Ay190 – Worksheet 8
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1 Problem 1

This code is designed to solve the equations of stellar structure:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM(r)}{r^2}\rho \tag{1}$$

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi\rho r^2 \tag{2}$$

for a white dwarf, along with the equation of state

$$P = K\rho^\Gamma \tag{3}$$

The skeleton code first defines the constants relative to the problem- $M_\odot$, G, Γ, and K. It next sets up the functions that will be used to implement the code, `tov_RHS()` and `tov_integrate_FE()` (more on these later). The implementation of the code starts with setting up the radius grid of 1000 points out to 2000 km, as well as the spacing dr. The M, P, and ρ arrays are also created with their central values. The code then loops through the radius grid and populates the M and P arrays at each point using the `tov_integrate_FE()`. This function takes the current radius, mass, pressure, denstity, and radius step size, and calculates the values at the next point by adding a value to the old point value. This value is calculated (depending on the scheme) by the `tov_RHS()` function. For example, in the forward Euler scheme, `tov_integrate_FE()` would calculate

$$y_{n+1} = y_n + \mathrm{d}r f(r, y_n) \tag{4}$$

where $y = [P, M]$ and $f(r, y_n)$ is the RHS of Equations 1 and 2. The value of d$rf(r, y_n)$ is calculated by `tov_RHS()`. At the end of each step, the density is calculated using Equation 3.

This process is repeated until the surface of the star is reached, which is defined by a threshold pressure of 10^{-10} times the central value. When the pressure threshold is crossed the first time, this r is defined as the surface. Any time later the threshold is crossed, the surface has already been found, so M,P, and ρ are set to their surface values. The program then prints the total mass and radius.

2 Problem 2

I filled in the missing parts of the code to implement the proceedure described in Section 1. I find $M = 1.45\ M_\odot$ and $R = 1502$ km.

3 Problem 3

I added an option to the code to select the scheme to be used- Euler or RK2. For the RK2 scheme, I used the equations in the notes to update the mass and pressure:

$$y_{n+1} = y_n + k_2 \tag{5}$$

where

$$k_2 = hf(r_n + \frac{1}{2}\mathrm{d}r, y_n + \frac{1}{2}k_1) \tag{6}$$

and

$$k_1 = \mathrm{d}rf(r_n, y_n) \tag{7}$$

Here, y_n is an array representing the pressure and mass. The density is calculated using the equation of state and the pressure at $r + \frac{1}{2}\mathrm{d}r$. With this method, I find $M = 1.46\ M_\odot$ and $R = 1538$ km. To test convergence, I ran both schemes with N=500, 1000, and 2000 points. The results are shown in Tables 1 and 2. I used the self-convergence formulas from the notes:

$$Q_S = \frac{h_3^n - h_2^n}{h_2^n - h_1^n} \tag{8}$$

$$Q_S = \frac{|y(h_3) - y(h_2)|}{|y(h_2) - y(h_1)|} \tag{9}$$

For forward Euler, Equation 8 gives $Q_S = 0.5$. For RK2, $Q_S = 0.25$. The results of Equation 9 are given in Table 3. Both schemes show the expected self-convergence rate of $\mathrm{d}r$ for Euler and $\mathrm{d}r^2$ for RK2.

Table 1: Forward Euler Self-Convergence

N	M	R
500	1.444090	1466.933
1000	1.450694	1501.502
2000	1.454043	1519.760

Table 2: RK2 Self-Convergence

N	M	R
500	1.457693	1535.070
1000	1.457490	1537.538
2000	1.457440	1536.768

Table 3: Self-Convergence Rates

	M	R
Euler Q_S	0.507	0.528
RK2 Q_S	0.246	0.312

4 Problem 4

Figure 1 shows pressure, density, and mass as a function of radius using RK2 with 2000 gridpoints. Pressure and density are scaled to their central values of 1.06×10^{28} dynes/cm^2 and 10^8 g/cm^3.

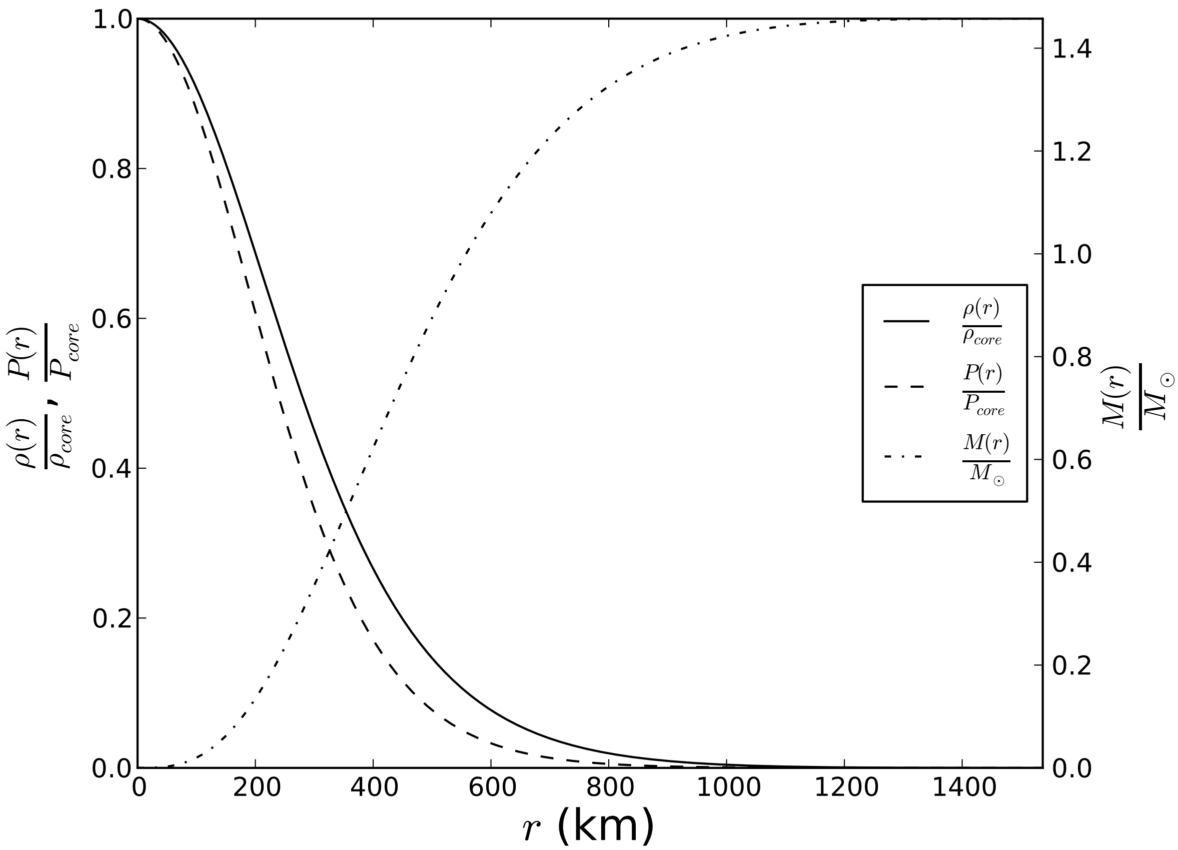

Figure 1: Density, pressure, and mass vs. radius for the model white dwarf.