Ay190 – Worksheet 16
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I worked with Daniel DeFellipis.

1 Problem 1

I downloaded and completed the code template. The code works by first defining a class containing all the relevant variables for the problem, making it easier to pass all these variables to other functions and update them. Next, the grid and initial conditions are set up, with x between -0.5 and 0.5:

$$\rho(x) = \begin{cases} 10 & x \leq 0 \\ 1 & x > 0 \end{cases} \tag{1}$$

$$\epsilon(x) = \begin{cases} 2.5 & x \leq 0 \\ 1.795 & x > 0 \end{cases} \tag{2}$$

Pressure is calculated using

$$P = (\gamma - 1)\rho\epsilon \tag{3}$$

and initial velocities are set to 0.

Next, a timestep is calculated that insurances stability everywhere on the grid, as in Worksheet 15. Then, the *primitive variables*, ρ, v, and ϵ, are converted to the *conserved variables*, ρ, ρv, and $\rho\epsilon + \frac{1}{2}\rho v^2$. The initial density profile is plotted, and the code enters a loop to run the simulation.

Each iteration starts by plotting the current density and calculating a new timestep. The conserved variables, q, are updated in two stages in an RK2-like scheme. First, they are updated by a half time-step:

$$q_{intermediate} = q_{old} + \frac{1}{2}\mathrm{d}t k_1 \tag{4}$$

where k_1 is given by the RHS of the Euler fluid equations. Values of the primitive variables are interpolated to the edges of each grid cell. These are used to calculate the net flux of the conserved variables across each cell interface. The result is assigned to k_1. q_{new} is then converted to intermediate primitive variables, and boundary conditions are applied. The intermediate primitive variables are used in the RHS of the Euler equations to calculate k_2. New conserved variables are then calculated using

$$q_{new} = q_{old} + \mathrm{d}t(\frac{1}{2}k_1 + \frac{1}{2}k_2) \tag{5}$$

The new conserved variables are converted to new primitive variables and the boundary conditions are again applied. Time is updated by the timestep, and the loop repeats.

The results of the simulation are shown in Figures 1-9.

2 Problem 2

I modified the code to use piecewise constant, TVD-minmod, and TVD-MC2 reconstruction. The results are shown in Figure 10 at $t = 0.2$. Surprisingly, all three methods give the same solution.

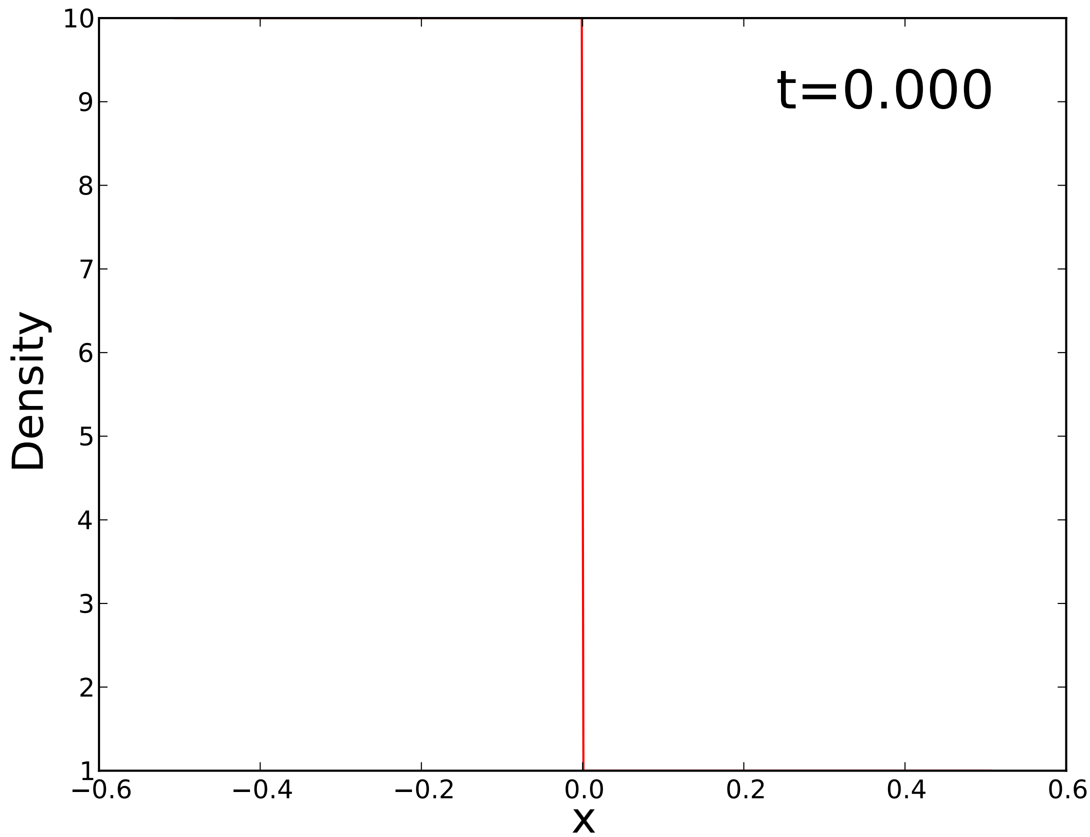

Figure 1: Initial condition for density.

3 Problem 3

Figure 11-14 show density, velocity, pressure, and energy at $t = 0.2$ for the current code and my Worksheet 15 results, modified to use the same number of SPH particles as Worksheet 16 gridpoints. I never ran the exact solution for Worksheet 15, but comparing my Worksheet 16 results with my answer for that part of Worksheet 15, the current results are much less attractive and much worse at soccer.

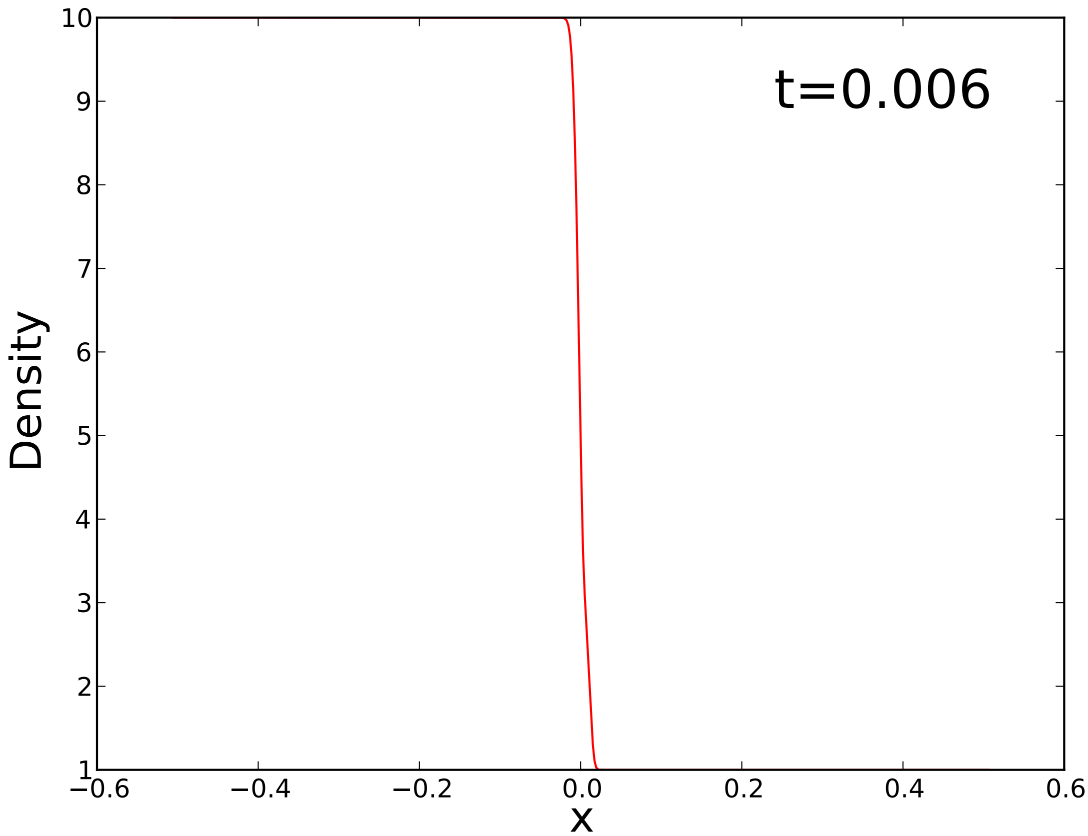

Figure 2: Density at $t = 0.006$. The density discontinuity has started to spread into a more gradual change.

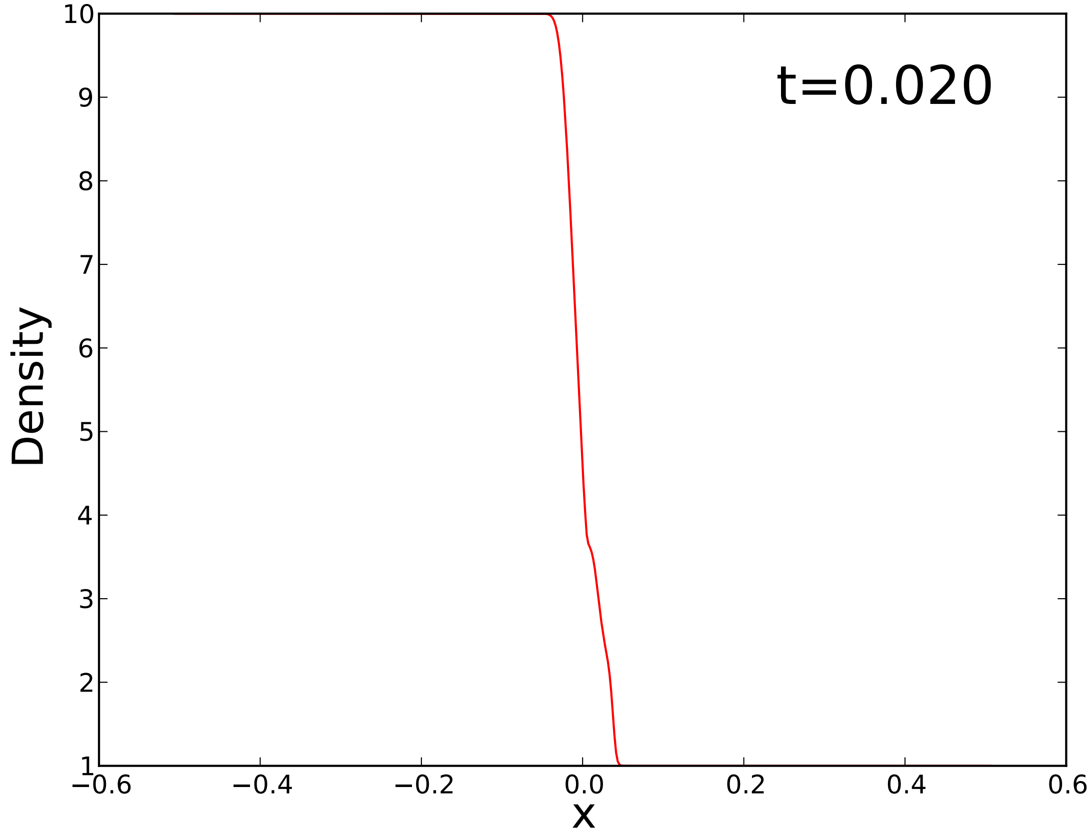

Figure 3: Density at $t = 0.020$. The step-like part of the Reimann problem solution is starting to develop in the density transition region.

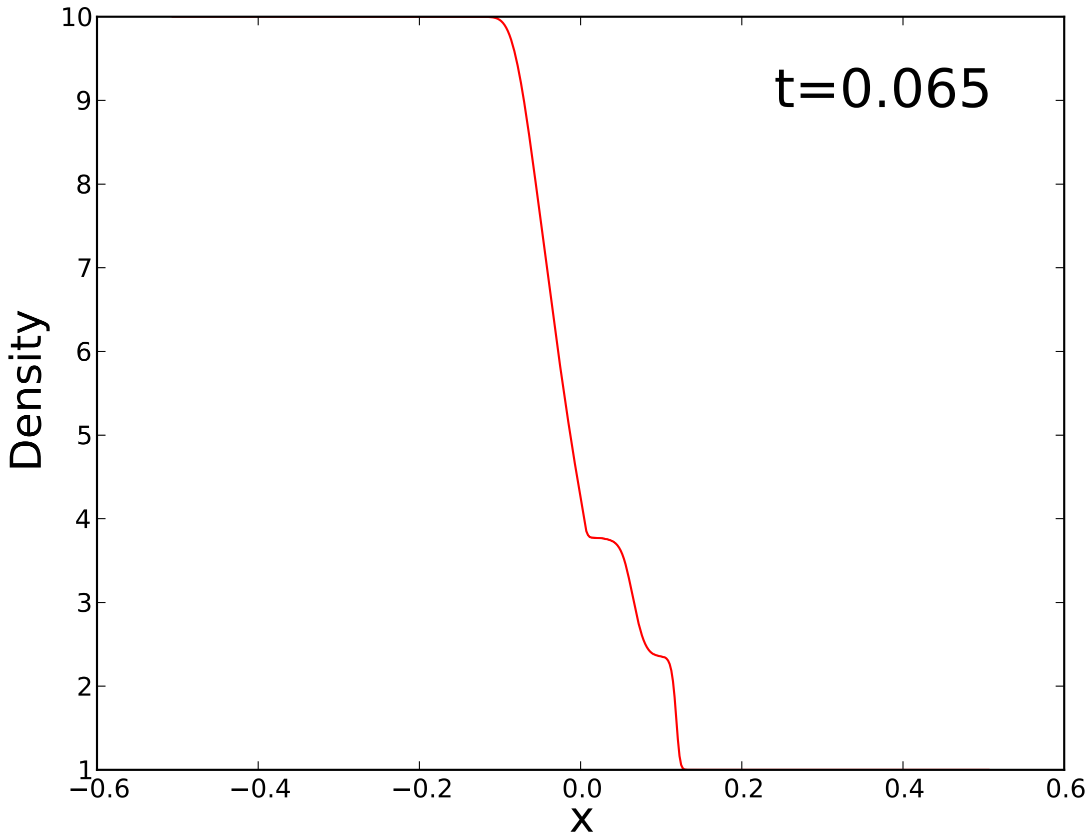

Figure 4: Density at $t = 0.065$. The step-like structure at the contact and shock is now visible.

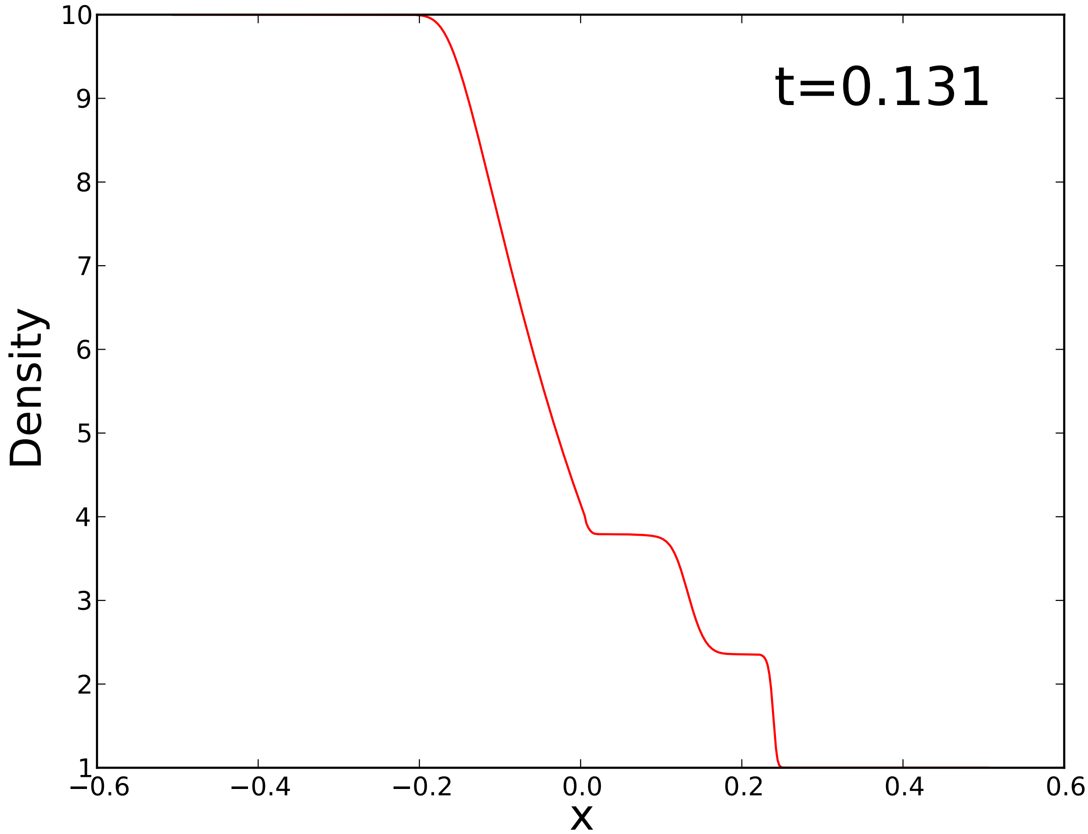

Figure 5: Density at $t = 0.131$. The transition region has expanded and the contact and shock continue to develop.

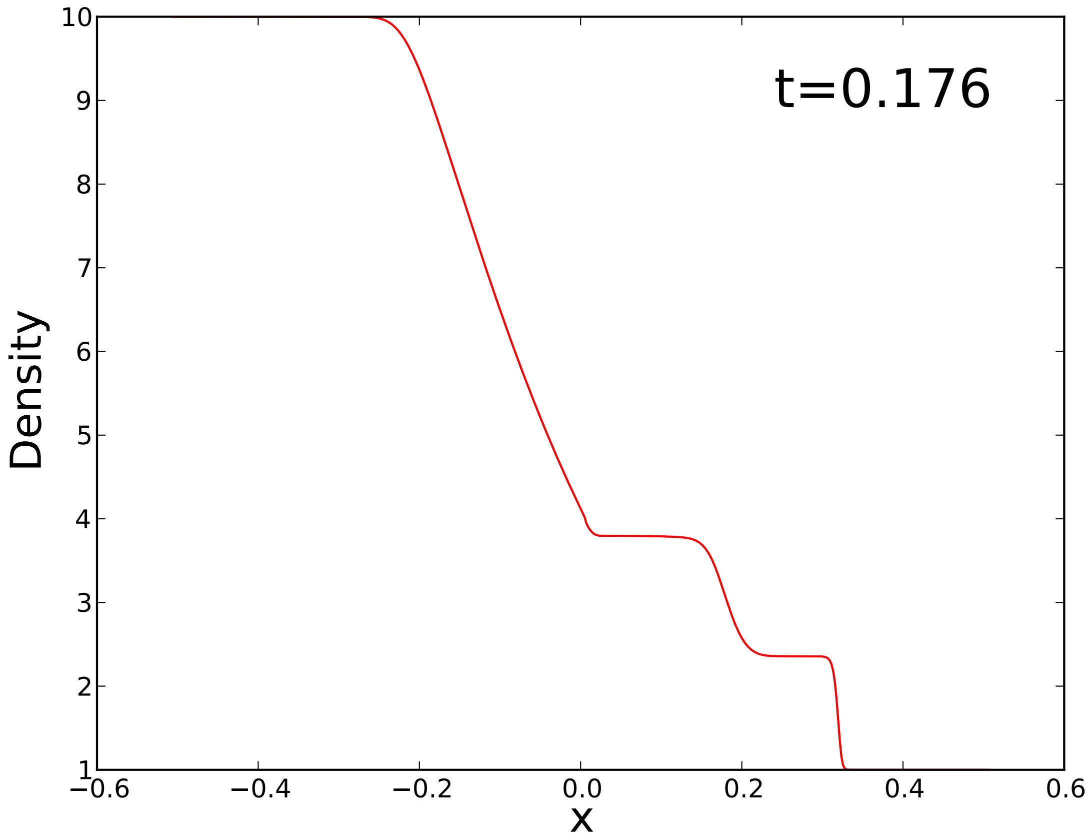

Figure 6: Density at $t = 0.176$. The solution continues to expand.

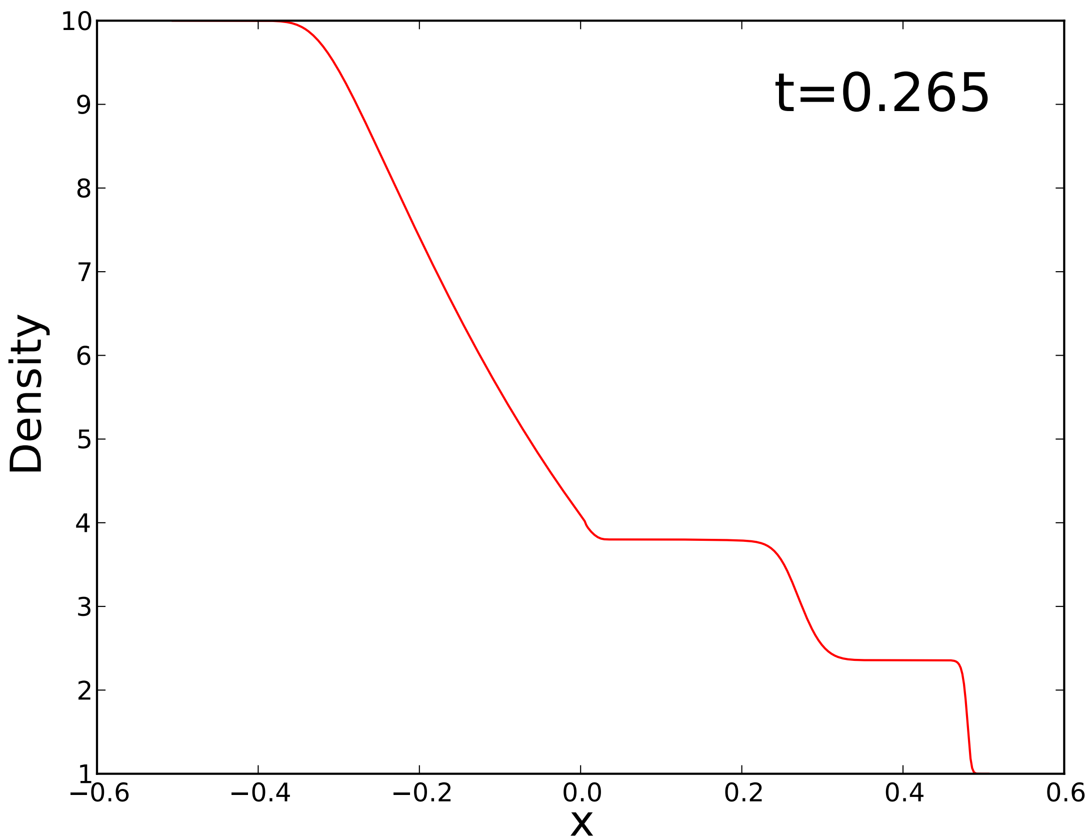

Figure 7: Density at $t = 0.265$. The rarefaction, contact, and shock are all clearly visible. The shock is approaching the right edge of the grid.

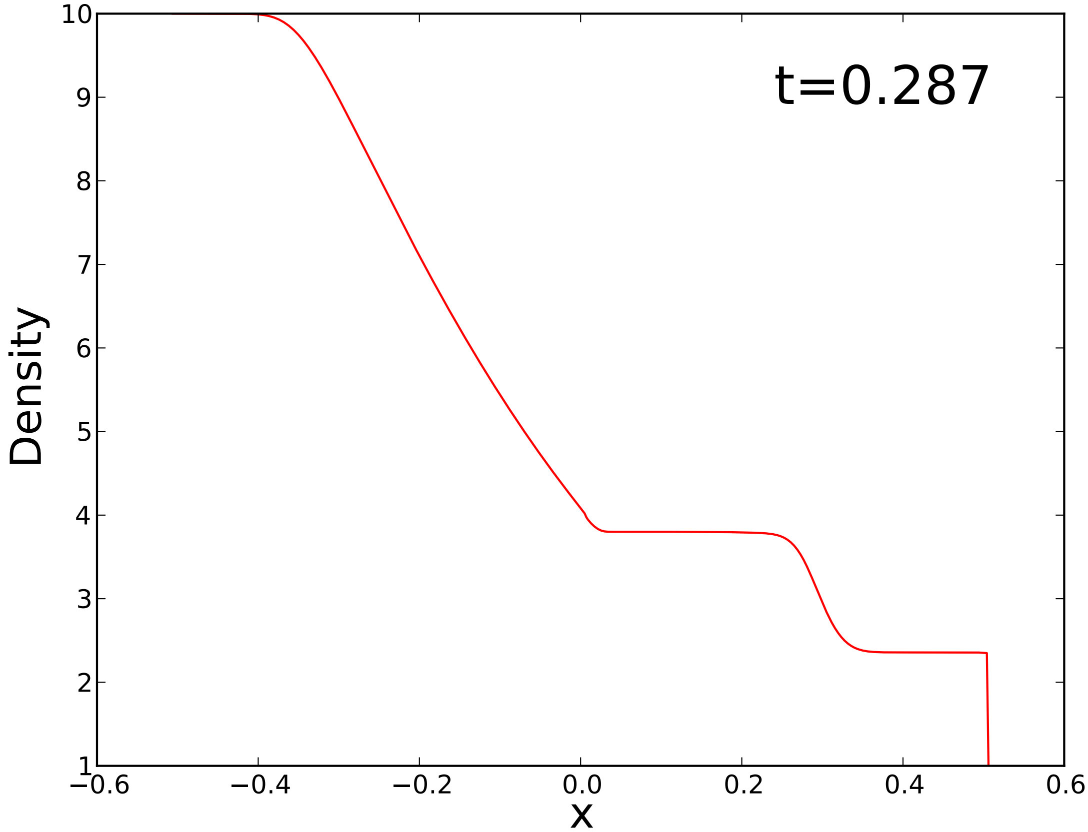

Figure 8: Density at $t = 0.287$. The shock has left the edge of the grid. The boundary conditions cause the sharp drop in density to 1.

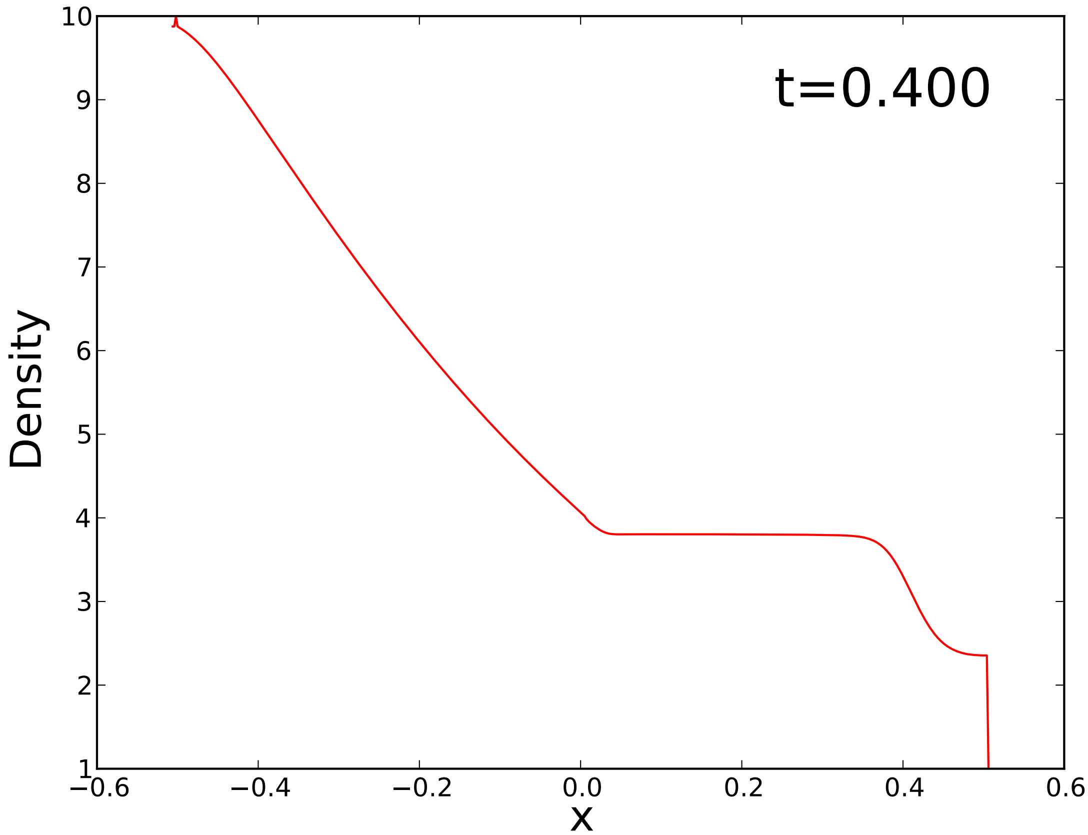

Figure 9: Density at $t = 0.400$, when the simulation ends. The shock is now completely off the grid and the contact is approaching the edge. On the left, the rarefaction has begun to leave the grid.

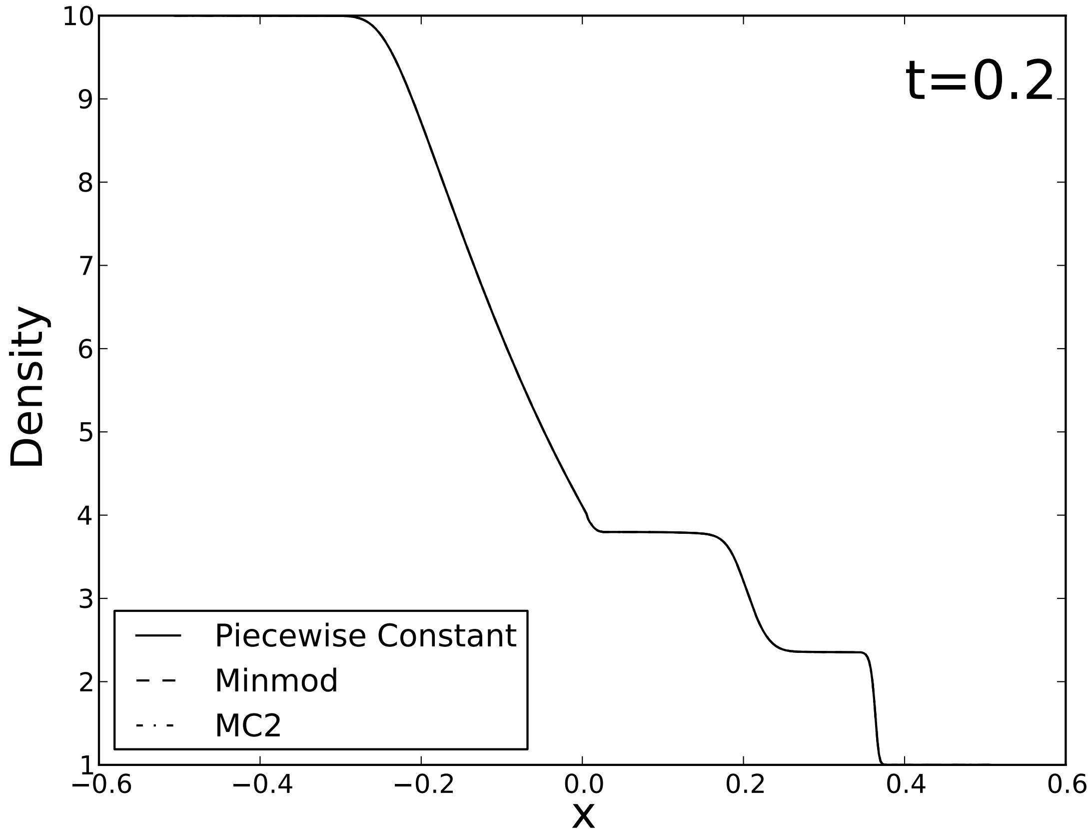

Figure 10: Results at $t = 0.2$ using piecewise constant (solid curve), TVD-minmod (dashed curve), and TVD-MC2 (dash-dotted curve).

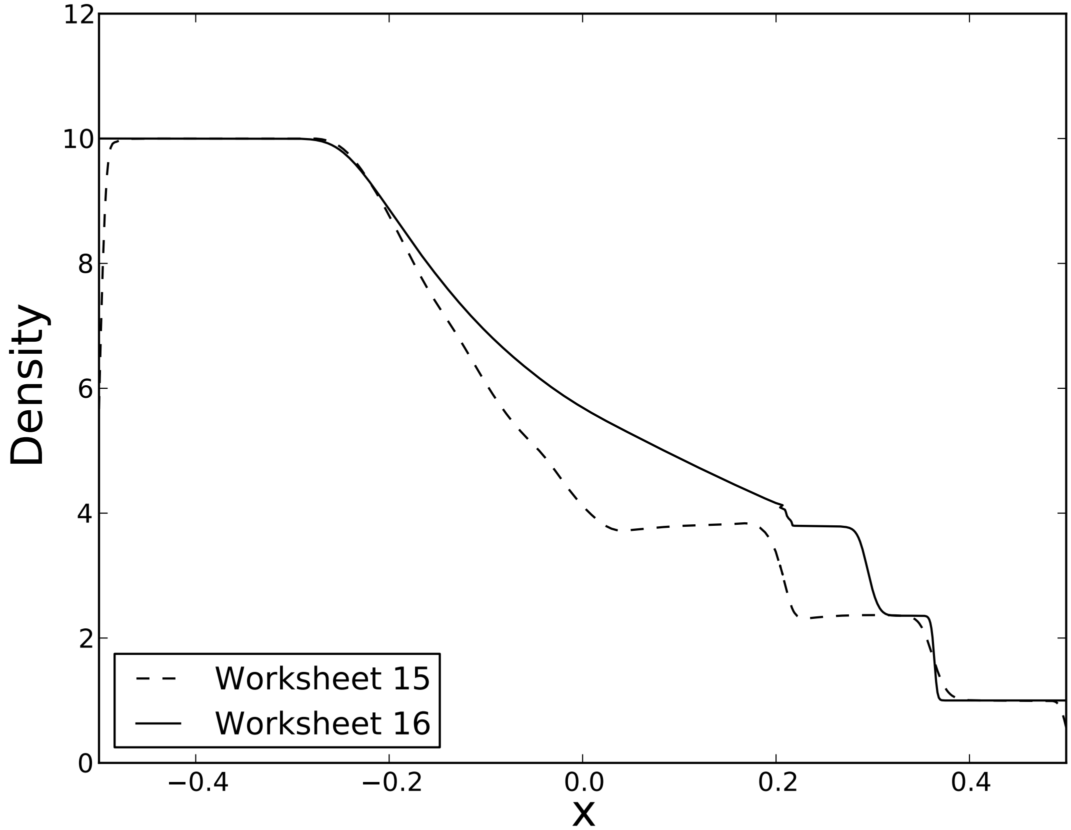

Figure 11: Density at $t = 0.2$ for Worksheet 15 (dashed curve) and Worksheet 16 (solid curve).

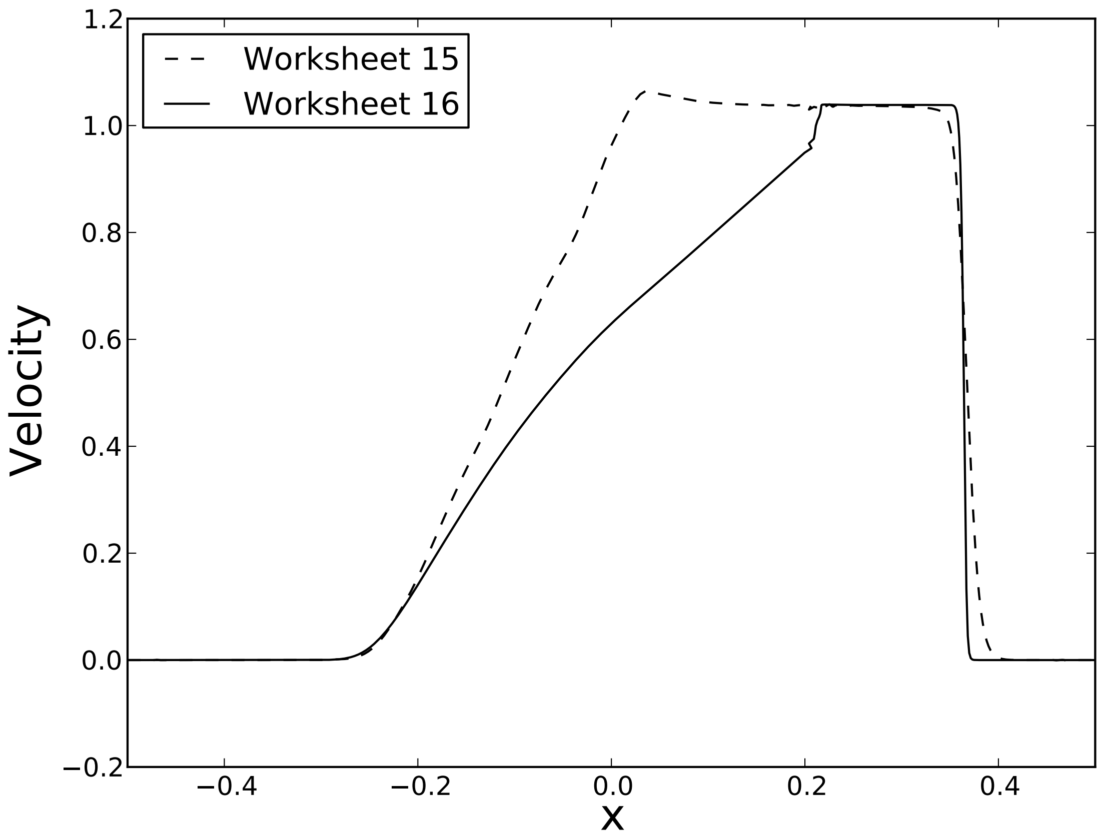

Figure 12: Velocity at $t = 0.2$ for Worksheet 15 (dashed curve) and Worksheet 16 (solid curve).

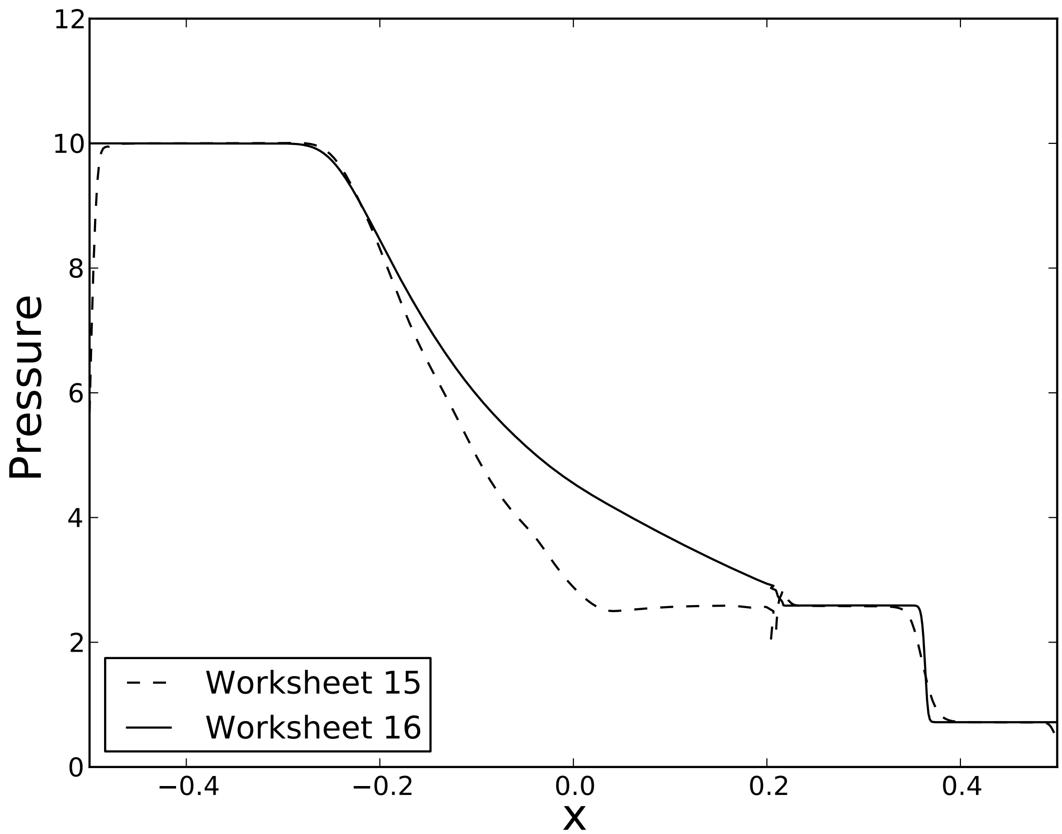

Figure 13: Pressure at $t = 0.2$ for Worksheet 15 (dashed curve) and Worksheet 16 (solid curve).

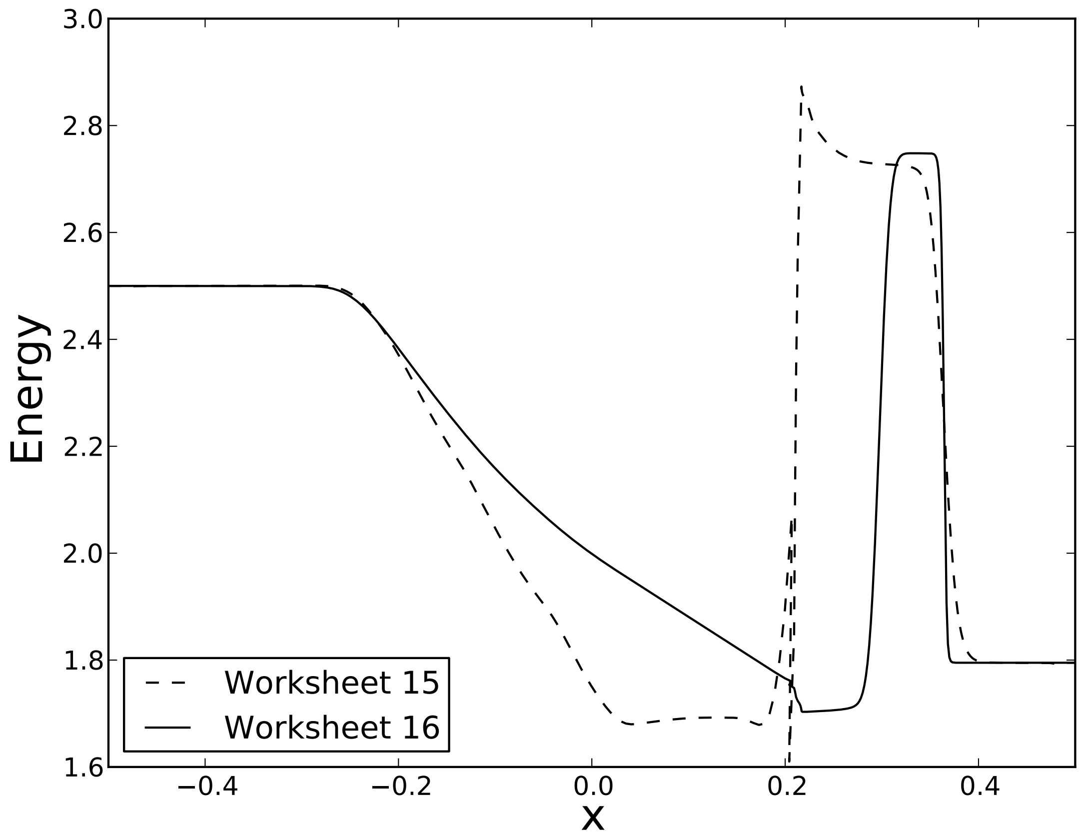

Figure 14: Energy at $t = 0.2$ for Worksheet 15 (dashed curve) and Worksheet 16 (solid curve).