Ay190 – Worksheet 5
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1 Problem 1

1.1 Part a)

See Figure 1 for plot.

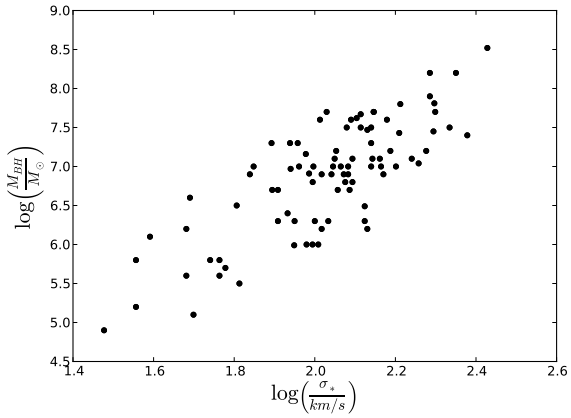

Figure 1: The M-σ relation, using the data of [1]

1.2 Part b)

To perform the fit of $\log\left(\frac{M_B H}{M_\odot}\right) = a_1 + a_2 \log\left(\frac{\sigma}{\mathrm{km/s}}\right)$, I used the equations from the notes:

$$a_1 = \frac{\Sigma y \Sigma x^2 - \Sigma x \Sigma x \Sigma y}{S\Sigma x^2 - (\Sigma x)^2} \tag{1}$$

$$a_2 = \frac{S\Sigma xy - \Sigma y \Sigma x}{S\Sigma x^2 - (\Sigma x)^2} \tag{2}$$

where

$$\begin{gathered} S = \sum_{i=1}^{N} \frac{1}{\sigma_i^2},\ \Sigma x = \sum_{i=1}^{N} \frac{x_i}{\sigma_i^2},\ \Sigma y = \sum_{i=1}^{N} \frac{y_i}{\sigma_i^2} \\ \Sigma x^2 = \sum_{i=1}^{N} \frac{x_i^2}{\sigma_i^2},\ \Sigma xy = \sum_{i=1}^{N} \frac{x_i y_i}{\sigma_i^2} \end{gathered} \tag{3}$$

For this fit with no uncertainties, the σ's are ignored (set equal to 1). The results are shown with the fit of [1] in Figure 2. I find $\log M_{BH} = 0.93 + 2.93 \log \sigma$.

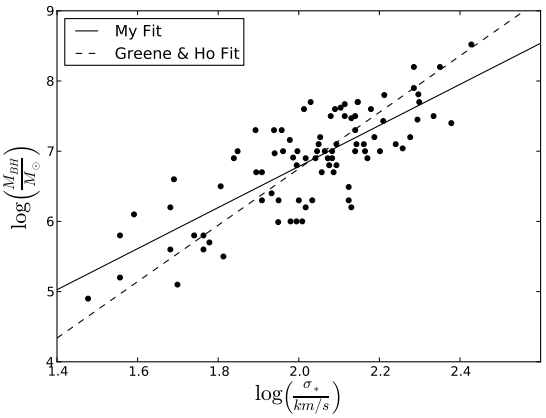

Figure 2: A linear fit to the M-σ relation, compared to a similar fit by [1].

1.3 Part c)

To find the uncertainty in $\log\sigma$, I propogated the given σ uncertainties.

$$\Delta \log\sigma = \frac{\mathrm{d}\log\sigma}{\mathrm{d}\sigma} = \frac{\Delta\sigma}{\sigma \ln 10} \tag{4}$$

I turned this into a $\log M_{BH}$ uncertainty using

$$\Delta \log M_{BH} = \frac{\mathrm{d}\log M_{BH}}{\mathrm{d}\log\sigma} \Delta \log sigma \tag{5}$$

where $\frac{\mathrm{d}\log M_{BH}}{\mathrm{d}\log\sigma}$ is given by the fit from part b)-

$$\frac{\mathrm{d}\log M_{BH}}{\mathrm{d}\log\sigma} = a_2 \tag{6}$$

For the $\log M_{BH}$ uncertainties, when two assymetric uncertainties were given, I chose the larger value to be conservative. I then used the equations from part b) to solve for a_1 and a_2. I find $\log M_{BH} = 0.82 + 2.92\log\sigma$. The fit is shown in Figure 3.

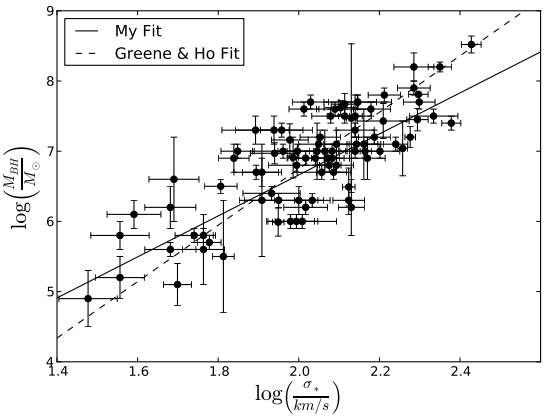

Figure 3: A linear fit to the M-σ relation, with uncertainties included. A similar fit by [1] is shown for comparison.

References

[1] J. E. Greene and L. C. Ho. The M_{BH}-σ_* Relation in Local Active Galaxies. *Astrophys. J. Lett.*, 641:L21–L24, April 2006.