Ay190 – Worksheet 14
Scott Barenfeld
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I worked with Daniel DeFellipis.

1 Problem 1

I used a similar code to worksheet 11 to solve Burger's equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0 \tag{1}$$

with initial condition

$$u_0(x, t = 0) = \frac{1}{8}\sin(\frac{2\pi x}{L}) \tag{2}$$

and $L = 100$. Instead of a constant v as in the advection equation, I use u at the current x and t to update each gridpoint. Using an upwind scheme and forward Euler integration, I then solve Berger's equation. For stability, I need

$$0 \leq \frac{u\mathrm{d}t}{\mathrm{d}x} \leq 1 \tag{3}$$

dx is set to be 0.1, so if d$t = 0.5$, Equation 3 is true as long as $0 \leq u \leq 0.2$. This is always the case for positive values of u, but the negative values do not meet this condition. I update the negative values using the downwind scheme, which requires

$$-1 \leq \frac{u\mathrm{d}t}{\mathrm{d}x} \leq 0 \tag{4}$$

This condition is met for $|u| \leq 0.2$, so the overall scheme is stable everywhere. My results are shown in Figures 1-5. The positive and negative values of u flow toward each other, and a shock at the boundary begins to form at around $t = 140$. By the end of the simulation at $t = 249$, the shock is clearly visible.

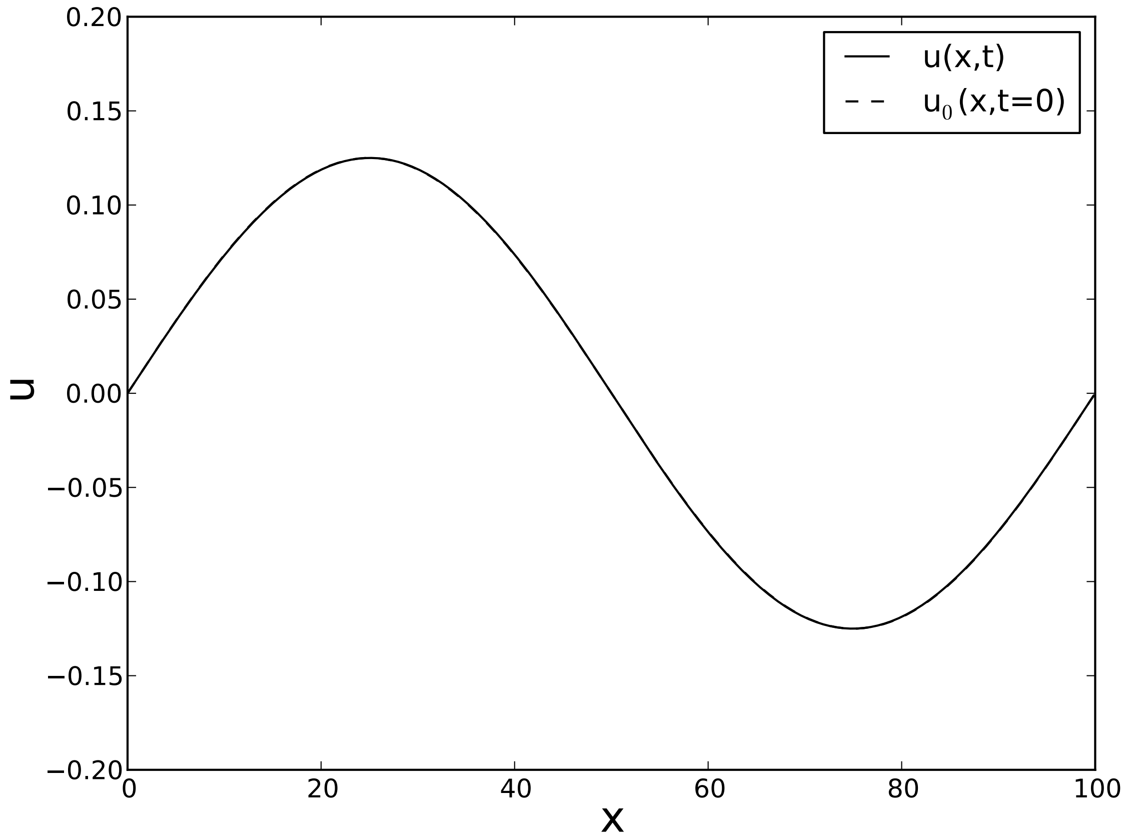

Figure 1: Initial condition for $u(x,t)$.

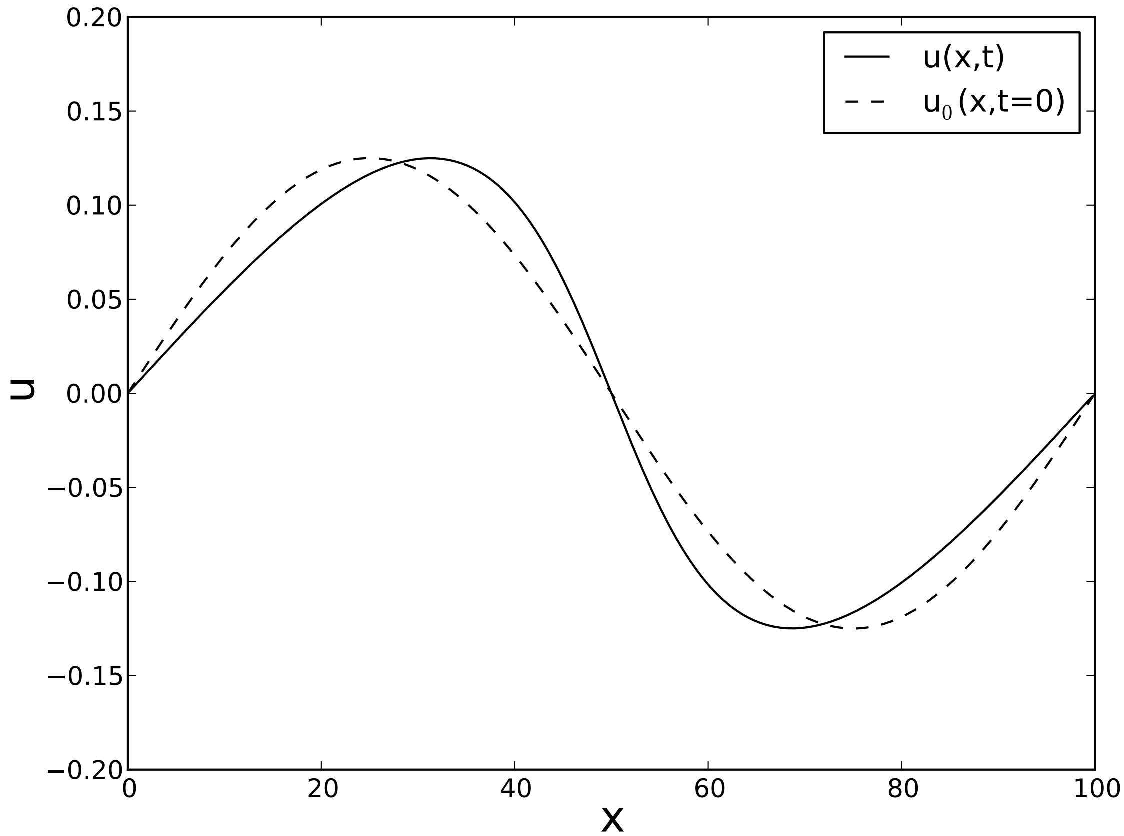

Figure 2: $u(x, t = 50)$. u is starting to collect in the center relative to it's initial form.

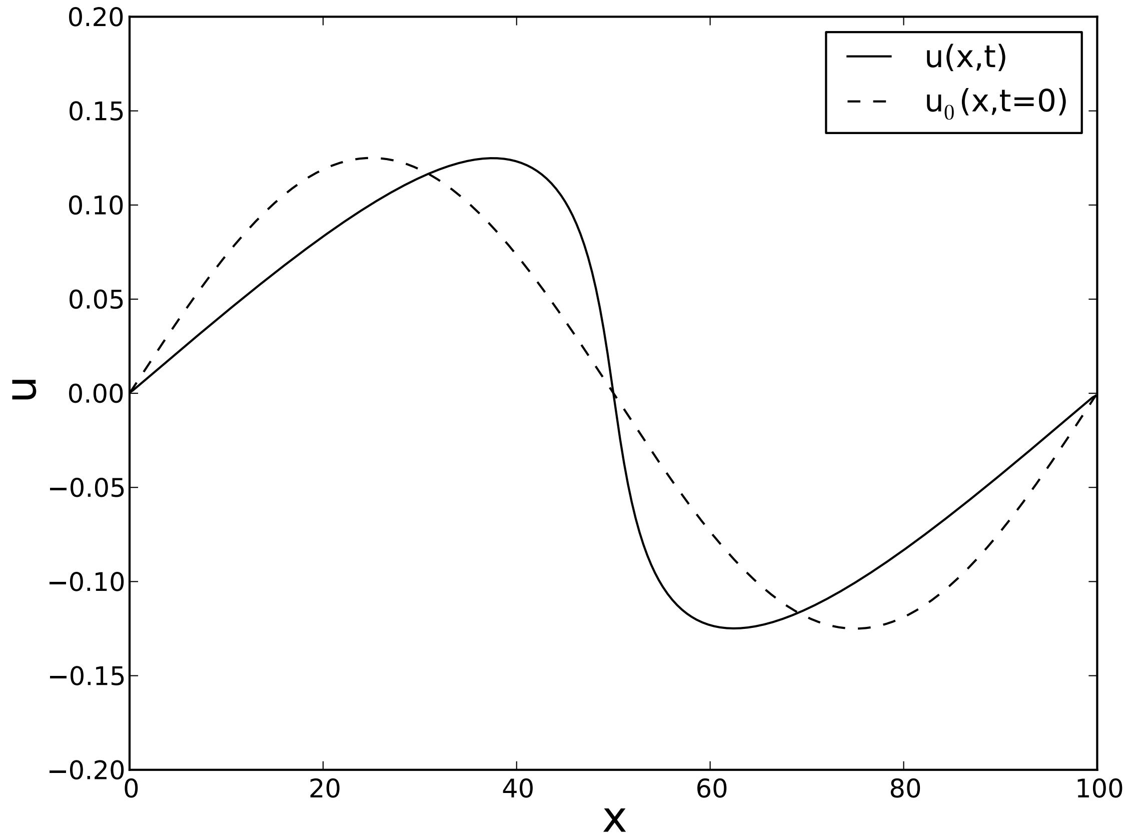

Figure 3: $u(x, t = 100)$. u continues to collect in the center. No shock is visible yet.

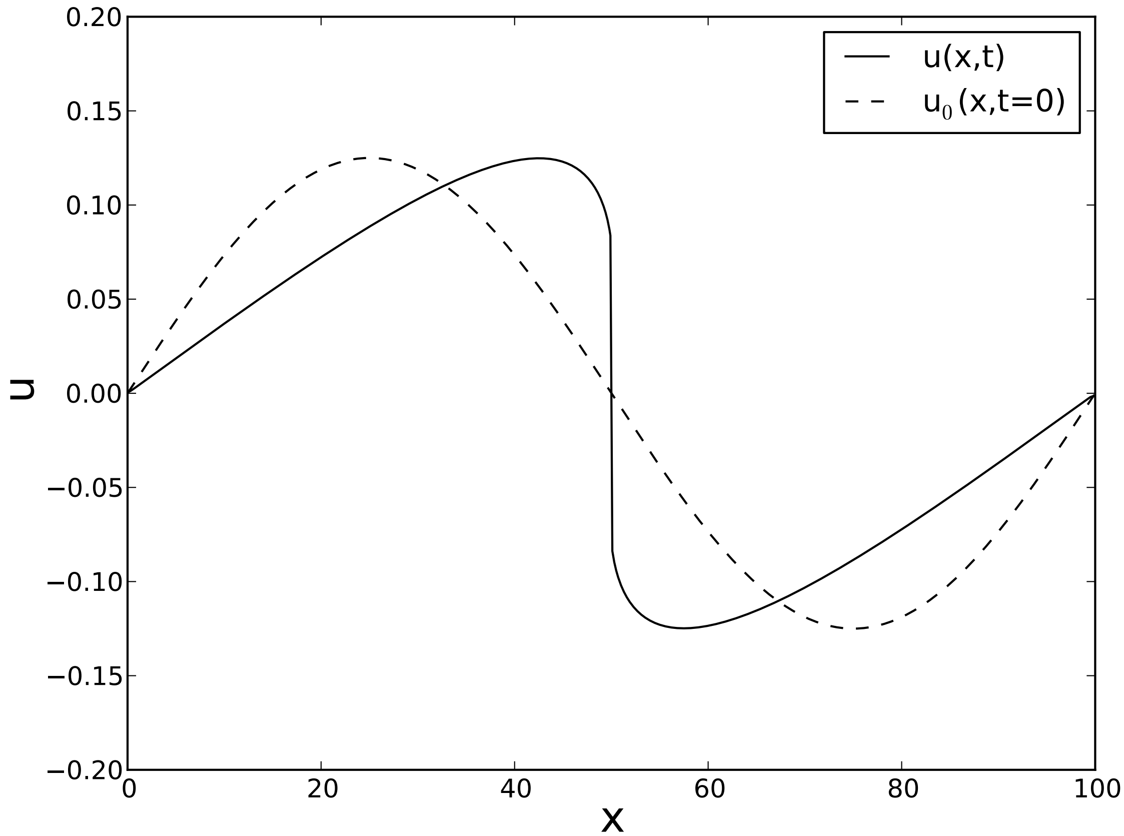

Figure 4: $u(x, t = 140)$. A shock has formed in the center.

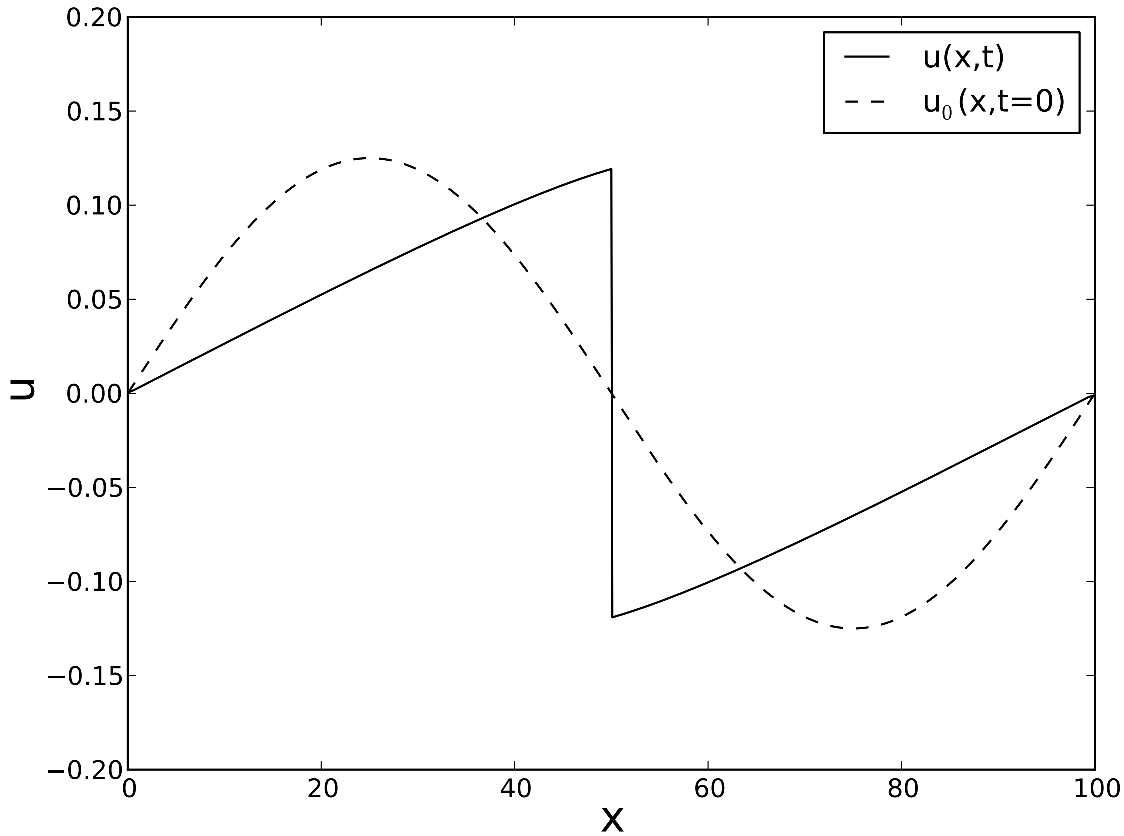

Figure 5: $u(x, t = 249)$. By the end of the simulation, a shock is clearly visible.