Ay190 – Worksheet 4
Scott Barenfeld
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I worked with Daniel DeFelippis.

1 Problem 1

1.1 Part a

To find the eccentric anomaly, E, I used

$$E - \omega t - e \sin E \tag{1}$$

where $\omega = 2\pi/T$ is the angular frequency, T is the period, e is the eccentricity, and t is the day of the orbit. I used Newton's method. For the Earth, $\omega \approx 0.01$, so for t of around 100, $\omega t \approx 1$. So for small e, $E = 1$ should be about right, so I used $E = 1$ as my initial guess. To find the x and y positions of the Earth, I use the equations

$$x = a \cos E \tag{2}$$

and

$$y = b \sin E, \tag{3}$$

where $a = 1$ AU and $b = a\sqrt{1-e^2}$.

My results are summarized in Table 1. For all three values of t, the solution converged in 4 steps. To make sure my code still works for bad initial guesses, I tried an initial guess of 100. My solution (Table 2) converged to the same values, this time in 5 steps.

Table 1: Newton's Method for Initial Guess of 1

t (days)	E	x (AU)	y (AU)
91	1.58	-0.01	1.00
182	3.13	-1.00	0.01
273	4.68	-0.03	-1.00

Table 2: Newton's Method for Initial Guess of 100

t (days)	E	x (AU)	y (AU)
91	1.58	-0.01	1.00
182	3.13	-1.00	0.01
273	4.68	-0.03	-1.00

1.2 Part b)

Let's say our good friends at the Johnson Space Center were tired of the local baseball team losing, so the decided to send the whole roster into orbit. The weight of all those losing seasons required a pretty big rocket, and unfortunately, the force of the rocket launch knocked the Earth onto a highly eccentric orbit. To see what would happen, I changed the eccentricity in my code to 0.99999. With an initial guess of $E = 2$, solving for $t = 91$ days and 182 days took 5 steps, while 273 days took 6 steps. Table 3 shows the results. For an initial guess of 100, E converged to the same values, but this time took 3139, 2276, and 115 steps for t of 91, 182, and 273 days. The high eccentricity makes convergence much slower, unless the initial guess is good. A good initial guess accelerates the convergence. Given the y-values of the orbit, we probably crash into the Sun on our way past it. So, getting back to the cause of all this, Houston, we have a problem.

Table 3: Newton's Method for $e = 0.99999$

t (days)	E	x (AU)	y (AU)
91	2.31	-0.67	3.32×10^{-3}
182	3.14	-1.00	2.42×10^{-5}
273	3.96	-0.68	-3.27×10^{-3}