Ay190 – Worksheet 10
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1 Problem 1

I used the shooting method (NRA approved?) to solve the diffential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 12x - 4,\ y(0) = 0,\ y(1) = 0.1 \tag{1}$$

This equation can be rewritten as two first order equations:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = u(x)\frac{\mathrm{d}u}{\mathrm{d}x} = 12x - 4 \tag{2}$$

which can be solved by guessing a value for $y'(0)$ and integrating numerically to find a solution $y_n(x)$. This can be done iteratively using root finding on $y_n(1) - y(1) = 0$ to satisfy the boundary condition at $x = 1$. I filled in the skeloton code provided to implement this process.

Figure 1 shows my solution using forward-Euler integration and 100 gridpoints, along with the true solution of

$$y(x) = 2x^3 - 2x^2 + 0.1x \tag{3}$$

Figure 2 shows the same for 1000 gridpoints. The average error from the true solution over the interval is 0.01 for 100 gridpoints and is 0.001 for 1000 gridpoints. This shows the expected convergence rate- the global error or forward-Euler goes as h, so cutting h by a factor of 10 should reduce the error by a factor of 10.

I also solved the problem using an RK2 integrator. The result is shown in Figure 3. In this case, because this is such a simple problem, the numerical solution is exact. This is because in RK2 integration, the error term is proportional to the third derivative of the function being solved for. In the case of u, this is 0, so u can be solved for exactly. u can then be used to solve for y exactly. Also, $y_n(1) - y(1)$ is linear as a function of the guess for $y'(0)$, so root-finding by Newton's method is exact. Hence, the final solution for y is exact.

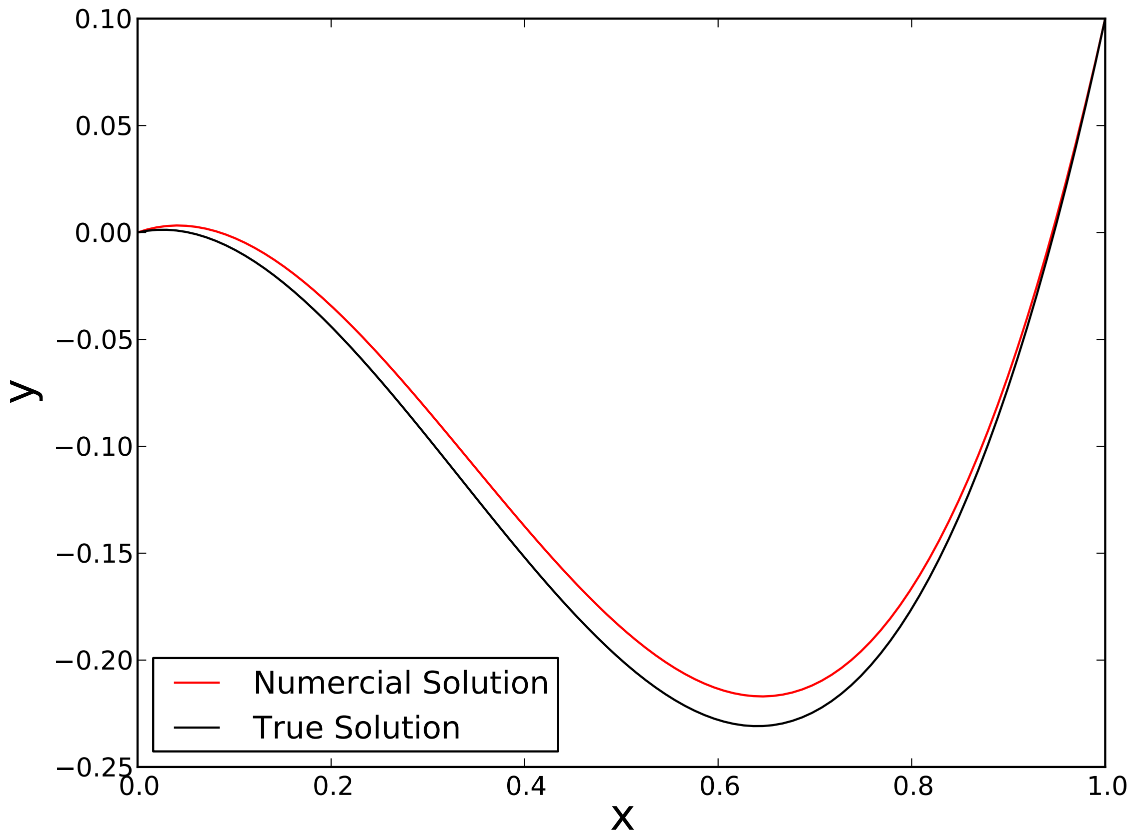

Figure 1: Solution to Equation 1 using forward-Euler integration and 100 gridpoints.

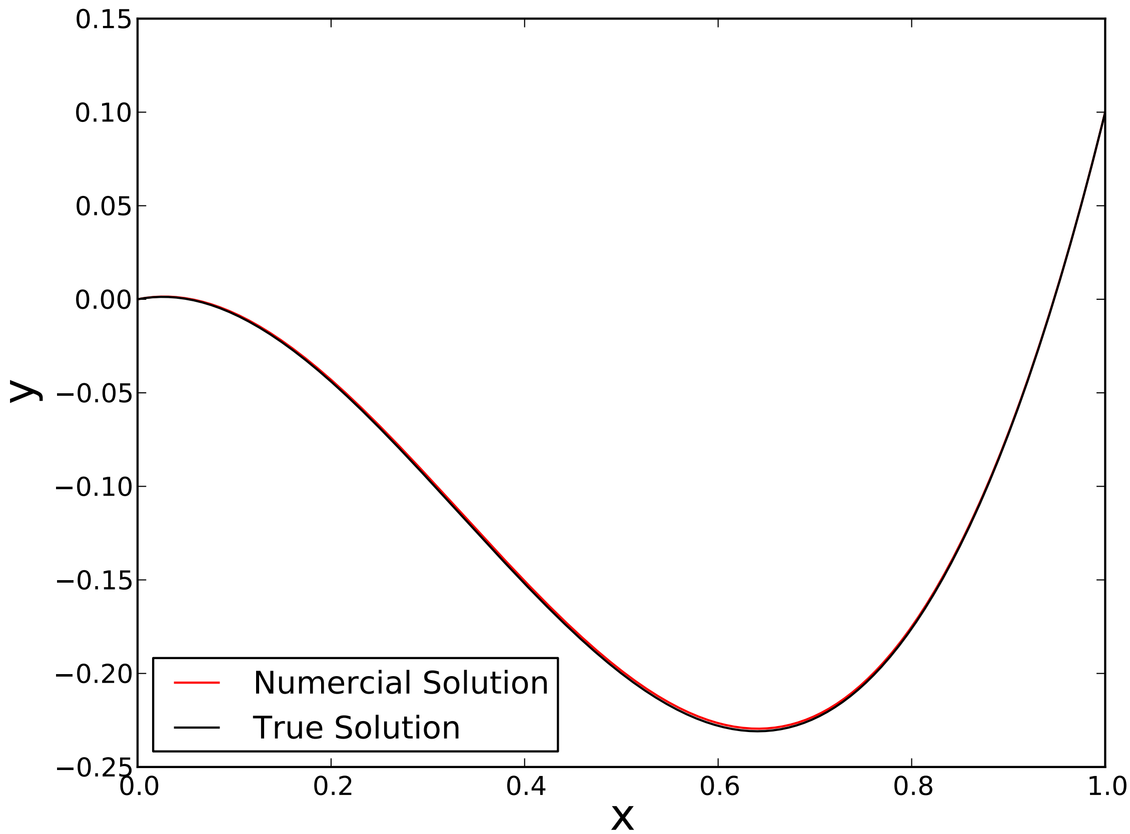

Figure 2: Solution to Equation 1 using forward-Euler integration and 1000 gridpoints.

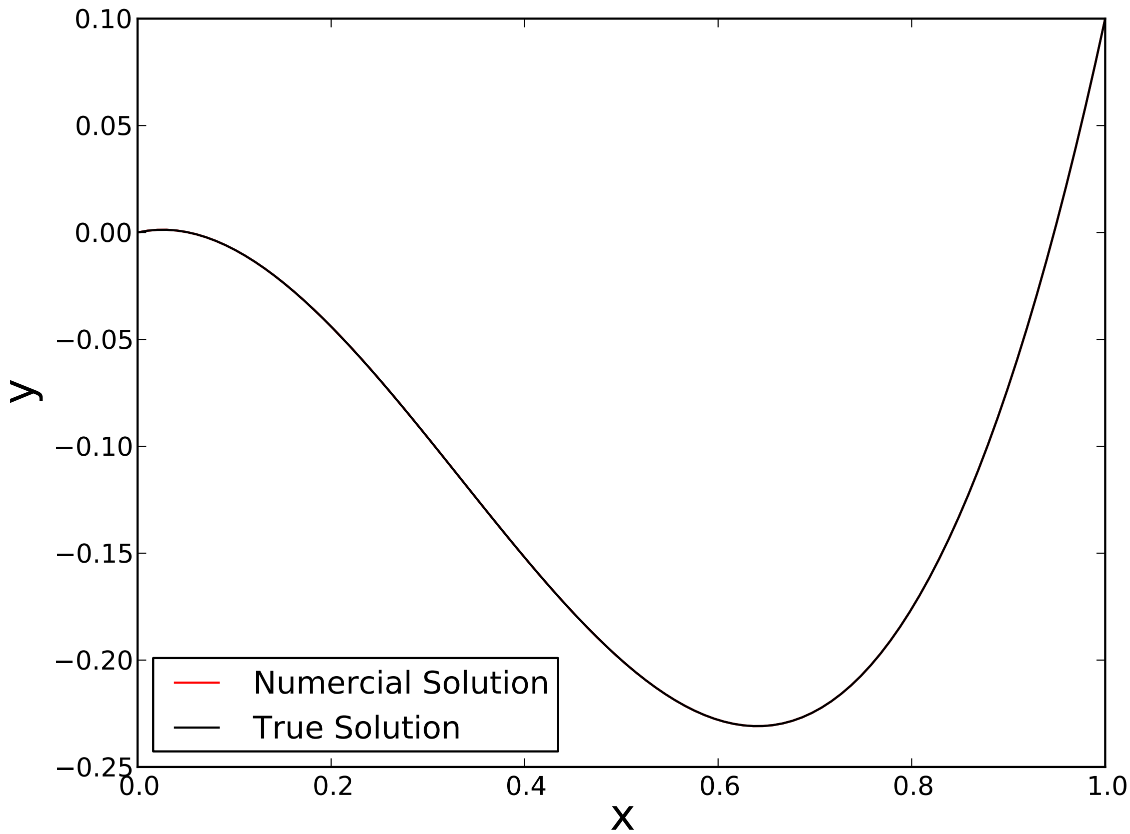

Figure 3: Solution to Equation 1 using RK2 integration and 100 gridpoints.