Minimum Wage and Non-Competes in a Search Model with Human Capital

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Abstract

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1 Model

1.1 Setting

Workers can be employed or unemployed. We imagine firms as being a from a partial equilibrium–i.e., match specific productivity is given by an exogenous distribution $F(X)$. The production of a worker with human capital $h_t \in \{0, h\}$, where h is to be estimated, matched with a firm with match specific productivity x is given by:

$$y_t = x + h_t.$$

Wages are given by w_t. We specify wages as coming from some bargaining process or else being the minimum wage, $\underline{w}$. In particular, when it is above the minimum

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wage, we let the wage w_t be the wage that solves the following Nash Bargaining problem:

$$\max_{w} \left(V(w, h_t) - U(h_t)\right)^{\alpha} \Pi(w, X, h_t)^{1-\alpha},$$

where $V(w, h_t)$ is the expected value of employment at wage w and human capital h_t, $U(h_t)$ is the value of unemployment with human capital h_t, and $\Pi(w, X, h_t)$ is the expected profit from hiring a worker at wage w and human capital h_t. We explore these objects in the latter sections, however to preempt questions, because $V(\cdot, \cdot)$ and $\Pi(\cdot, \cdot, \cdot)$ are quasilinear in w, this will provide a clean characterization of w_t.

For clarity, let

$$w_\alpha \triangleq \arg\max_{w} \left(V(w, h_t) - U(h_t)\right)^{\alpha} \Pi(w, X, h_t)^{1-\alpha},$$

therefore, w_t will be a function of X and h_t given by:

$$w_t(X, h_t) = \begin{cases} w_\alpha & \text{if } w_\alpha \geq \underline{w} \\ \underline{w} & \text{if } w_\alpha < \underline{w} \end{cases}.$$

For simplicity of notation, however, we will often write simply w_t rather than $w_t(X, h_t)$.

1.2 WORKERS

Workers encounter firms at some Poisson rate λ_0 when unemployed and λ_1 when employed. Worker's human capital evolves probabilistically. While employed, a worker's human capital can increase from 0 to h with a poisson rate of ξ_1, and while unemployed can depreciate from h to 0 with a poisson rate of ξ_0. A match is destroyed at some exogenous rate ρ. The discount rate is given by r. While unemployed, workers get flow utility of b. While employed, workers simply get their wage.

1.2.1 UNEMPLOYMENT

We can express the value of unemployment by each human capital type, 0 and h.

$$U(0) = b\Delta t + \frac{1}{1 + r\Delta t}\{(\lambda_0\Delta t + o(\Delta t)) \int_{S_X} \max\{V(w_t, 0), U(0)\}dF(X)$$
$$+ (1 - (\lambda_0\Delta t + o(\Delta t))U(0)\}$$

$$U(0)\{\frac{\Delta tr}{1 + r\Delta t}\} = b\Delta t + \frac{1}{1 + r\Delta t}\{(\lambda_0\Delta t + o(\Delta t)) \int_{S_X} \max\{V(w_t, 0), U(0)\}dF(X)$$
$$+ (-\lambda_0\Delta t + o(\Delta t))U(0)\}$$

$$U(0)\{\frac{r}{1 + r\Delta t}\} = b + \frac{1}{1 + r\Delta t}\{(\lambda_0 + o(\Delta t)/\Delta t) \int_{S_X} \max\{V(w_t, 0), U(0)\}dF(X)$$
$$+ (-\lambda_0 + o(\Delta t)/\Delta t)U(0)\}$$

$$U(0)(r + \lambda_0) = b + \lambda_0 \int_{S_X} \max\{V(w_t, 0), U(0)\}dF(X),$$

where the final line is obtained by taking the limit as $\Delta t \to 0$.

Starting with $h_t = h$ and walking through similar steps leads us to a similar expression:

$$U(h) = \Delta tb + \frac{1}{1 + r\Delta t}\{(\lambda_0\Delta t + o(\Delta t)) \int_{S_X} \max\{V(w_t, h), U(h)\}dF(X)$$
$$+ (\xi_0\Delta t + o(\Delta t))U(0) + (1 - (\lambda_0\Delta t + o(\Delta t)) - (\xi_0\Delta t + o(\Delta t)))U(h)\},$$

which then reduces to:

$$U(h)(r + \lambda_0 + \xi_0) - \xi_0 U(0) = b + \lambda_0 \int_{S_X} \max\{V(w_t, h), U(h)\}dF(X).$$

Equivalently, we can write a general characterization:

$$U(h_t)(r + \lambda_0 + \xi_0) - \xi_0 U(0) = b + \lambda_0 \int_{S_X} \max\{V(w_t, h_t), U(h_t)\}dF(X).$$

1.2.2 EMPLOYMENT

Everything is similar while employed, and we get the following general expression:

$$V(w_t, h_t)(r + \lambda_1 + \xi_1 + \delta) - \xi_1 V(w_t, h) = w_t + \lambda_1 \int_{S_X} \max\{V(w_t(X', h_t), h_t), V(w_t, h_t)\}dF(X') + \delta U(h_t).$$

1.3 FIRMS

We assume a free entry condition so that the value of a vacancy is zero. Firms flow profits are given by the difference between production and wages $X + h_t - w_t$.

In the case when $h_t = h$, everything is standard as we will show. However, when the human capital of the individual is low (0), there is some probability that the productivity will increase from 0 to h.

$$\Pi(w_t, X, h) = \Delta t(X + h - w_t) + \frac{1}{1 + r\Delta t}\{(\delta\Delta t + o(\Delta t)) \cdot 0$$
$$+ (1 - (\delta\Delta t + o(\Delta t)))\Pi(w_t, X, h)\}$$

which reduces to:

$$\Pi(w_t, X, h)(r + \delta) = X + h - w_t.$$

In the case where $h_t = 0$, we have instead:

$$\Pi(w_t, X, 0) = \Delta t(X - w_t) + \frac{1}{1 + r\Delta t}\{(\delta\Delta t + o(\Delta t)) \cdot 0$$
$$+ (\xi_1\Delta t + o(\Delta t))\Pi(w_t, X, h) + (1 - (\delta\Delta t + o(\Delta t)) - (\xi_1\Delta t + o(\Delta t)))\Pi(w_t, X, 0)\}$$

which reduces to:

$$\Pi(w_t, X, 0)(r + \delta + \xi_1) = X - w_t + \xi_1\Pi(w_t, X, h).$$

Again, we can write a general characterization as:

$$\Pi(w_t, X, h_t)(r + \delta + \xi_1) = X + h_t - w_t + \xi_1\Pi(w_t, X, h).$$

The main point of this section is to emphasis the following: when $h_t = 0$, it is possible that a firm may employ a worker at a wage $w_t > X$ because of the expected future profits that may accrue if h_t transitions to h: $\xi_1\Pi(w_t, X, h)$. This is, to our knowledge, an unexplored feature of human capital in search models, especially in the context of minimum wages. We also seek to extend our model to include choices by the firm to implement non-compete clauses as a way to increase the expected future profits associated with hiring a worker–without such a clause, a worker will be unable to commit to a staying at the firm. With the commitment device, however, workers and firms are able to enjoy the surplus of the match.

1.4 WAGE DETERMINATION

First, we consider the case without a minimum wage for simplicity, and then we return to that feature.

The first thing to note is that for all h_t, $V(w_t, h_t)$ is increasing in w_t. Second, whenever the minimum wage is not binding, w_t is increasing in X.[1]

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[1]This is obvious, but needs some justification.

With this in mind, let $\underline{X}(h_t)$ be the match specific productivity such that $V(w_t(\underline{X}, h_t), h_t) = U(h_t)$.[2] With this notation, we can simplify the value of unemployment:

$$U(h_t)(r + \lambda_0 + \xi_0) - \xi_0 U(0) = b + \lambda_0 \int_{\underline{X}(h_t)}^{\infty} V(w_t, h_t) dF(X) + \lambda_0 F(\underline{X}(h_t)) U(h_t).$$

Theoretically, the result of the Nash Bargaining process could result in some wage contract with stipulations (e.g., a non-compete) with some value C. Above we explored what this would look like if the contract could be summarized by an hourly wage rate. However workers have quasilinear utility, therefore any contract with a value C such that:

$$C_t = V(w^*, h_t)$$
$$\text{where}$$
$$w^* = \arg\max_w \left(V(w, h_t) - U(h_t)\right)^{\alpha} \Pi(w, X, h_t)^{1-\alpha}$$

would be equivalent from a welfare perspective.[3] The main advantage of the alternative contracts C_t are if the standard contract $V(w^*, h_t)$ is not feasible because $w^* < \underline{w}$. The firm may be able to offer some contract C_t such that $C_t = V(w^*, h_t)$.[4]

In the non-compete case, C would take the form of ensuring a length of time of employment.

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[2]Is it the case that $\underline{X}(0) = \underline{X}(h)$? I do not think so.
[3]What edits to the model are necessary for non-competes to have bite?
[4]Note that if $\Pi(\underline{w}, X, h) < 0$, then no contract C is possible.