Ay190 – Worksheet 11
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I worked with Daniel DeFellipis.

1 Problem 1

1.1 Part a)

We are trying to solve the equation

$$\frac{\partial \Psi}{\partial t} + v\frac{\partial \Psi}{\partial x} = 0 \tag{1}$$

with initial condition

$$\Psi_0 = \Psi(x, t=0) = e^{\frac{-(x-x0)^2}{2\sigma^2}} \tag{2}$$

with $x_0 = 30$ and $\sigma = \sqrt{15}$. The analytical solution is

$$\Psi(x,t) = e^{\frac{-(x-vt-x0)^2}{2\sigma^2}} \tag{3}$$

To implement this analytical solution, I filled in the code provided by Christian.

1.2 Part b)

I implemented the upwind scheme and tested it for α =0.1, 0.5, and 1. In all three cases, the solution was stable and matched the analytical solution reasonably well. I also tested α =1.1 and 2, which quickly became unstable. To measure the error in the scheme, I computed the absolute difference in the peak of the Gaussian for the numerical and analytical solutions, since the central section of the Gaussian is the part of the solution we'd be most likely to care about. I did this for $\alpha = 0.5$ (Figure 1) and $\alpha = 2.0$ (Figure 2). For $\alpha = 0.5$, the error is stable, increasing linearly with time. For $\alpha = 2.0$, the error starts out increasing linearly, but then becomes unstable and increases exponentially. For a Gaussian with σ a factor of 5 smaller, the error in the numerical solution is larger by an order of magnitude. The numerical solution becomes a shorter, wider Gaussian than the analytical solution.

1.3 Part c)

I replaced the upwind scheme with an unstable FTCS scheme. The growth of the instability is shown in Figures 3 through 6. The scheme starts our appearing stable, but an instability develops to the left of the peak. This instability spreads to the rest of the solution as time increases.

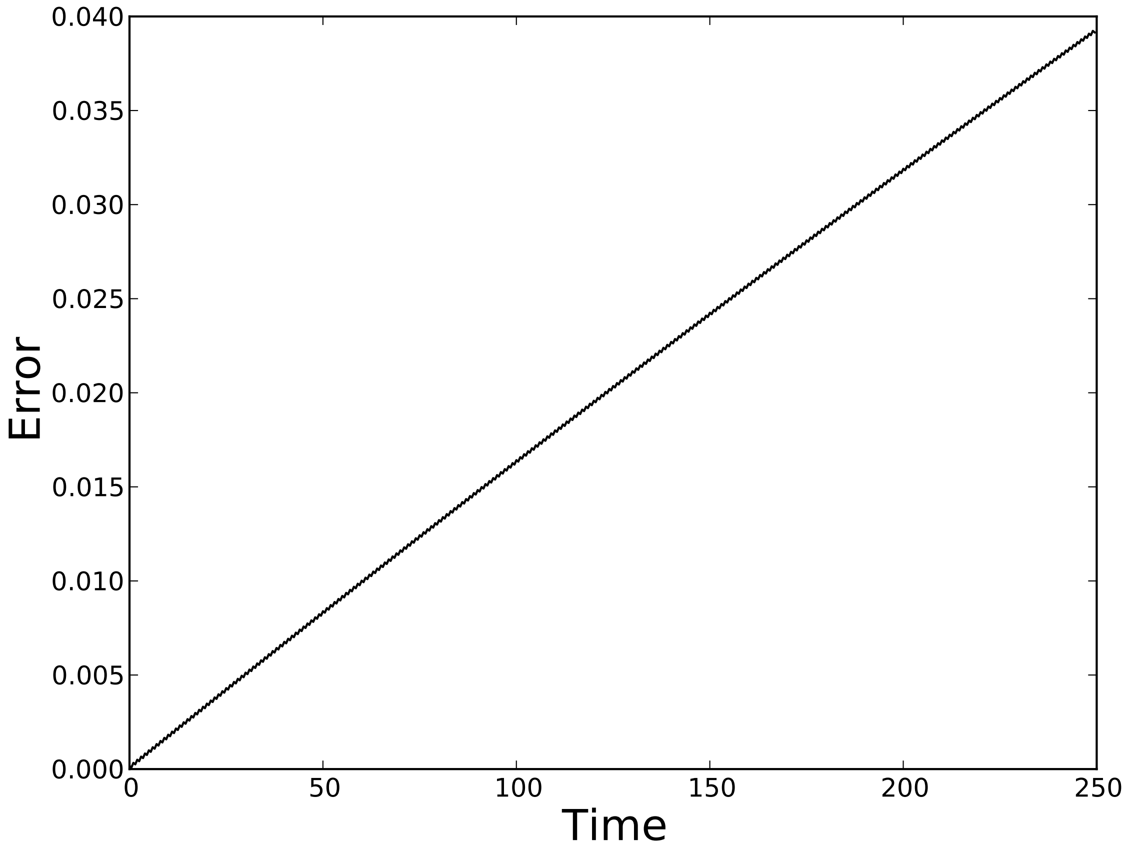

Figure 1: Absolute error in the peak of the numerical solution for $\alpha = 0.5$, which increases linearly with time.

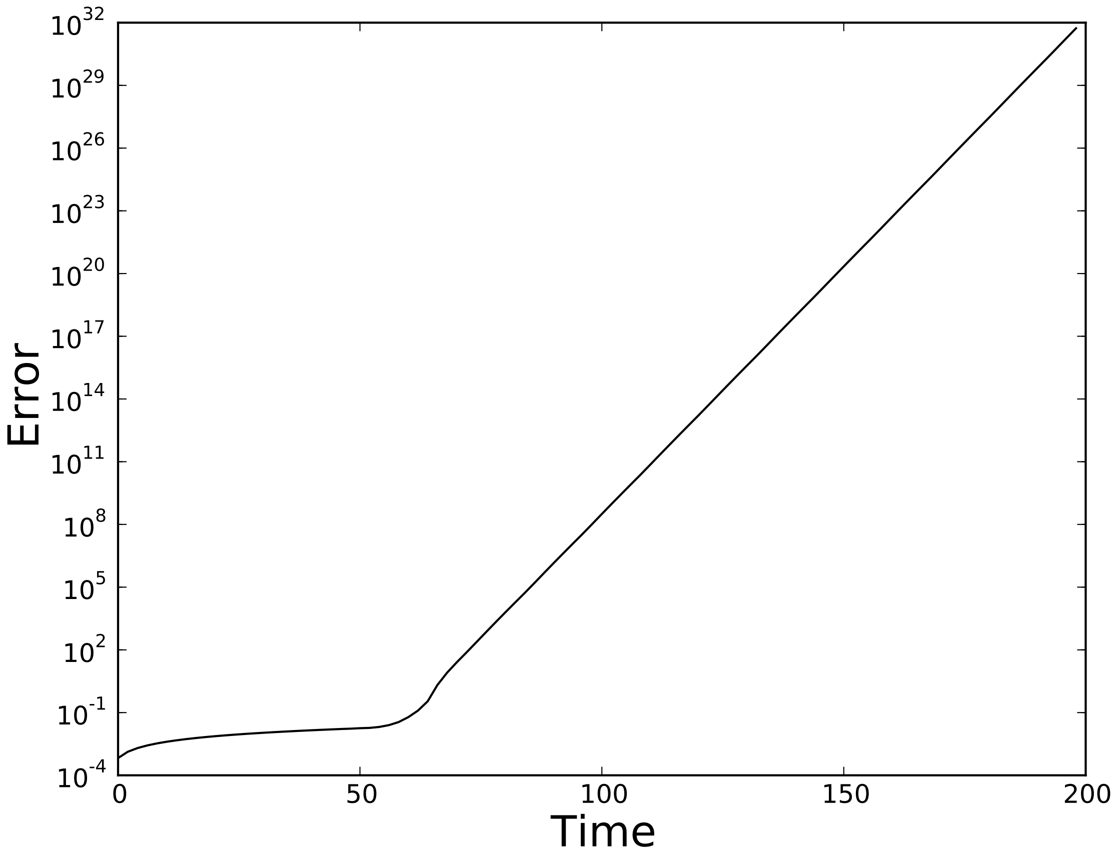

Figure 2: Absolute error in the peak of the numerical solution for $\alpha = 2.0$. Early on, the scheme is stable, but just past t of 50, an instabilility occurs and the error increases exponentially.

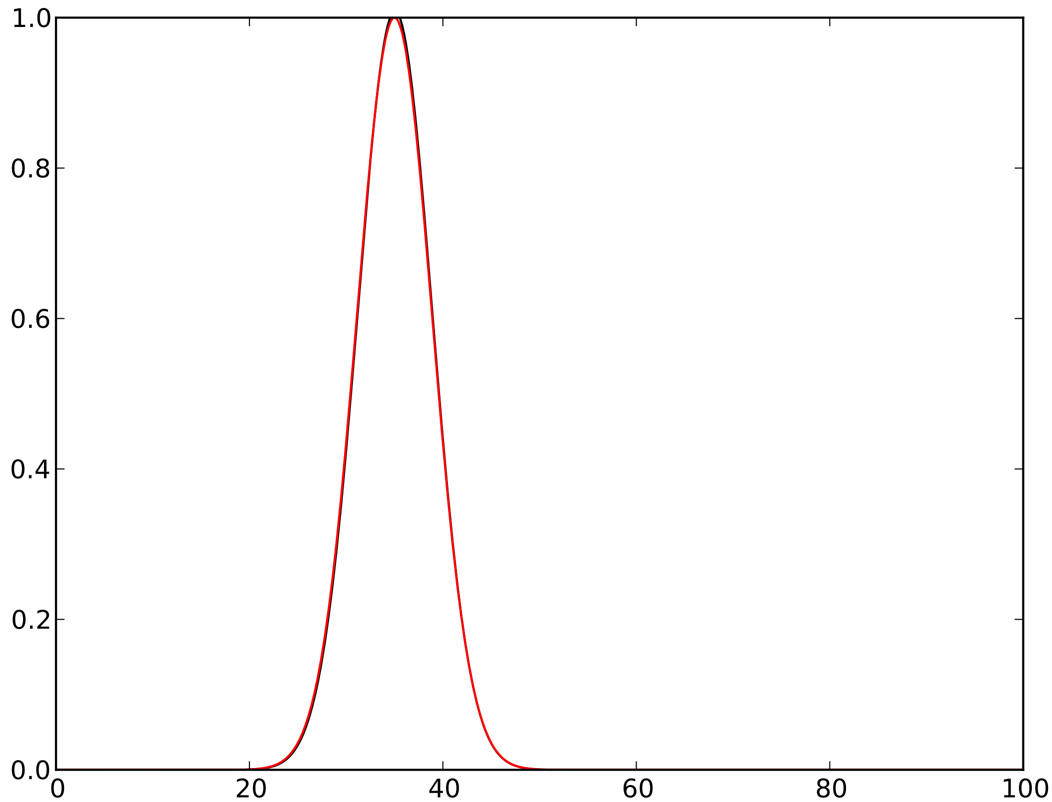

Figure 3: Numerical (black) and analytical (red) solution at 100 iterations using the FTCS scheme. At this point, the solution still appears stable.

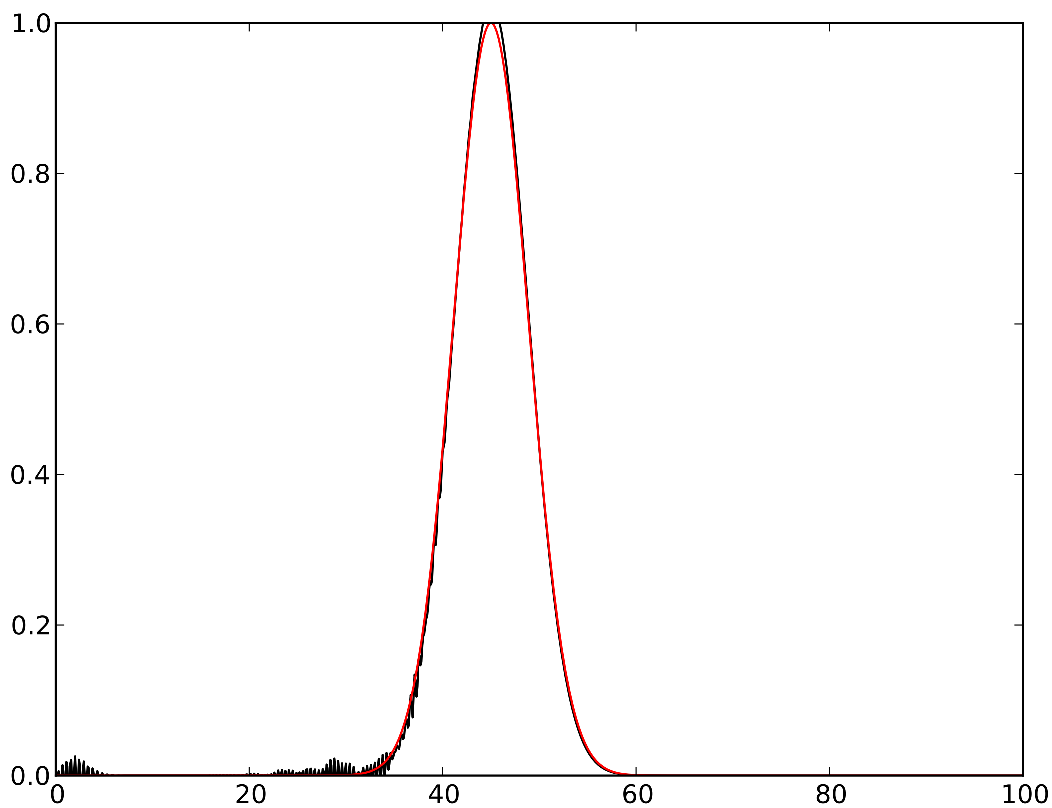

Figure 4: Numerical (black) and analytical (red) solution at 300 iterations using the FTCS scheme. An instability has developed to the left of the peak.

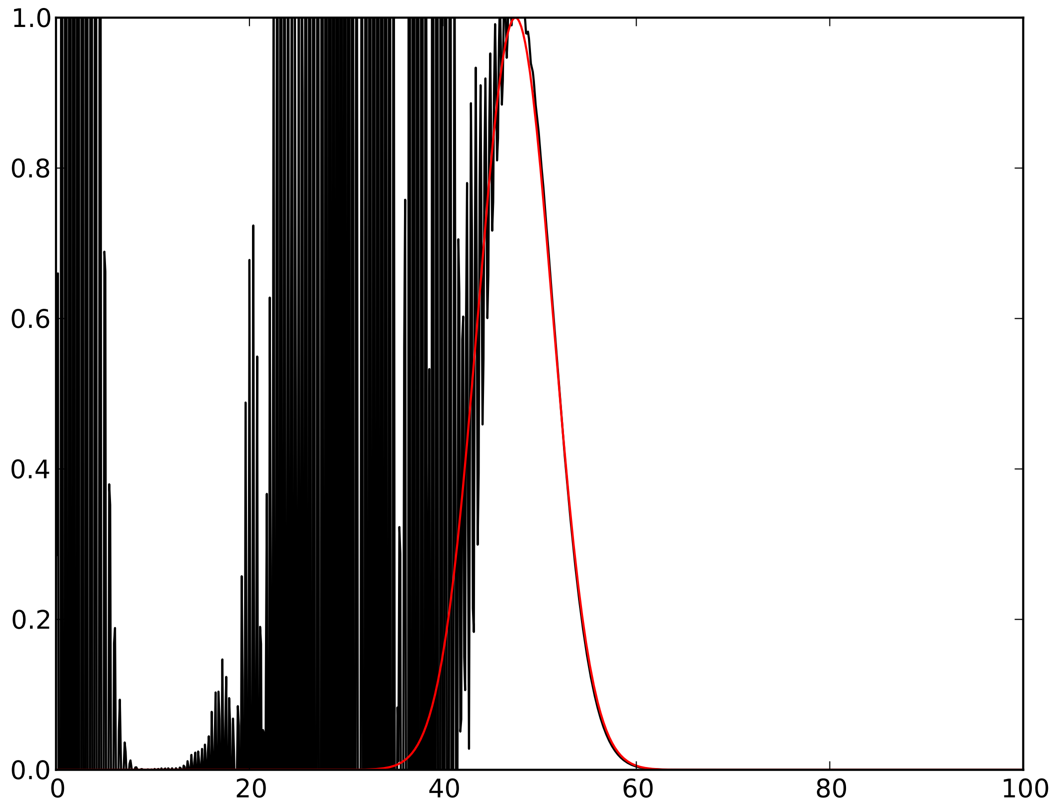

Figure 5: Numerical (black) and analytical (red) solution at 350 iterations using the FTCS scheme. The instability has grown and spread to the peak itself.

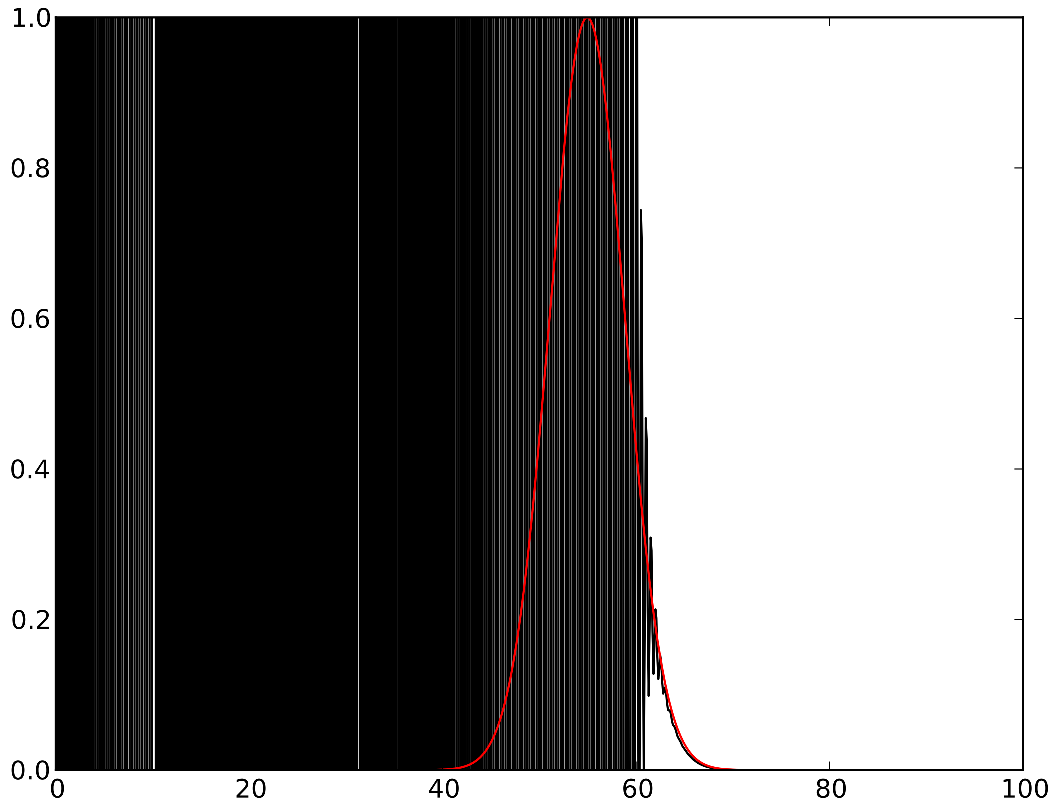

Figure 6: Numerical (black) and analytical (red) solution at 499 iterations using the FTCS scheme. At the end of the computation, the instability includes most of the solution.