Exponential distribution

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Introduction

In this project you will investigate the exponential distribution. We ilustrate its properites via simpulations in R. We use the follwing packages:

Exponential distribtion

Exponential distriubution is given by the formula:

$$f(x) = \lambda e^{-\lambda x},$$

for $\lambda > 0$ where $x \geq 0$. Its mean is λ^{-1}, and its variance is λ^{-2}.

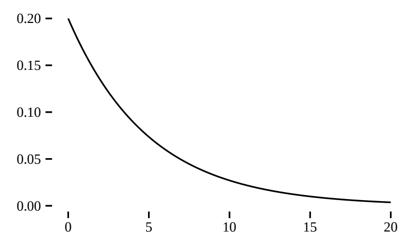

Figure 1: Exponental distribution with lambda equal to 0.2

Sample mean and variance

Now we see how the sample mean approximate the theoretical one.

```
N <- 10000
set.seed(1110)
exps <- rexp(N, lambda)
means <- sapply(1:N, function(n) mean(exps[1:n]))
ggplot() + geom_line(aes(x = 1:N, y = means)) +
    geom_hline(yintercept = 1/lambda, color = "red") +
    labs(x = "") + theme_tufte()
```

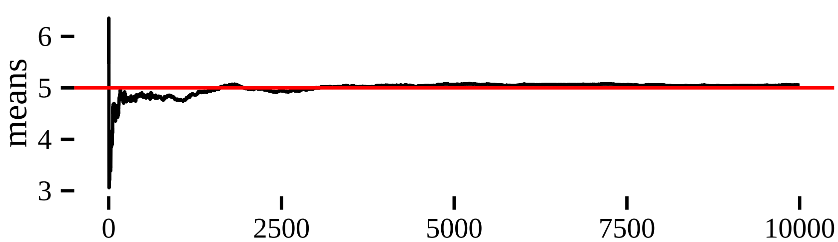

Figure 2: Sample means converging to the theoretical one.

And now we see how the sample variance approximate the theoretical one.

```
vars <- sapply(1:N, function(n) var(exps[1:n]))
ggplot() + geom_line(aes(2:N, vars[2:N])) + geom_hline(yintercept = 1/lambda^2,
    color = "red") + labs(x = "", y = "variances") +
    theme_tufte()
```

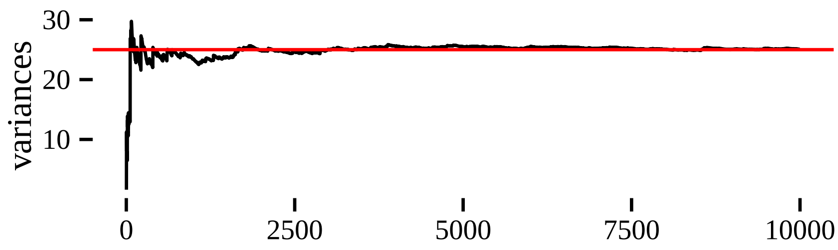

Figure 3: Sample variances converging to the theoretical one.

Sum of independent random variable with exponential distribution

Exponential distribution is a special case of *gamma distribution*

$$f_{\alpha,\nu}(x) = \frac{1}{\Gamma(\nu)}\alpha^{\nu}x^{\nu-1}e^{-\alpha x}, \nu > 0, x > 0.$$

with $\alpha = \lambda$ and $\nu = 1$. Let us also recall that:

$$\Gamma(t) = \int_0^{\infty} x^{t-1}e^{-x}dx.$$

It is also worth noting that if X_1 and X_2 are two independent random variable that have gamma distribution respectively $f_{\alpha,\mu}$, $f_{\alpha,\nu}$ with the same parameter α, then the sum $X_1 + X_2$ has the distribution $f_{\alpha,\mu+\nu}$. Hence sum k of identicaly distributed random variables with expotential distribribution is a gamma distribution $f_{\alpha,k}$. Moreover, if X is a Gamma distribution with a parametrs λ, ν, then X/k is Gamma distribution with paramenter $k\lambda, \nu$. Therefore, the density of $(X_1 + \ldots + X_k)/k$ is $f_{k\lambda,k}$. So lets compare Gamma distribution $f_{8,40}$ that correspons to the distribution of the mean of 40 exponential samples with $\lambda = 0.2$ and the normal distribution that should approximate this mean.

```
x <- seq(2, 8, 0.01)
k <- 40
e40.m <- 1/lambda
e40.sd <- 1/(lambda * sqrt(40))
ggplot() + geom_path(aes(x, dgamma(x, shape = k,
    rate = k * lambda), colour = "blue")) + geom_path(aes(x = x,
    y = dnorm(x, e40.m, e40.sd), colour = "red")) +
    labs(x = "", y = "") + scale_colour_discrete(name = "Distribution",
    labels = c("Normal", "Gamma")) + theme_tufte()
```

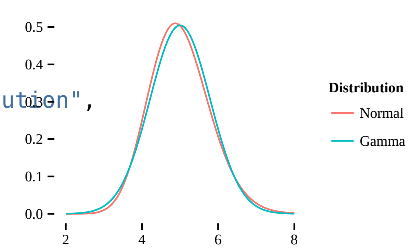

Figure 4: Gamma and normal distribution

Simulation

```
N <- 1000
means <- c()
```

```
set.seed(10)
for (n in 1:N) {
    exps <- rexp(k, lambda)
    means <- c(means, mean(exps))
}
e40.dt <- data.table(means = means)
x <- seq(e40.m - 3.5 * e40.sd, e40.m + 3.5 * e40.sd,
    e40.sd/10)
ggplot() + geom_histogram(data = e40.dt, aes(x = means,
    y = ..density..), fill = "white", colour = "black") +
    geom_path(aes(x = x, y = dnorm(x, e40.m, e40.sd),
        colour = "red")) + theme_bw() + geom_path(aes(x,
    k * dgamma(k * x, shape = k, scale = 5), colour = "blue")) +
    scale_colour_discrete(name = "Distribution",
        labels = c("Simulation", "Normal", "Gamma")) +
    theme_tufte()
```

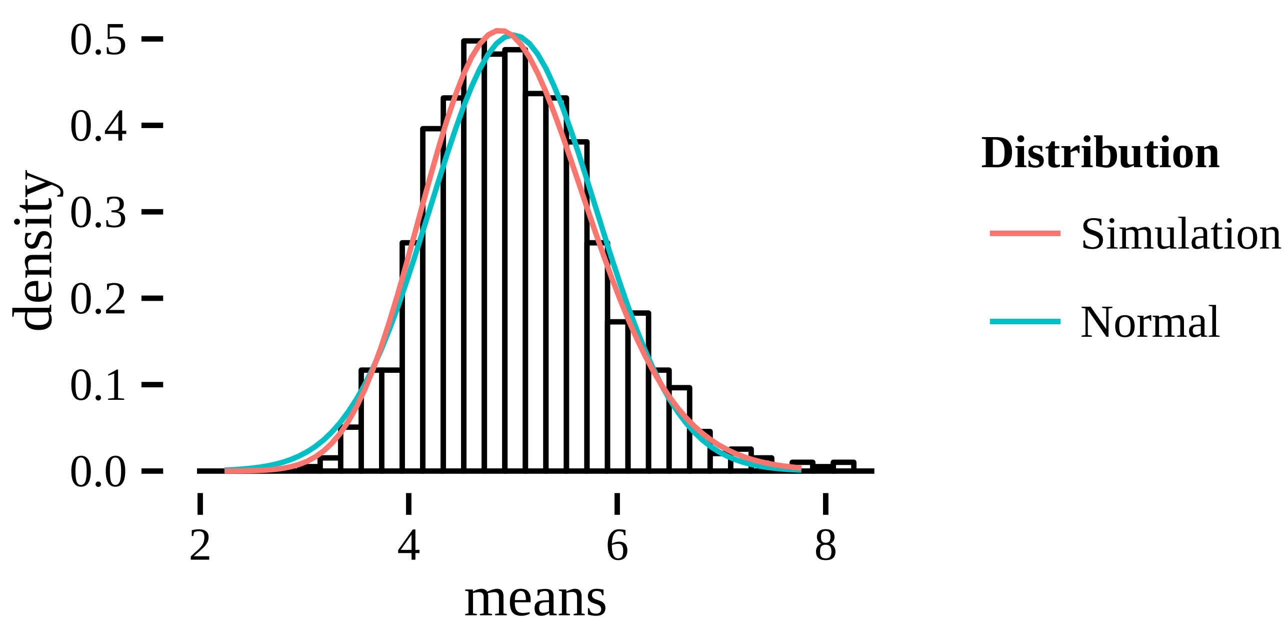

Figure 5: Histogram of 1000 sample means and corresponding normal and Gamma distribution.