## Camera Design: Pinhole

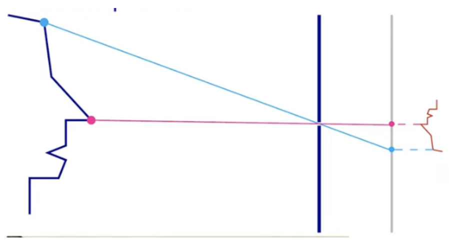

**Figure 1:** Representation of pinhole camera

- Simplest camera: put sensor in front of object, covered so that only a small hole lets light through.

## Pinhole camera geometry

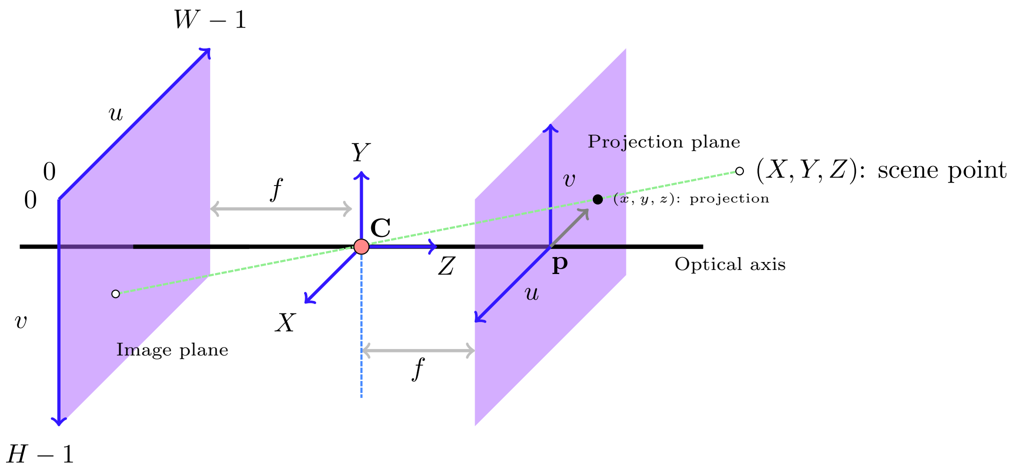


**Figure 2:** Camera geometry

- Image plane at $Z = -f$, "virtual" projection plane at $Z = f$
- Center of projection $\mathbf{p}$ at $(0, 0, f)$, with $u = 0, v = 0$

## Perspective equations

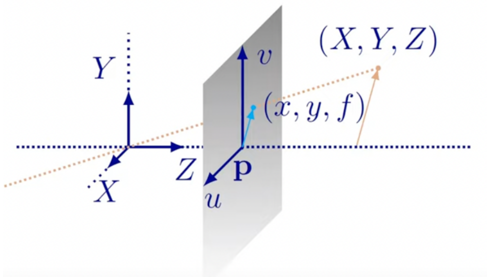


**Figure 3:** Projection from $(X, Y, Z)$

We can understand the projection plane by looking at the triangles formed by $(X, Y, Z)$ and $(x, y, f)$.

**$x$ is the scaling of the $X$ by a factor of $\frac{f}{z}$.**

(1) From similar triangles:

$$(X, Y, Z) \mapsto \left(f\frac{X}{Z}, f\frac{Y}{Z}, f\right)$$

(2) Projection plane (image) coordinates:

$$(X, Y, Z) \mapsto (x, y)$$

where $x = f\frac{X}{Z}, y = f\frac{Y}{Z}$

> ### Notation
>
> $(X, Y, Z)$ is a point in 3D world. $(x, y)$ is a point in 2D image plane (projection).

> ### Perspective effect
>
> Same object appears larger if closer (because $Z$)is smaller

## Projections is a dimensionality reduction

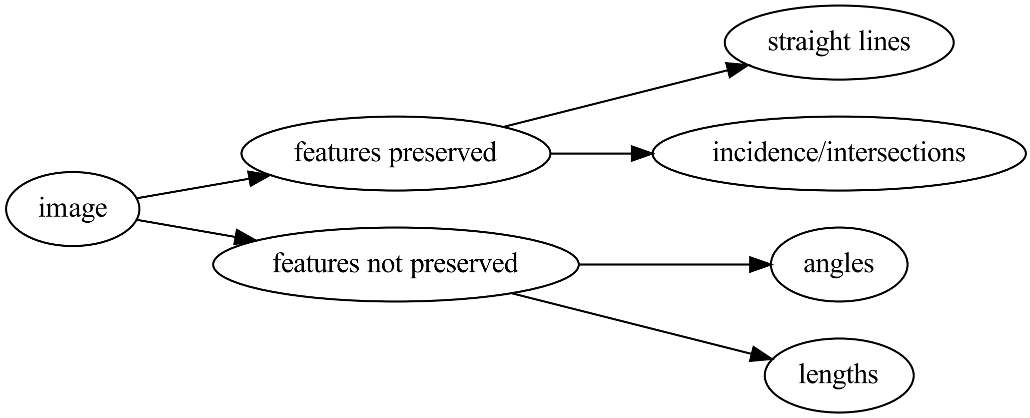


**Figure 4**

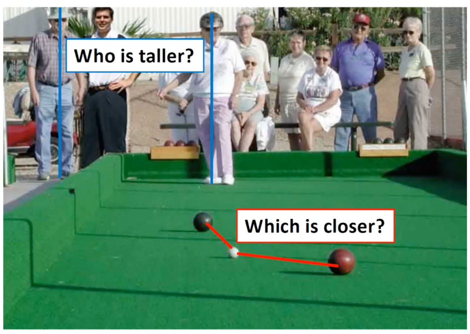


**Figure 5:** Source: D. Hoiem

### Fronto-parallel plane

**Fronto-parallel plane** Projection of a plane parallel to the image plane (perpendicular to the optical axis)

**What happens to the projection of a pattern on a fronto-parallel plane?** All points are at a fixed depth $Z$, but the pattern gets scaled by $\frac{f}{Z}$ while the angles and ratios of lengths/areas are preserved.

## Vanishing points

Projections of all parallel (in 3D) lines converge to a *vanishing point*, **except those parallel to the image plane.**

Each direction in 3D *maps* to a vanishing point in the image.

## Vanishing lines

**Fact** the set of vanishing points for all lines in a plane corresponds to a line in the image plane

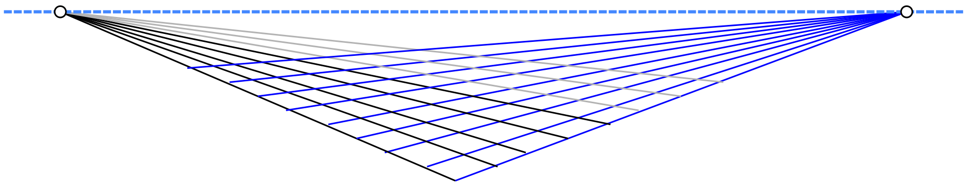

**Figure 6:** Vanishing lines

**Horizon** The *horizon* is the vanishing line for the ground plane. Points higher than the camera project above the horizon. Points lower than the horizon project below the horizon.

## Perspective distortion on a sphere

Consider the occluding contour of the sphere; it lies in a plane. If the plane is not fronto-parallel, we have distortion.

## Modeling the projection

Recall we are mapping from $(X, Y, Z)$ to $(x, y)$ where $(x, y) = f\frac{X}{Z}, f\frac{Y}{Z}$. But this is a non linear transformation.

A trick to make it linear is to add one more coordinate. Then

$$(x, y) \Rightarrow \underbrace{\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}}_{\text{homogeneous image coordinates}} \quad \text{and} \quad (X, Y, Z) \Rightarrow \overbrace{\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}}^{\text{homogeneous scene coordinates}}$$

1. To homogeneous coordinates: **just add 1**
2. From homogeneous coordinates: divide by the last coordinate

$$\begin{pmatrix} x \\ y \\ w \end{pmatrix} \Rightarrow \left(\frac{x}{w}, \frac{y}{w}\right) \text{ and } \begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix} \Rightarrow \left(\frac{X}{W}, \frac{Y}{W}, \frac{Z}{W}\right)$$

> Perspective projection collapses an entire ray $\mathbf{C} + \lambda \cdot (X, Y, Z)$ to a point $\left(f\frac{X}{Z}, f\frac{Y}{Z}\right)$

Using homogeneous coordinates, projection is a linear operation.

$$\begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} \Rightarrow \left(f\frac{X}{Z}, f\frac{Y}{Z}\right)$$

In practice, need to handle different coordinate systems and the fact that pixrl coordinates are different from projection plane coordinates

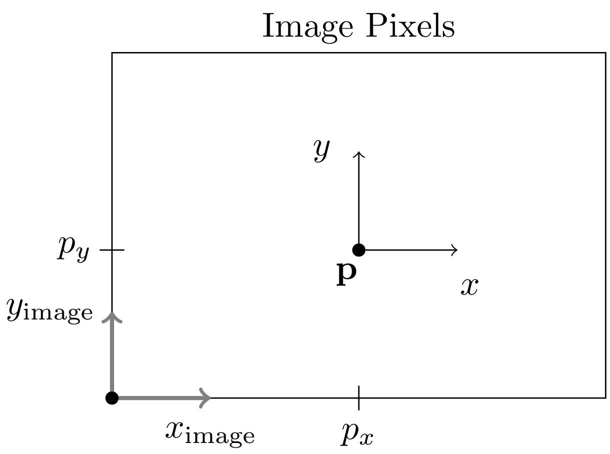


**Figure 7:** Pixel Coordinates

- Principal point $p = p_x, p_y$: point in the image where the optical axis crosses the projection (image) plane
- Normalized coordinate system: origin at $\mathbf{p}$
- Image coordinates: origin is **the corner**
- Want the principal point to **map to** $(p_x, p_y)$ **not** $(0, 0)$