Ay190 – Worksheet 12
Scott Barenfeld
Date: February 15, 2014

I worked with Daniel DeFellipis.

1 Problem 1

Radius is in Column 2 (with column numbers starting at 0), based on the range of values in this column. Density is should be $\sim 10^8$ g/cm^3 on average in the core, which is close to Columns 3 and 4. However, Figure 1 shows that Column 3 shows has a large jump at $\sim 6 \times 10^7$ cm, which is unphysical for density. So this column is temperature, and density of Column 4. Figure 2 plots density vs. radius.

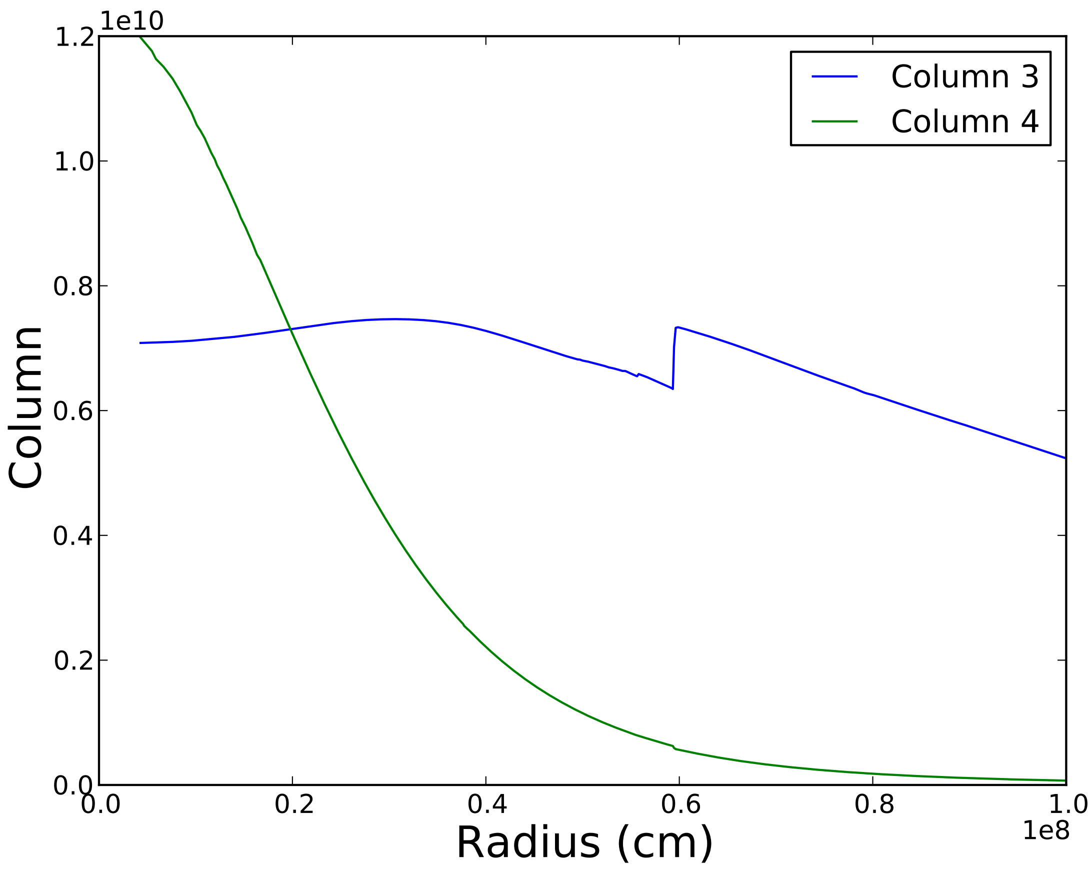

Figure 1: Columns 3 and 4 of the data table. Column 3 has an increase which would be unphysical for density.

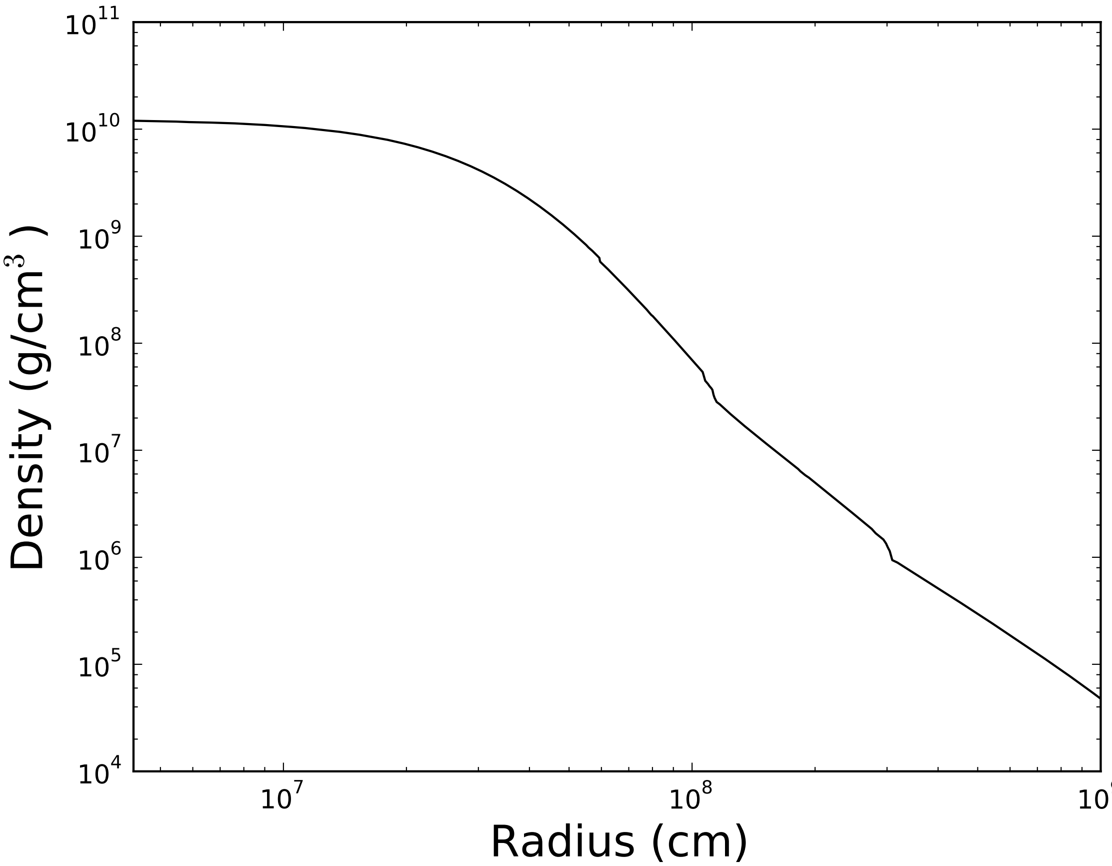

Figure 2: Data table density vs. radius.

2 Problem 2

I used Python's built-in cubic spline interpolation. The result is shown in Figure 3.

3 Problem 3

I solve Poisson's equation:

$$\nabla^2 \Phi = 4\pi G\rho \tag{1}$$

For spherical symmetry, this can be written as two first-order ODEs:

$$\frac{\mathrm{d}\Phi}{\mathrm{d}r} = z \tag{2}$$

$$\frac{\mathrm{d}z}{\mathrm{d}r} + \frac{2}{r}z = 4\pi G\rho \tag{3}$$

with boundary conditions

$$z = 0 \tag{4}$$

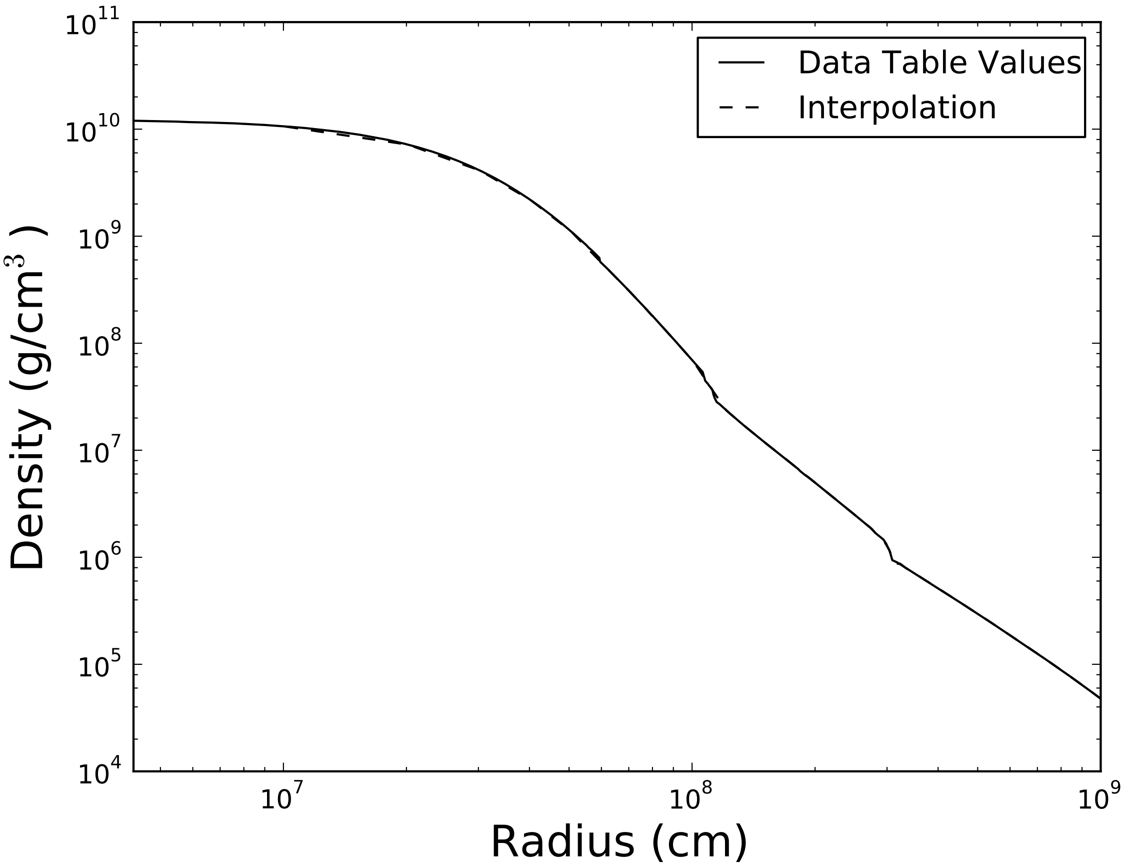

Figure 3: Density interpolated onto an evenly spaced radius grid.

$$\Phi(R_{outer}) = -\frac{GM_{tot}}{R_{outer}} \tag{5}$$

To solve this numerically, I set $\Phi(0) = 0$ and integrate outwards using forward Euler. Since gravitational potential is only defined relative to an arbitrary constant, I then offset my solution to match the outer boundary condition. Also, I shift my radius grid over by 0.5dr to prevent dividing by 0 in Equation 3.

I test my code on a constant density sphere, for which the solution should be

$$\Phi(r) = \frac{2}{3}\pi G\rho(r^2 - 3R_{outer}^2) \tag{6}$$

Figure 4 shows my results for 1000 gridpoints, which match the analytical results. To test convergence, I use 100 and 1000 gridpoints. Globally, forward Euler integration has an error that goes as $\frac{1}{N}$, so my relative error should decrease by a factor of 10. This can be seen in Figures 5 and 6.

Finally, I apply my code to the data table. The result is shown in Figure 7.

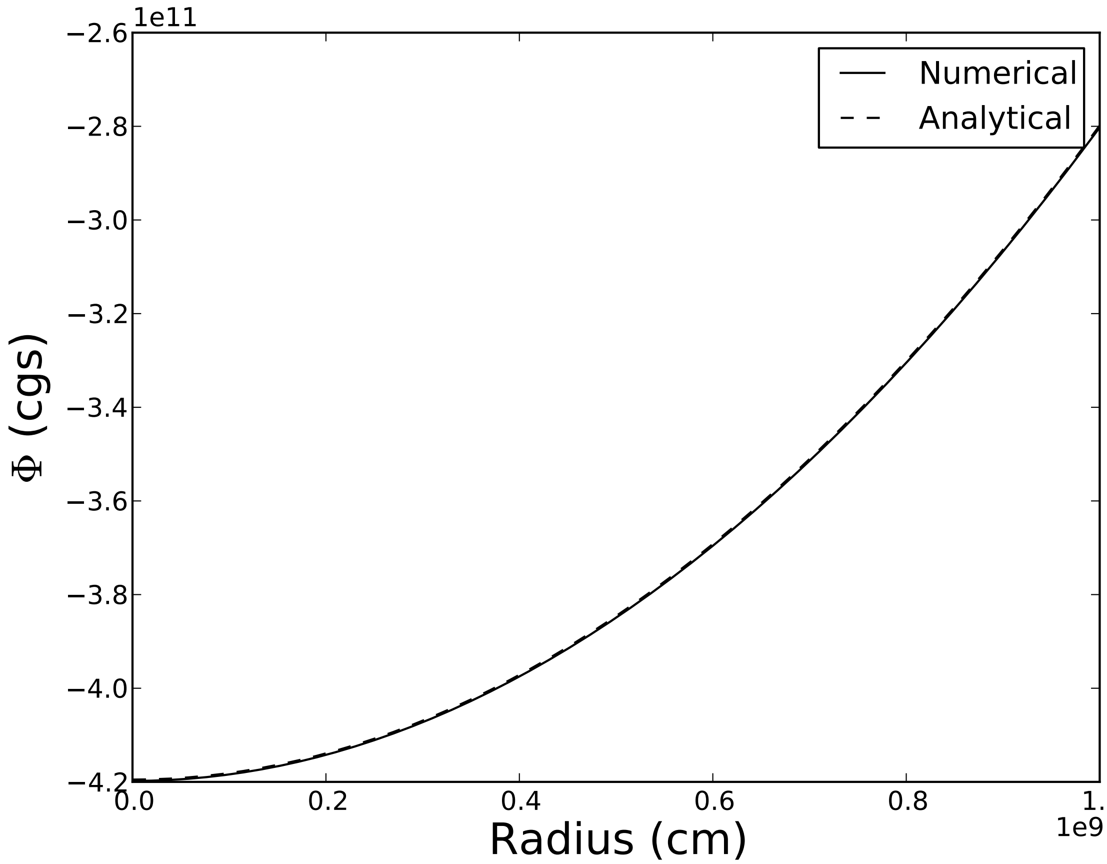

Figure 4: Gravitational potential in a uniform density sphere.

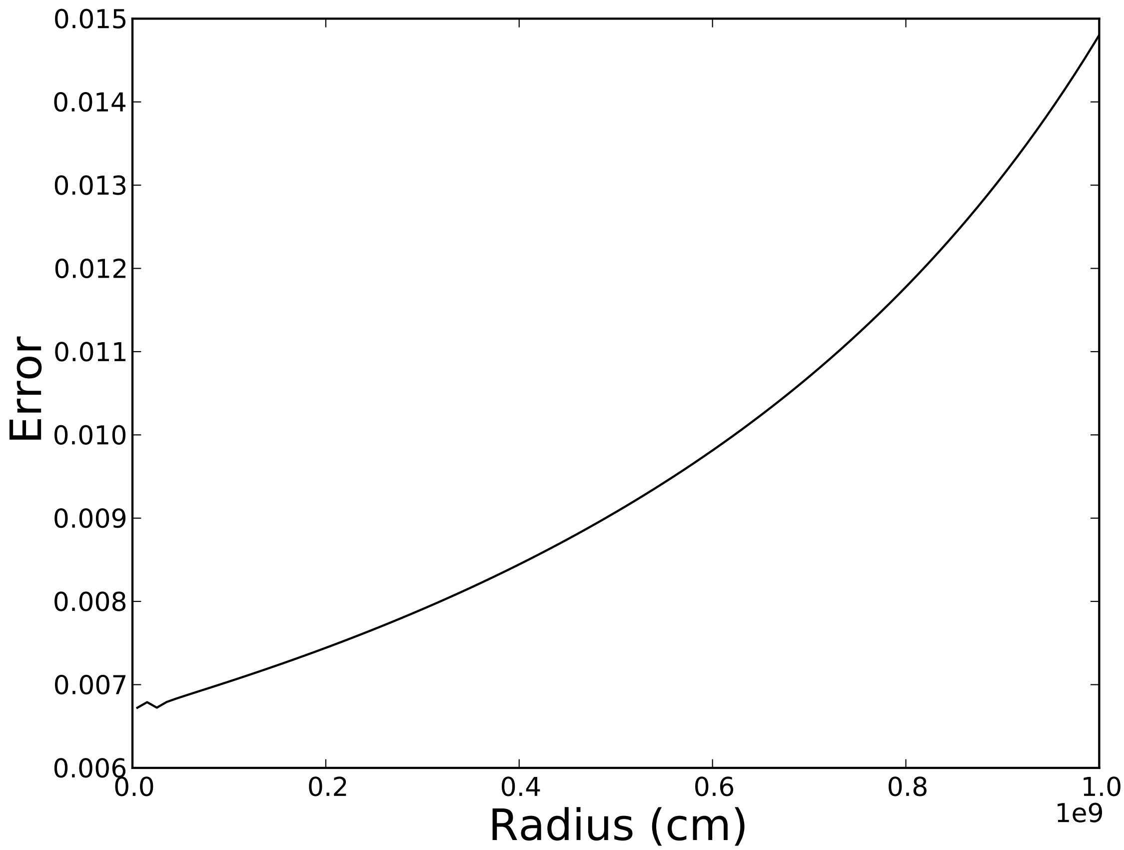

Figure 5: Relative error for 100 gridpoints.

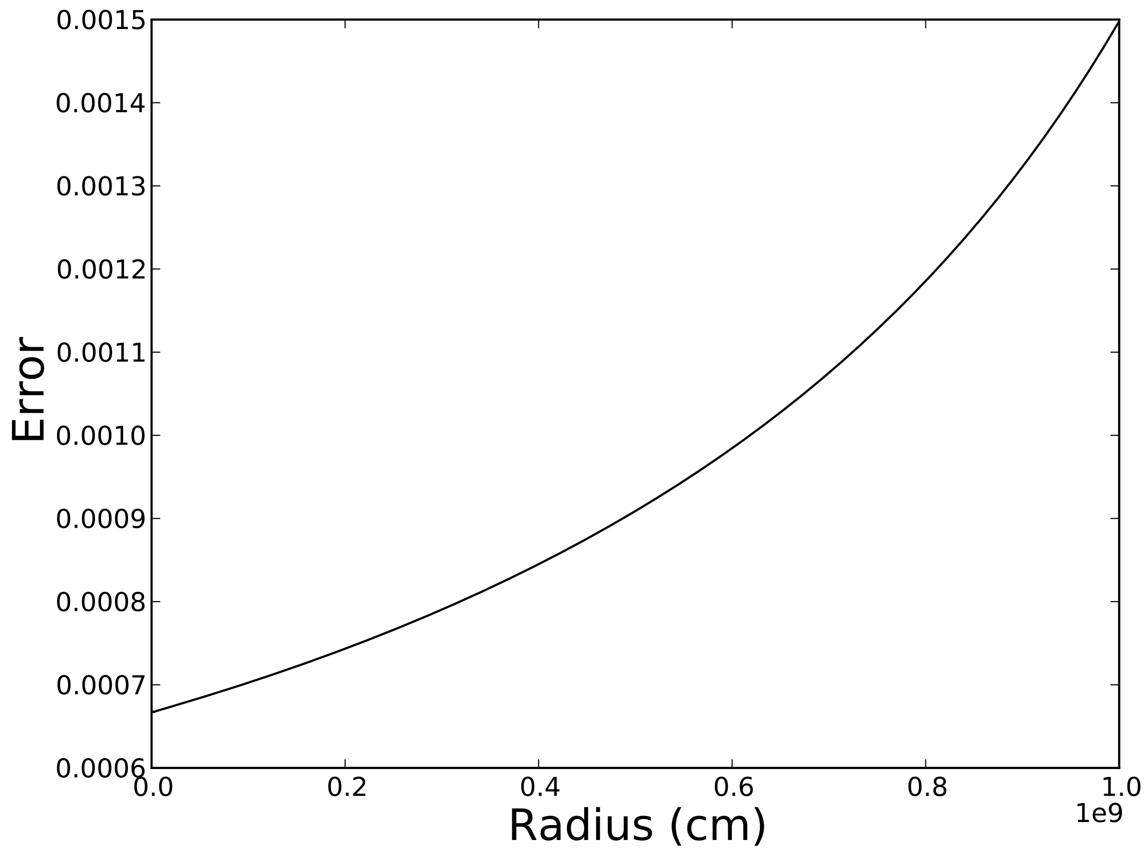

Figure 6: Relative error for 1000 gridpoints. The error is approximately a factor of 10 less than that for 100 gridpoints.

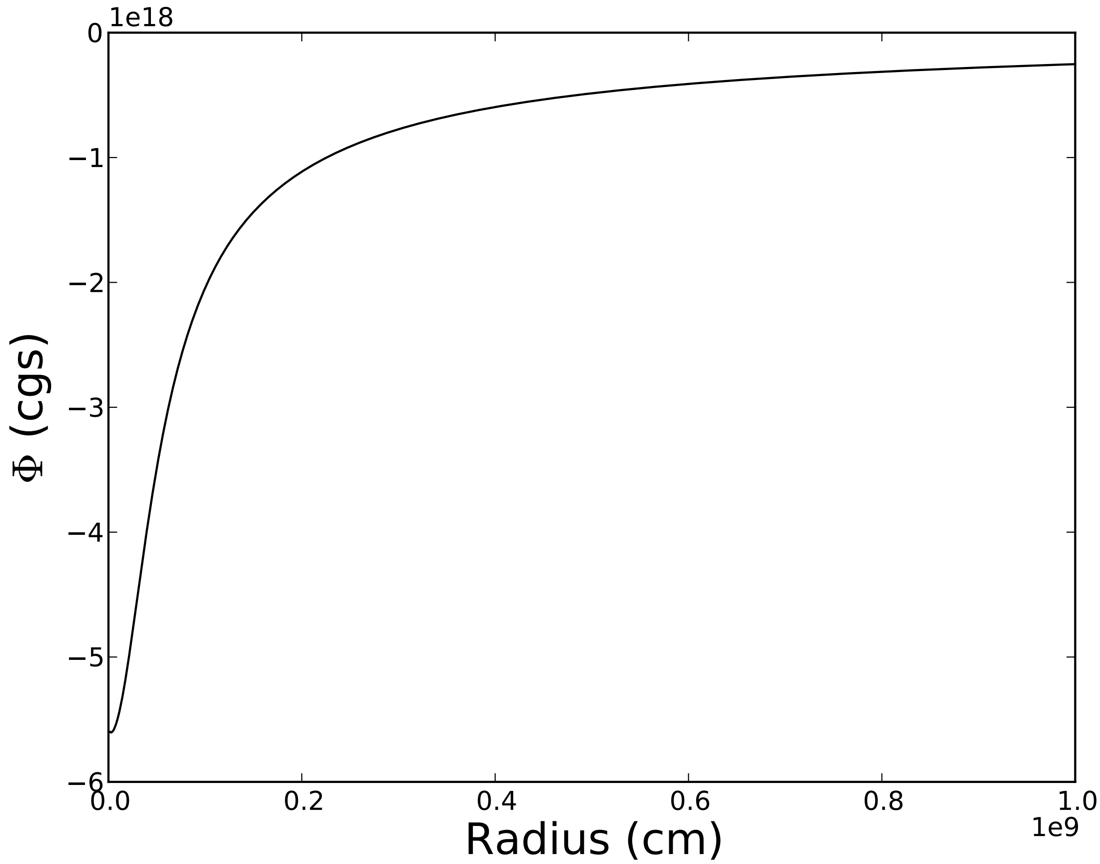

Figure 7: Gravitational potential for the presupernova model.