Ay190 – Worksheet 15
Scott Barenfeld
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I worked with Daniel DeFellipis

1 Problem 1

I filled in the skeloton code provided. The code works by first sets up the problem by creating a grid of particles evenly spaced on [-0.5,0.5] and specifies the initial conditons:

$$\rho(x) = \begin{cases} 1 & x \leq 0 \\ 0.25 & x > 0 \end{cases} \tag{1}$$

$$\epsilon(x) = \begin{cases} 2.5 & x \leq 0 \\ 1.795 & x > 0 \end{cases} \tag{2}$$

Initial velocities are set to 0.

The mass of each particle is defined as

$$m_i = \rho_i \mathrm{d}x \tag{3}$$

The initial time step is calculated as

$$\mathrm{d}t = \min(\mathrm{d}t_{\mathrm{old}}, \mathrm{CFL}\frac{h}{\max(c_s)}) \tag{4}$$

c_s is calculated as

$$c_s^2 = (\gamma - 1)\rho\epsilon + \frac{P}{\rho^2}(\gamma - 1)\rho \tag{5}$$

$$P = (\gamma - 1)\rho\epsilon \tag{6}$$

At each time step, positions, velocities, accelarations, energies, densities, pressures, and sound speeds are updated. The Leap-Frog method is used to update the velocities, which are spaced at half-time steps:

$$v_i^{n+1/2} = v_i^{n-1/2} + \Delta t a_i^n \tag{7}$$

$v_i^{-1/2}$ is set to be equal to v_i^0. Accelarations are calculated using:

$$a_i = -\sum_j m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \prod_{ij} \right) \nabla_i W(r_{ij}, h) \tag{8}$$

The summation is performed over the nearest neighbors to particle i, those within a distance of $2h$, where h is the smoothing length used to group particles together and calculate average quantities. $\prod_{ij}$ is the artificial viscosity between particles i and j, used to broaden shocks and increase entropy.

The artificial viscosity depends on v_i^n, which is approximated as

$$v_i^{n+1} = v_i^{n_1/2} + \frac{1}{2}\Delta t a_i^n \tag{9}$$

$W(r_{ij}, h)$ is the smoothing kernal, defined as

$$W(r,h) = \frac{2}{3h} \begin{cases} 1 - \frac{3}{2}\left(\frac{r}{h}\right)^2 + \frac{3}{4}\left(\frac{r}{h}\right)^3 & 0 \le \frac{r}{h} < 1 \\ \frac{1}{4}\left(2 - \left(\frac{r}{h}\right)^3\right) & 1 \le \frac{r}{h} < 2 \\ 0 & \frac{r}{h} \ge 2 \end{cases} \tag{10}$$

Internal energy is updated using

$$\epsilon_i^{n+1} = \epsilon_i^n + \Delta t \frac{\mathrm{d}\epsilon_i}{\mathrm{d}t} \tag{11}$$

where

$$\frac{\mathrm{d}\epsilon_i}{\mathrm{d}t} = \frac{1}{2}\sum_j m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \prod_{ij}\right) v_{ij} \nabla_i W(r_{ij}, h) \tag{12}$$

Here, the half-time step relative velocity between particles v_{ij} is used. Thus summation is again performed using the nearest neighbors.

Positions are updated using

$$r_i^{n+1} = r_i^n + \Delta t v_i^{n+1/2} \tag{13}$$

Density is averaged as the total mass the smoothing length. Finally, pressure and sound speed are calculated as before, and a new time step is chosen.

The results of the code are shown in Figures 1-5, which plot density vs. position. At $t = 0.2$ (Figure 5), the density profile matches the exact solution for this problem (Figure 6). Regions of constant density are separated by a rarefaction wave (gradual decrease) and contact and shock discontinuities (sharp decreases).

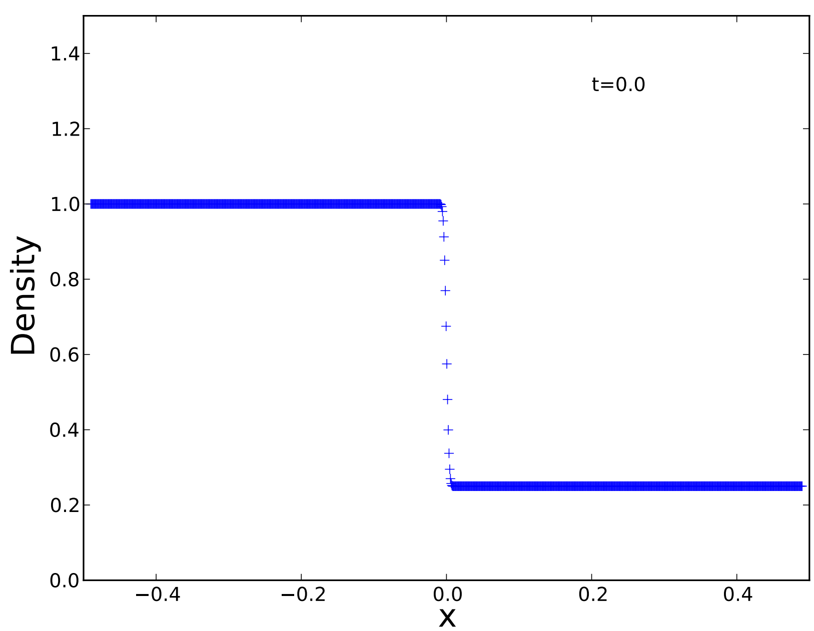

Figure 1: Initial density profile.

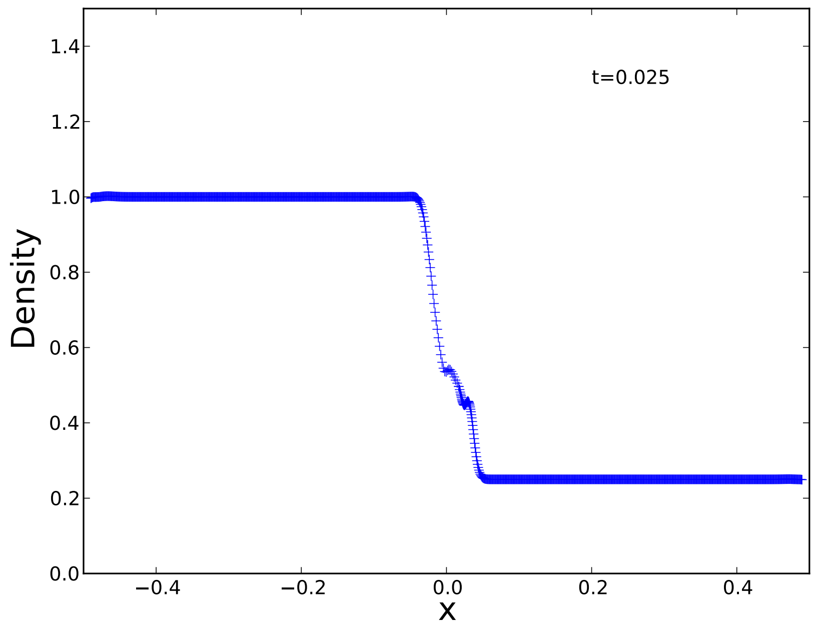

Figure 2: Density profile after at $t = 0.025$. Some structure has started to develop in the transition region.

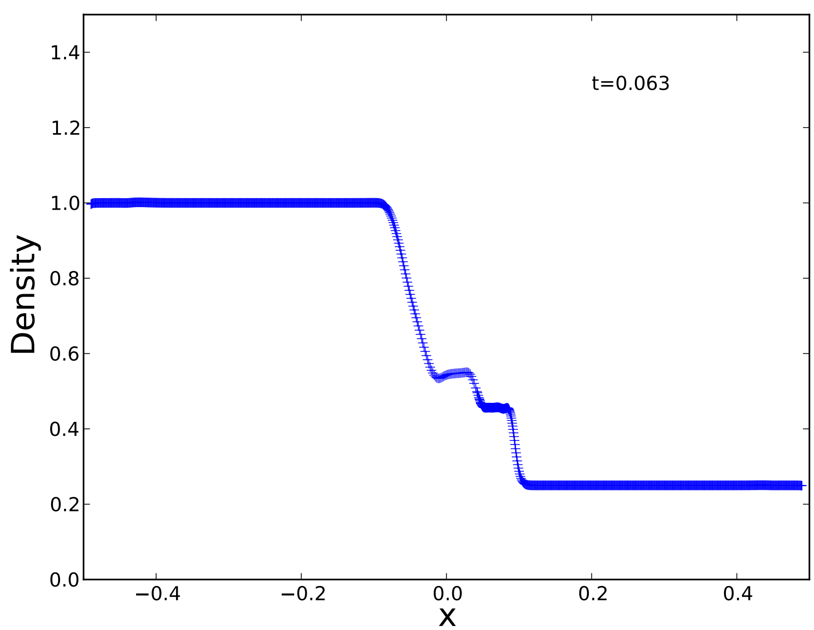

Figure 3: Density profile at $t = 0.063$. The structure continues to evolve.

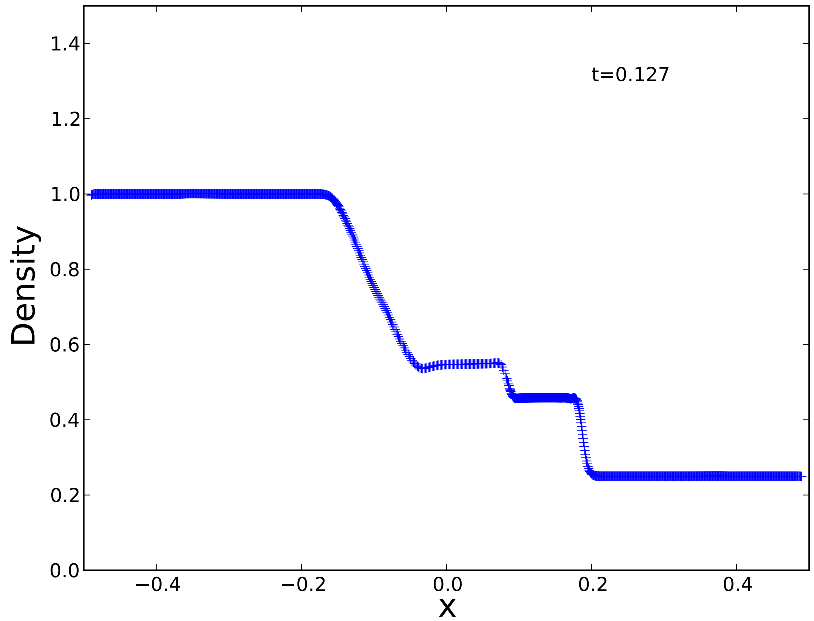

Figure 4: Density profile at $t = 0.127$. The solution is beginning to match the analytical solution.

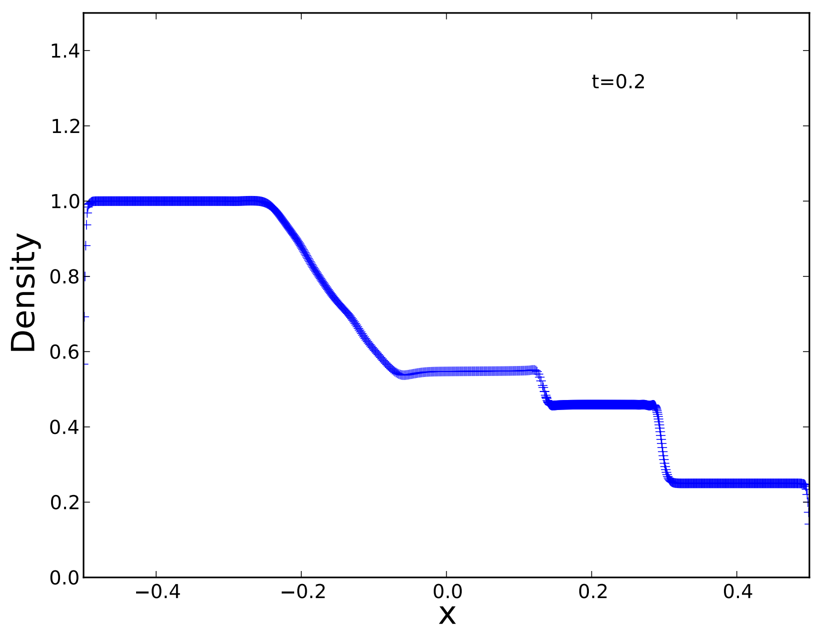

Figure 5: Final density profile at $t = 0.2$. The transition region has expanded relative to $t = 0.127$. The solution matches the exact solution reasonably well.

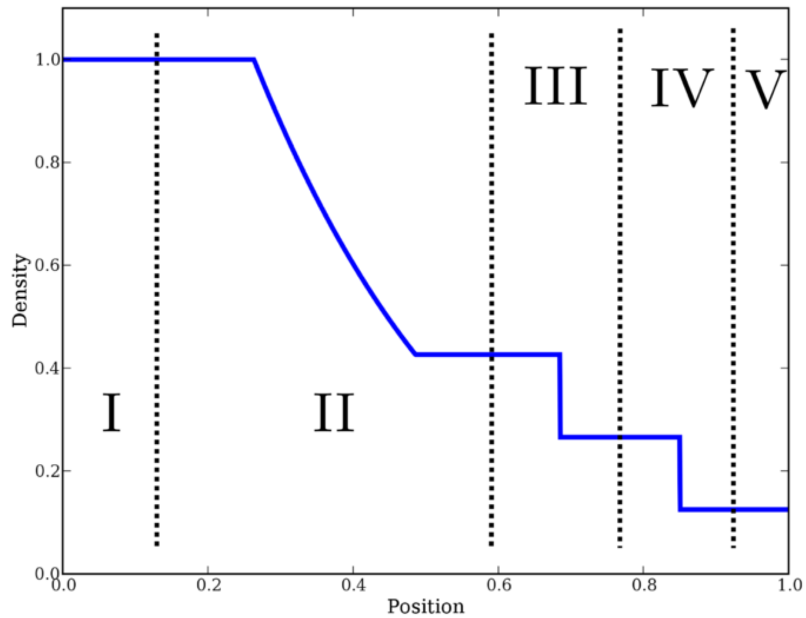

Figure 6: Exact solution from Wikipedia, on a shifted x-scale.

2 Problem 2

Let's face it- I don't want to deal with Fortran and you don't want to be grading this problem. So instead, here's a picture of Ryan Giggs to brighten your day. :)

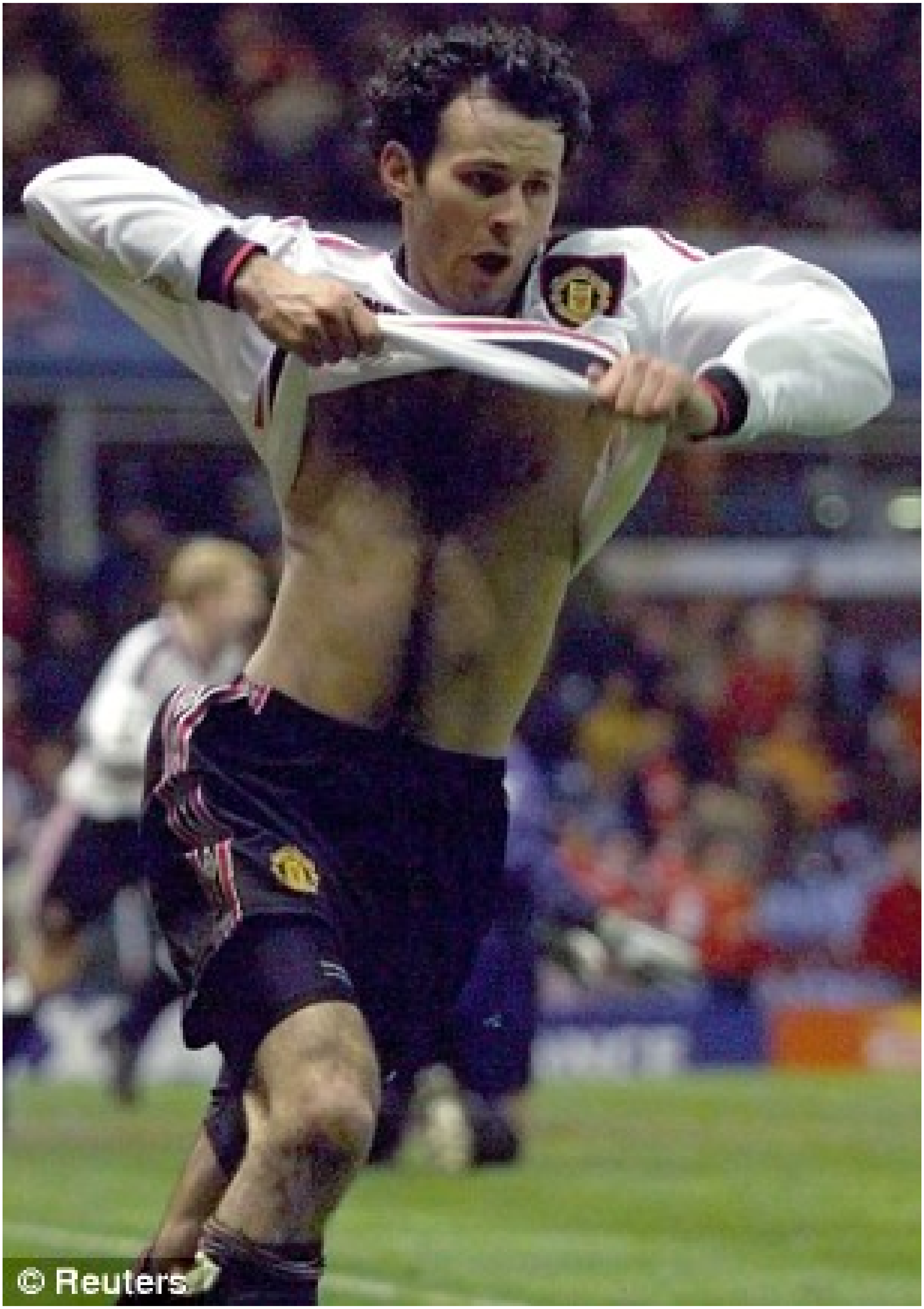

Figure 7: Giggs. What a man.