac{1}{\sqrt{N}}$, as it should.

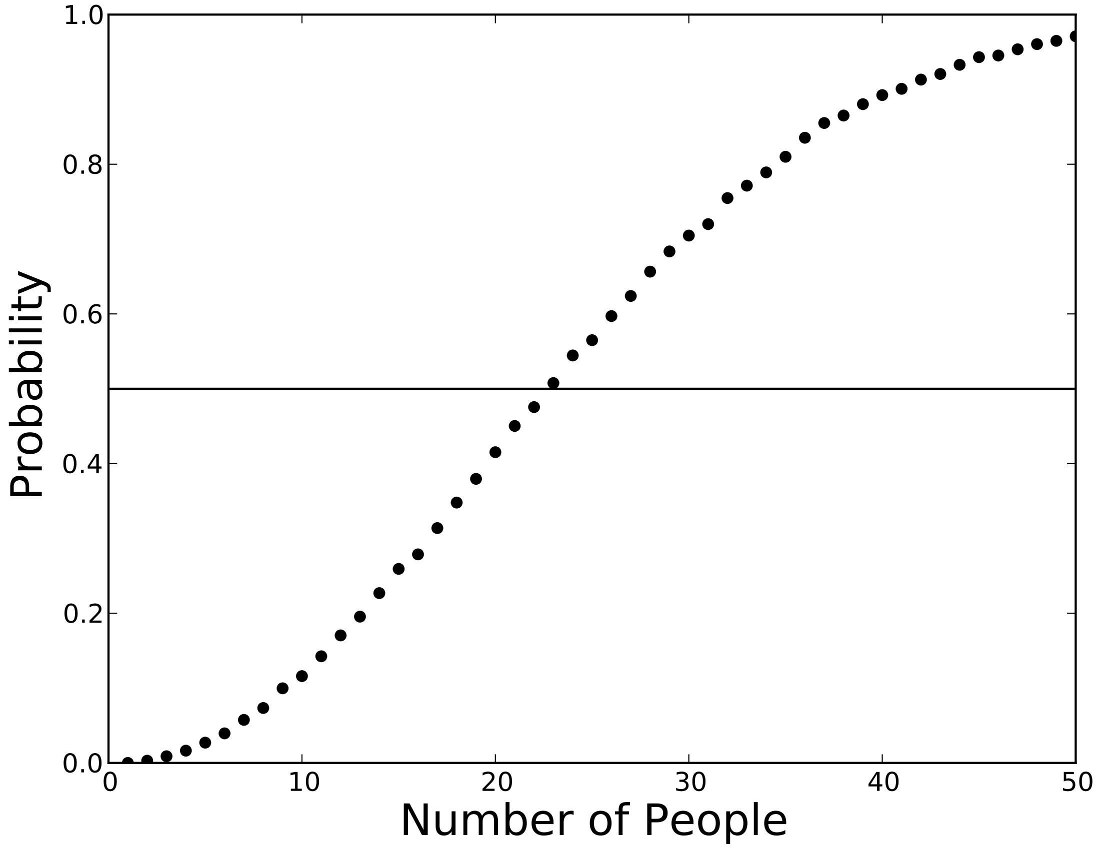

Figure 2: Probability that in a group of N people, two people will share the same birthday. The horizontal line shows $P = 0.5$. The first point above this line is $N = 23$.