Problem 1

$$
\begin{aligned}
& min \mid A - BX \mid_F \\
&= tr\left((A - BX)^T (A - BX)\right) \\
&= tr\left(A^T A + X^T B^T BX - X^T B^T A - A^T BX\right) \\
&= tr\left(A^T A\right) + tr\left(BXX^T B^T\right) - tr2\left(A^T BX\right)
\end{aligned}
$$

Then, $\left(A^T A\right)$ and $\left(BXX^T B^T\right)$ where $XX^T = I$ are both constant terms

Meaning that we can maximize $tr\left(A^T BX\right)$ to solve the optimization. We need to apply SVD

$$
\begin{aligned}
& tr\left(AB^T X^T\right) \\
&= tr\left(\mathbf{U}\Sigma\mathbf{V}^T X^T\right) \\
&= tr\left(\Sigma\mathbf{V}^T X^T\mathbf{U}\right) \text{ with } \mathbf{Z} = \mathbf{V}^T X^T\mathbf{U} \\
&= tr\left(\Sigma\mathbf{Z}\right) = \sum_i Z_{i,i}\Sigma_{i,i}
\end{aligned}
$$

$\mathbf{Z}$ is an orthogonal matrix, then the optimum is achieved when $\mathbf{Z} = \mathbf{I}$

$$X = \mathbf{U}\mathbf{V}^T$$

Problem 12

```
from run import Engine

engine = Engine()
```

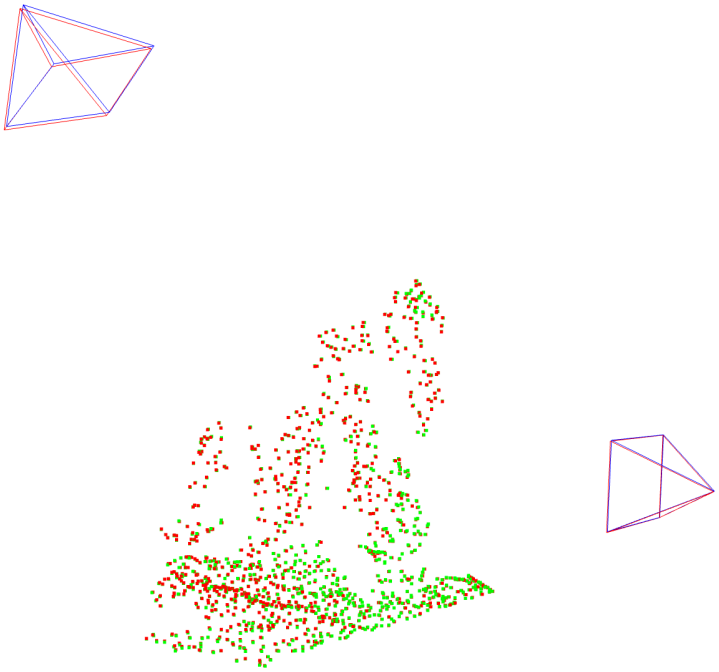
8-point algorithm

```
engine.sfm_pipeline(use_noise=True, use_BA=True, final_vis=True, normalize_eight_point=False
```

```
condition number 386779150735.66705
step 0, err: 7.160732705871644
step 1000, err: 0.5214589665466234
step 2000, err: 0.5633329289869181
step 3000, err: 0.5623017607094797
step 4000, err: 0.5698268473504396
step 5000, err: 0.5678945413205159
step 6000, err: 0.5560209380827632
step 7000, err: 0.5827503155670568
step 8000, err: 0.591579259857796
step 9000, err: 0.5298454106034809
[Open3D DEBUG] GLFW init.
[Open3D WARNING] GLFW Error: Cocoa: Failed to find service port for display
```

```
[Open3D WARNING] GLFW Error: Cocoa: Failed to find service port for display
[Open3D DEBUG] Add geometry and update bounding box to [(-22.3970, 2.1672, -28.3910) - (22.3
[Open3D DEBUG] Add geometry and update bounding box to [(-22.6612, 2.0966, -28.3910) - (22.4
[Open3D DEBUG] Add geometry and update bounding box to [(-22.6612, 2.0966, -28.3910) - (28.3
[Open3D DEBUG] Add geometry and update bounding box to [(-22.6612, 2.0966, -28.3910) - (70.6
[Open3D DEBUG] Add geometry and update bounding box to [(-22.6612, 2.0966, -28.3910) - (70.6
[Open3D DEBUG] Add geometry and update bounding box to [(-22.6612, 2.0966, -28.3910) - (70.7
[Open3D DEBUG] Global colormap init.
```

Condition number was huge: 386779150735.66705. The error didn't decrease much after the first 1,000 steps, with a final error of 0.5298



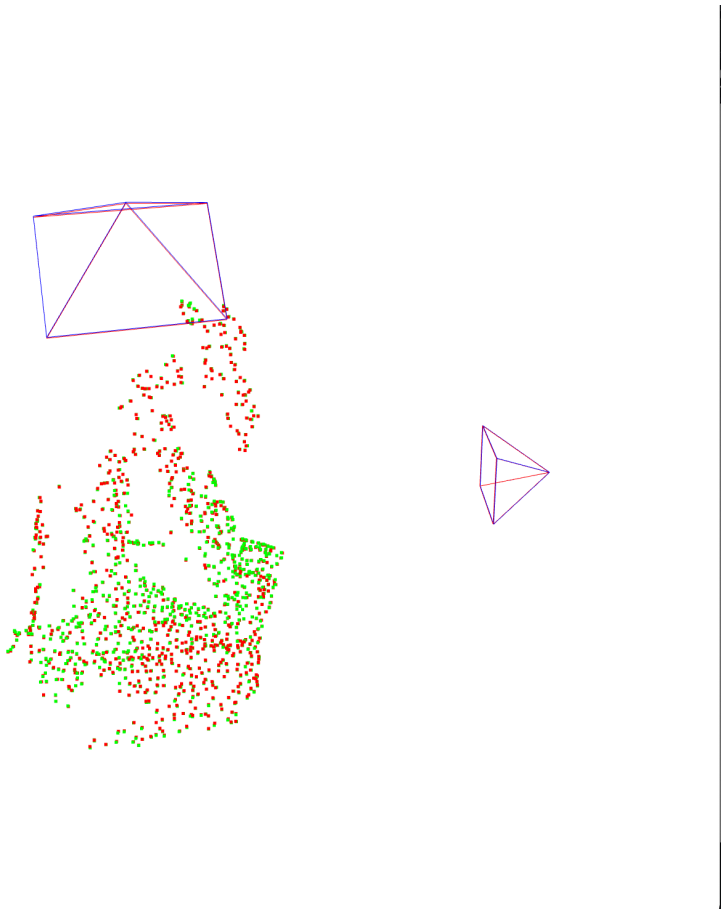
Normalized 8-point algorithm

```
engine.sfm_pipeline(use_noise=True, use_BA=True, final_vis=True, normalize_eight_point=True)
```

```
condition number 1331.178438294837
```

```
step 0, err: 0.39939844880027875
step 1000, err: 0.5463882982681467
step 2000, err: 0.5056460621908008
step 3000, err: 0.5696202979531187
step 4000, err: 0.512972216224318
step 5000, err: 0.5190394232732399
step 6000, err: 0.4941910075008047
step 7000, err: 0.5145344731266677
step 8000, err: 0.522201767267177
step 9000, err: 0.5102767320059329
[Open3D DEBUG] GLFW init.
[Open3D WARNING] GLFW Error: Cocoa: Failed to find service port for display
[Open3D WARNING] GLFW Error: Cocoa: Failed to find service port for display
[Open3D DEBUG] Add geometry and update bounding box to [(-22.3970, 2.1672, -28.3910) - (22.3
[Open3D DEBUG] Add geometry and update bounding box to [(-22.3970, 1.9839, -28.3910) - (22.4
[Open3D DEBUG] Add geometry and update bounding box to [(-22.3970, 1.9839, -28.3910) - (28.3
[Open3D DEBUG] Add geometry and update bounding box to [(-22.3970, 1.9839, -28.3910) - (70.6
[Open3D DEBUG] Add geometry and update bounding box to [(-22.3970, 1.9839, -28.3910) - (70.6
[Open3D DEBUG] Add geometry and update bounding box to [(-22.3970, 1.9839, -28.3910) - (70.7
[Open3D DEBUG] Global colormap init.
```

Condition number improved significantly, decreasing to 1331.178. The error stayed on similar values, with marginal improvement reaching a final error of 0.51027



2% random correspondence flipping

```
engine.sfm_pipeline(use_noise=True, use_BA=True, final_vis=True, normalize_eight_point=True,
```

```
condition number 1328.3728785148517
step 0, err: 0.4116162877018157
step 1000, err: 0.6057451191145822
step 2000, err: 0.5955350924768572
step 3000, err: 0.5970830444593593
step 4000, err: 0.5546594614810048
step 5000, err: 0.5515708751662294
step 6000, err: 0.5379695943709637
step 7000, err: 0.5486317460873212
step 8000, err: 0.5751791939462941
step 9000, err: 0.5377793162386013
[Open3D DEBUG] GLFW init.
[Open3D WARNING] GLFW Error: Cocoa: Failed to find service port for display
```

```
[Open3D WARNING] GLFW Error: Cocoa: Failed to find service port for display
[Open3D DEBUG] Add geometry and update bounding box to [(-22.3970, 2.1672, -28.3910) - (22.3
[Open3D DEBUG] Add geometry and update bounding box to [(-22.3970, 2.1318, -28.3910) - (22.4
[Open3D DEBUG] Add geometry and update bounding box to [(-22.3970, 2.1318, -28.3910) - (28.3
[Open3D DEBUG] Add geometry and update bounding box to [(-22.3970, 2.1318, -28.3910) - (70.6
[Open3D DEBUG] Add geometry and update bounding box to [(-22.3970, 2.1318, -28.3910) - (70.6
[Open3D DEBUG] Add geometry and update bounding box to [(-22.3970, 2.1318, -28.3910) - (70.8
[Open3D DEBUG] Global colormap init.
```

Clearly we were going to receive a worse error than previously. The last step error was 0.53, worse than the previous two experiments. The condition number stayed in line with the normalized_8_point algorithm, as we keeped it enabled