Ay190 – Worksheet 2
Scott Barenfeld
Date: January 15, 2014

This week, I worked with Daniel DeFelippis and Donal O'Sullivan.

1 Problem 1

The values for the recurrence relation and exact relation are shown in Table 1. For $n = 15$, the absolute error is 3.66 and the relative error is 5.25×10^7.

Table 1: Problem 1 recurrence relation

Recurrence Relation	Exact Relation
1.00	1.00
0.33	0.33
0.11	0.11
3.70×10^{-2}	3.70×10^{-2}
1.23×10^{-2}	1.23×10^{-2}
4.12×10^{-3}	4.12×10^{-3}
1.39×10^{-3}	1.37×10^{-3}
5.13×10^{-4}	4.57×10^{-4}
3.76×10^{-4}	1.52×10^{-4}
9.44×10^{-4}	5.08×10^{-5}
3.59×10^{-3}	1.69×10^{-5}
1.43×10^{-2}	5.65×10^{-6}
5.72×10^{-2}	1.88×10^{-6}
0.23	6.27×10^{-7}
0.91	2.09×10^{-7}
3.66	6.97×10^{-8}

2 Problem 2

Figure 1 shows the absolute error in computing the derivative of

$$f(x) = x^3 - 5x^2 + x \tag{1}$$

using forward differencing. Figure 2 shows the absolute error if central differencing is used. The error in forward differencing goes as h, so halfing h should half the error. To show this, half the value of the $h = 0.1$ absolute is plotted in Figure 1 with crosses. It matches up almost exactly with the absolute error for $h = 0.05$, as it should. Similarly, Figure 2 shows $1/4$ of the $h = 0.1$ absolute error, since central differencing error goes as h^2. This also matches almost exactly.

Since both schemes really on surrounding points to calculate the derivative, the derivative is not calculated at $x = 6$ for forward differencing or for $x = -2$ and $x = 6$ for central differencing.

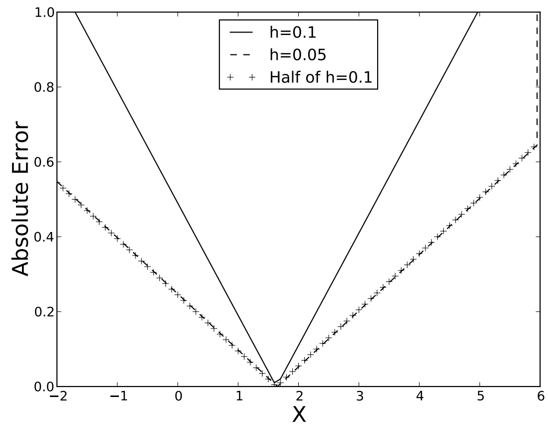

Figure 1: Absolute error of forward differencing. The solid line shows the absolute error for h=0.1. The dashed line shows absolute error for h=0.05. Half of the h=0.1 absolute error is plotted with crosses, as discussed in the text.

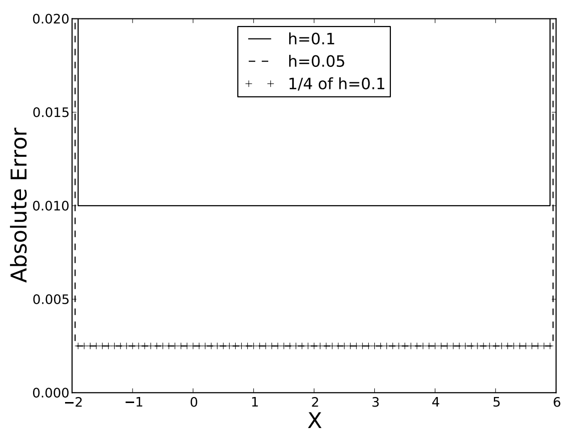

Figure 2: Absolute error of central differencing. The solid line shows the absolute error for h=0.1. The dashed line shows absolute error for h=0.05. One quarter of the h=0.1 absolute error is plotted with crosses, as discussed in the text.

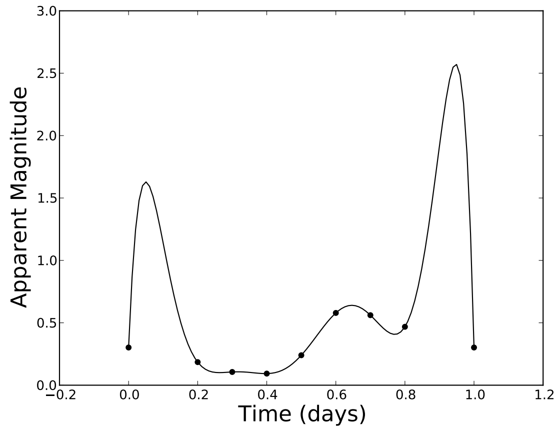

Figure 3: Lagrange interpolation of the Cepheid lightcurve. The points represent the data, while the solid curve represents the interpolated $p(x)$. The interpolation looks reasonable in the middle, but exhibits Runge's phenomenon at the edges of the data.

3 Problem 3

Start with the Taylor expansion of $f(x_0 + h)$:

$$f(x_0 + h) = f(x_0) + hf'(x_0) + \frac{h^2}{2} f''(x_0) + \frac{h^3}{6} f'''(x_0) + \mathcal{O}(h^4) \tag{2}$$

and of $f(x_0 - h)$:

$$f(x_0 - h) = f(x_0) - hf'(x_0) + \frac{h^2}{2} f''(x_0) - \frac{h^3}{6} f'''(x_0) + \mathcal{O}(h^4) \tag{3}$$

Adding these two equations gives:

$$f(x_0 + h) + f(x_0 - h) = 2f(x_0) + h^2 f''(x_0) + \mathcal{O}(h^4) \tag{4}$$

Solving for $f''(x_0)$ gives the *central difference* estimate for the second derivative:

$$\boxed{f''(x_0) = \frac{f(x_0 + h) + f(x_0 - h) - 2f(x_0)}{h^2} + \mathcal{O}(h^2)} \tag{5}$$

4 Problem 4

4.1 Part a

Using the formulae in the notes, I calculated $p(x)$ using Lagrange interpolation. The result is shown in Figure 3.

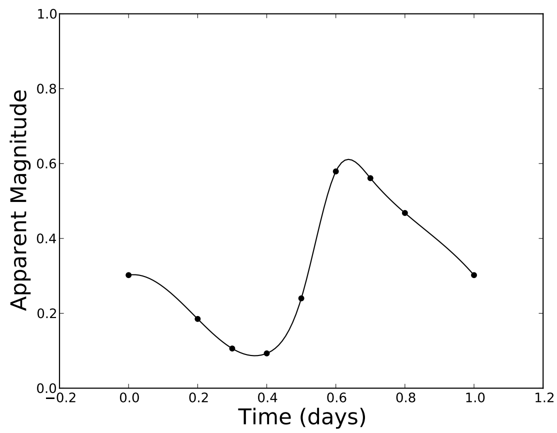

Figure 4: Spline interpolation of the Cepheid light curve. The points represent the data, while the solid curve represents the interpolated $p(x)$. The oscillations seen in the Lagrange interpolation are not present.

4.2 Part b

I did not have time to do this part, although I expect that the interpolation will be more accurate since piecewise interpolation using low degree polynomials avoids the large oscillations of Runge's phenomenon.

5 Problem Five

5.1 Part a

I also did not have time for this part. Again, since this is a piecewise interpolation, I expect the oscillations seen in the Lagrange interpolation to not be present.

5.2 Part b

I used Python's built in spline interpolator (scipy.interpolate.splrep) to do a natural cubic spline interpolation. The result is shown Figure 4. Like the other piecewise methods, there are no large oscillations. Figure 5 shows the Lagrange and spline interpolations together for comparison.

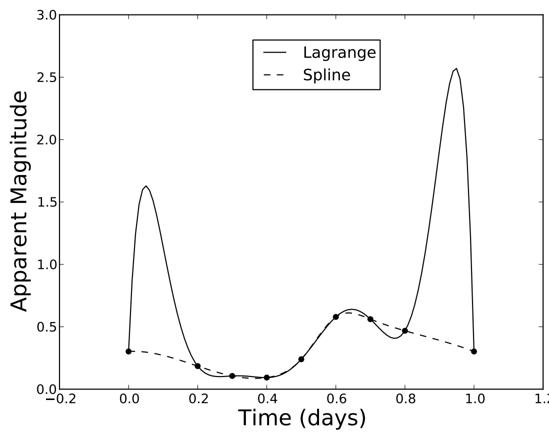

Figure 5: Lagrange (solid line) and spline (dashed line) interpolations together, for comparison. The solid points represent the data.