

Regression:

Regression is generally used for curve fitting task. Here we will demonstrate regression task for the following.

- 1) Fitting of line (one variable learning)
- 2) Fitting of line (two variable learning)
- 3) Fitting of a plane (two variable)
- 4) Fitting of M-dimensional hyperplane (M-dimension, both in matrix inversion and gradient descent)
- 5) Polynomial regression
- 6) Practical example of regression task (salary prediction)

1) Fitting of line

a) Generation of line data ($y = w_1 x + w_0$)

i) Generate x, 1000 points from 0-1.

ii) Take $w_0 = 10$ and $w_1 = 1$ and generate y

iii) Plot (x,y)

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt

x=np.linspace(0,1,1000)
w1=1
w0=10
# write your equation here
y=# insert the code for the given equation here

plt.plot(x,y)
```

```
File "<ipython-input-1-4c08b97c1423>", line 8
    y=# insert the code for the given equation here
                                           ^
```

SyntaxError: invalid syntax

b) Corrupt the data using uniformly sampled random noise.

i) Generate random numbers uniformly from (0-1) with same size as y.

ii) Corrupt y and generate y_{cor} by adding the generated randomsamples with a weight of 0.1.

iii) Plot (x, y_{cor}) (use scatter plot)

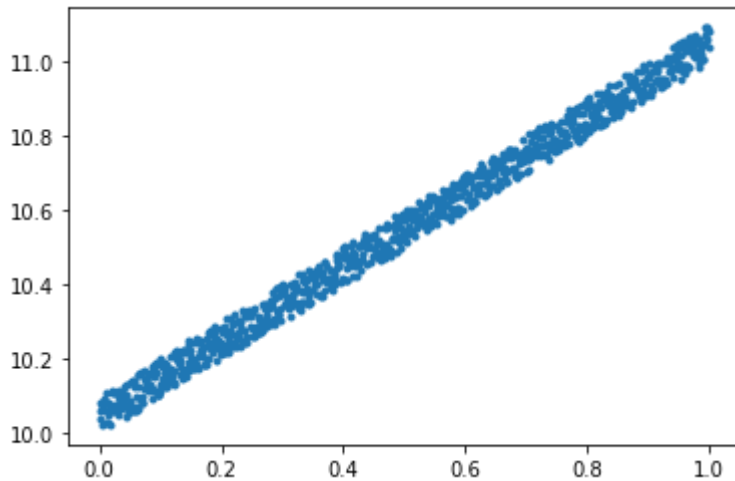
In []:

```
rnd_nos=np.random.random(y.shape)
y_cor= # insert your code here #(y+0.1*random noise)
print(rnd_nos.shape)
plt.plot(x,y_cor,'.')
```

(1000,)

Out[]:

[<matplotlib.lines.Line2D at 0x7f177fe5bfd0>]



c) Curve prediction using hurestic way.

i) Keep $w_0 = 10$ as constant and find w_1 ?

ii) Create a search space from -5 to 7 for w_1 , by generating 1000 numbers between that.

iii) Find y_{pred} using each value of w_1 .

iv) The y_{pred} that provide least norm error with y , will be decided as best y_{pred} .

$$error = \frac{1}{m} \sum_{i=1}^M (y_i - y_{pred_i})^2$$

v) Plot error vs srch_ w_1

vi) First plot the scatter plot (x, y_{cor}) , over that plot $(x, y_{bestpred})$.

In []:

```
# implementation of heuristic search for 1 variable case
def hurestic_srch(x,y_cor):
    srch_wl=np.linspace(-5,7,1000)
    srch_wl=np.expand_dims(srch_wl,axis=1)
    x=np.expand_dims(x,axis=1)

    y_pred=srch_wl @ x.T+w0    # @ used for matrix multiplication , */np.multiply
point wise multiplication, ## for array type initialization
    #print(x.shape)

    y_cor_rep=np.tile(y_cor,(x.shape[0],1))
    #print(y_cor_rep.shape)

    error=np.sum((np.power((y_cor_rep-y_pred),2)),axis=1)/(x.shape[0]) # row wise
sum

    #print(error.shape)
    idx = np.where(error == np.min(error))
    w1_opt=srch_wl[idx]
    return w1_opt,error,srch_wl,idx

w1_opt,error,srch_wl,idx=# insert your code here (call hurestic srch function)

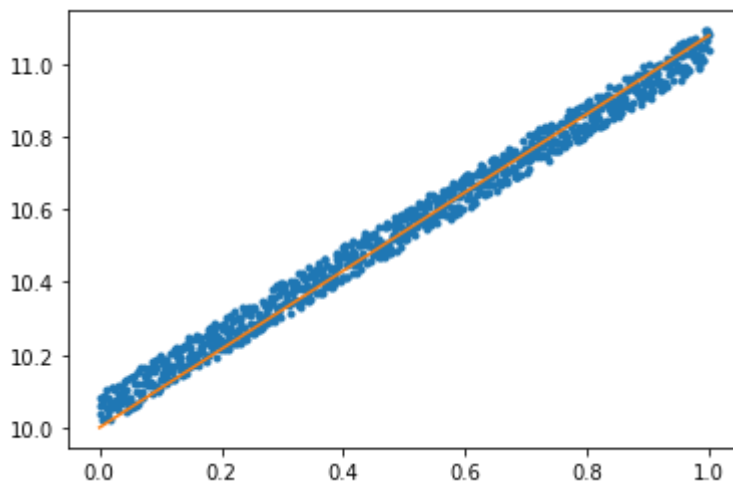
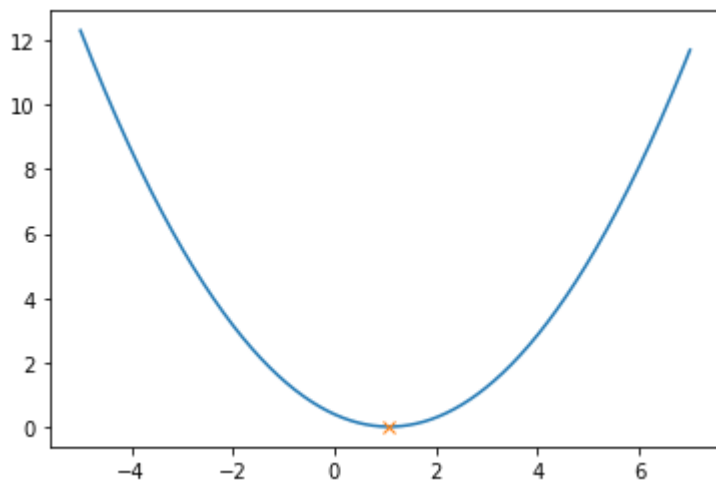
print(w1_opt)

# error surface plot
plt.plot(srch_wl,error)
plt.plot(w1_opt,error[idx],'x')
plt.figure()
# plotting
#print(x.shape)
y_bestpred=w1_opt*x+w0
#print(y_bestpred.shape)
plt.plot(x,y_cor,'.')
plt.plot(x,y_bestpred.T)
```

```
[[1.07807808]]
```

```
Out[ ]:
```

```
[<matplotlib.lines.Line2D at 0x7f177f985128>]
```



d) Gradient descent

$$\text{i) } Error = \frac{1}{m} \sum_{i=1}^M (y_i - y_{pred_i})^2 = \frac{1}{m} \sum_{i=1}^M (y_i - (w_0 + w_1 x_i))^2$$

$$\text{ii) } \nabla Error|_{w_1} = \frac{-2}{M} \sum_{i=1}^M (y_i - y_{pred_i}) \times x_i$$

$$\text{iii) } w_1|_{new} = w_1|_{old} - \lambda \nabla Error|_{w_1} = w_1|_{old} + \frac{2\lambda}{M} \sum_{i=1}^M (y_i - y_{pred_i}) \times x_i$$

In []:

```
import matplotlib.pyplot as plt

def f(w1):
    return (w1*x+w0)

# Gradient computation
def grad_computation(y_actual, w1_old, lr, x):
    w1_new = # write your update equation here (hint: y_pred=f(w1_old))
    return w1_new

def err(w1,y):
    return np.mean(np.power(y-f(w1),2))

# srch_w1=np.linspace(-10,10,1000)
# error=err(srch_w1,y_cor)
# print(error.shape)
plt.figure()
plt.plot(srch_w1,error)
# Gradient descent
w1_init = -4 # initialization
w0 = 10
lr = 0.1 # learning rate (0.1,2)
eps = 0.0000001

for i in list(range(1000)):
    if i == 0:
        w1_old = w1_init
        w1 = grad_computation(y_cor, w1_old, lr, x)
    else:
        w1_old = w1
        w1 = grad_computation( y_cor, w1_old, lr, x)

    dev = np.abs(err(w1,y_cor) - err(w1_old,y_cor))
    # print(dev)
    #plt.plot(w1,err(w1,y_cor), 'x')
    plt.plot([w1_old,w1],[err(w1_old,y_cor),err(w1,y_cor)],color='k')

    if dev <= eps:
        break

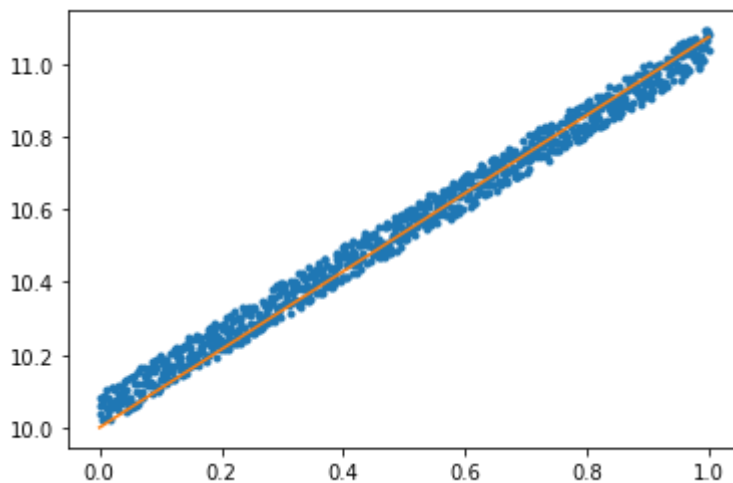
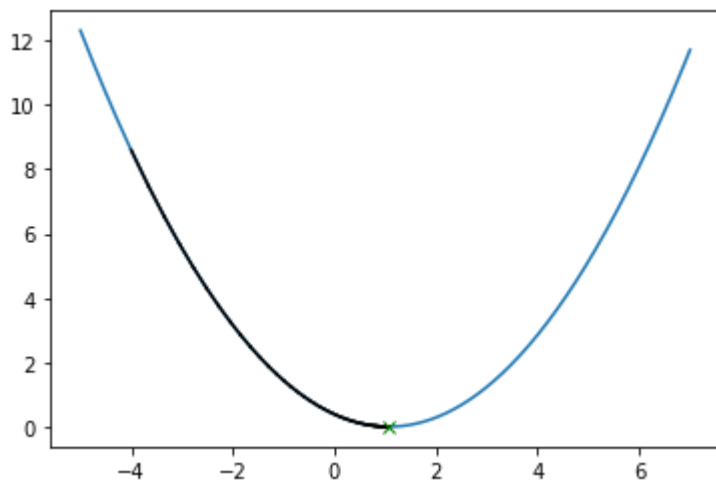
print(w1)
plt.plot(w1,err(w1,y_cor), 'x',color='g')

plt.figure()
# plotting
#print(x.shape)
y_bestpred=w1*x+w0
#print(y_bestpred.shape)
plt.plot(x,y_cor, '.')
plt.plot(x,y_bestpred)
```

1.0740457320045673

Out[]:

[<matplotlib.lines.Line2D at 0x7f177f8d62b0>]



2) Fitting line with two unknown variables

a) Generation of line data ($y = w_1 x + w_0$)

i) Generate x, 1000 points from 0-1.

ii) Take $w_0 = 5$ and $w_1 = 1.5$ and generate y

iii) Plot (x,y)

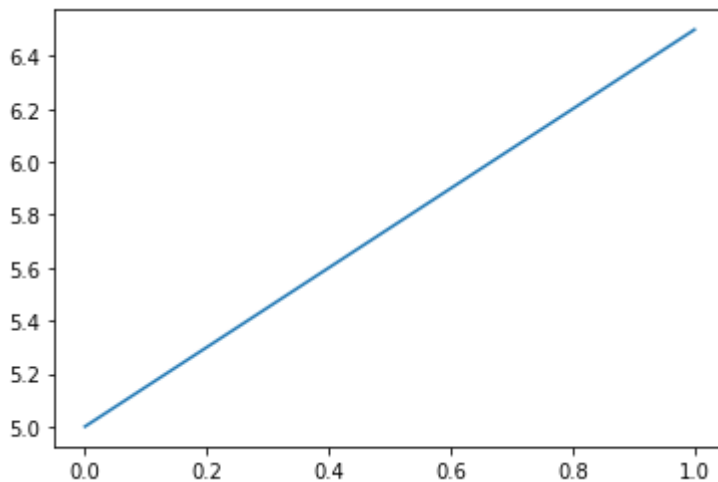
In []:

```
import numpy as np
import matplotlib.pyplot as plt

x = np.linspace(0,1,1000)
w0 = 5
w1 = 1.5
# write your equation here
y = w1*x + w0
plt.plot(x,y)
```

Out[]:

[<matplotlib.lines.Line2D at 0x7f177f673908>]



b) Corrupt the data using uniformly sampled random noise.

i) Generate random numbers uniformly from (0-1) with same size as y.

ii) Corrupt y and generate y_{cor} by adding the generated randomsamples with a weight of 0.1.

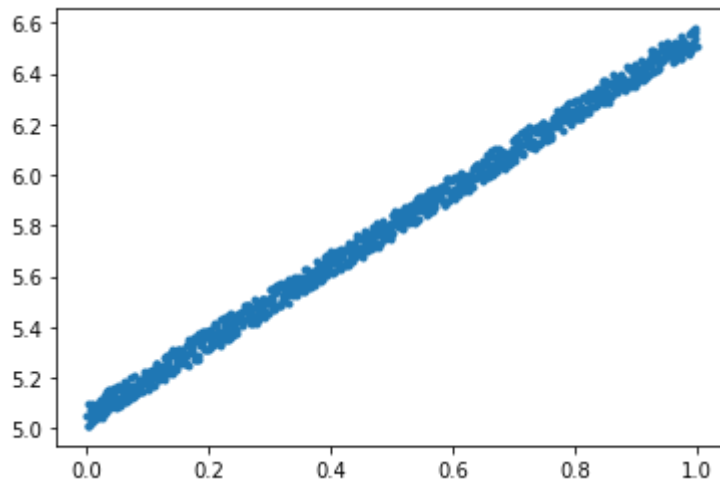
iii) Plot (x, y_{cor}) (use scatter plot)

In []:

```
rnd_nos = np.random.random(y.shape)
y_cor = y + 0.1*rnd_nos
# print(rnd_nos.shape)
plt.plot(x,y_cor,'.')
```

Out[]:

[<matplotlib.lines.Line2D at 0x7f177f655668>]



c) Plot the error surface

we have all the data points available in y_{cor} , now we have to fit a line with it. (i.e from y_{cor} we have to predict the true value of w_1 and w_0)

i) take w_1 and w_0 from -10 to 10, to get the error surface.

In []:

```
# c

def f(w1, w0,x):
    return (w1*x + w0)

srch_w1=np.linspace(-10,10,100)
srch_w0=np.linspace(-10,10,100)

S_w1,S_w0=np.meshgrid(srch_w1,srch_w0)
print(S_w1.shape)

def error(w1,w0,x,y):
    if len(w1.shape)==0:
        return np.mean(np.power(y-(f(w1,w0,x)),2))
    else:
        err=np.zeros(w1.shape)
        for x_i,y_i in zip(x,y):
            err1=np.power((np.tile(y_i,w1.shape)-(f(w1,w0,x_i))),2)
            err=err+err1
        return err/x.shape[0]
err=error(S_w1,S_w0,x,y_cor)
print(err.shape)

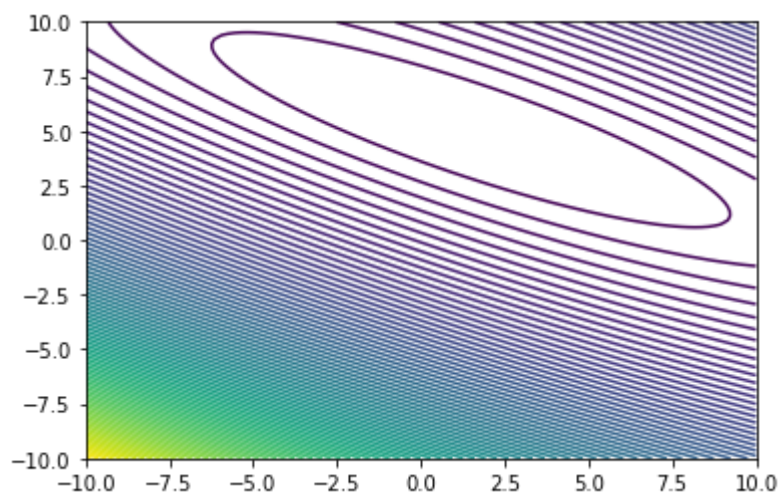
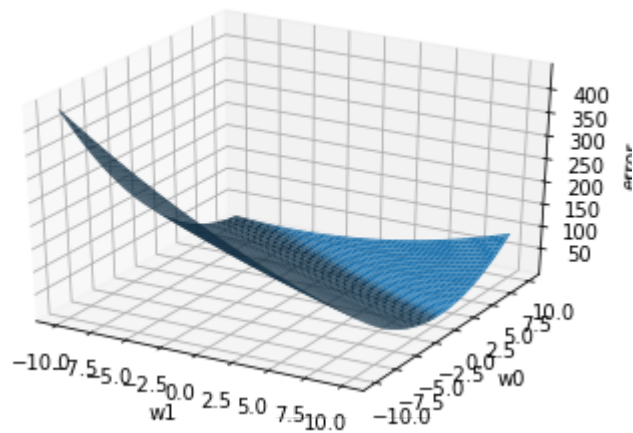
plt.figure()
ax = plt.axes(projection='3d')
ax.plot_surface(S_w1, S_w0, err)
ax.set_xlabel('w1')
ax.set_ylabel('w0')
ax.set_zlabel('error');

plt.figure()
plt.contour(S_w1, S_w0, err,100)
```

```
(100, 100)  
(100, 100)
```

```
Out[ ]:
```

```
<matplotlib.contour.QuadContourSet at 0x7f177cd3c358>
```



d) Gradient descent:

In []:

```
# Gradient descent
w1_init = -7 # initialization
w0_init = -5
lr = 0.6 # learning rate (0.9 diverges, 0.6 quite interesting)
eps = 0.000001

# Gradient computation
def grad_computation(y_actual, w0_old, w1_old, lr, x):
    wo_new = # write your update equation (w0_old + lr*avg(2*(y_actual - y_pre
d)))
    w1_new = # write your update equation (w1_old + lr*avg(2*(y_actual - y_pred
*x))
    return wo_new, w1_new

plt.figure()
plt.contour(S_w1, S_w0, err,100)

for i in list(range(1000)):
    if i == 0:
        w0_old = np.array([w0_init])
        w1_old = np.array([w1_init])
        y_pred = f(w1_old, w0_old,x)
        w0, w1 = grad_computation(y_cor, w0_old, w1_old, lr, x)
    else:
        w0_old = w0
        w1_old = w1
        y_pred = f(w1_old, w0_old,x)
        w0, w1 = grad_computation(y_cor, w0_old, w1_old, lr, x)

    dev = np.abs(error(w1,w0,x,y_cor) - error(w1_old,w0_old,x,y_cor))
    # # print(dev)
    plt.plot([w1_old,w1],[w0_old,w0],color='k')

    if dev <= eps:
        break

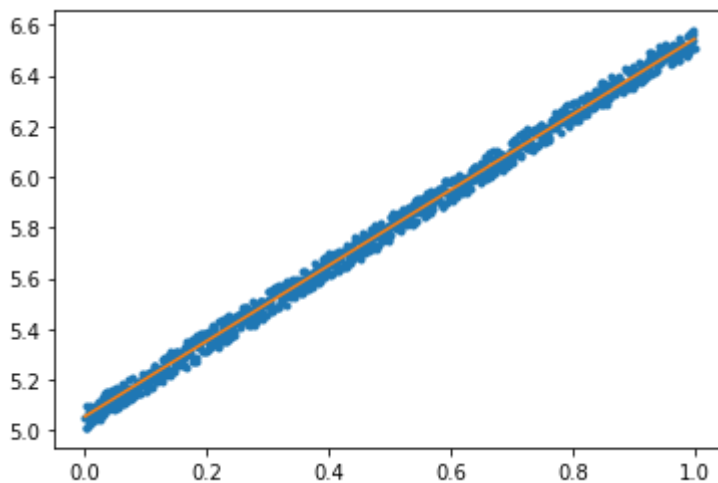
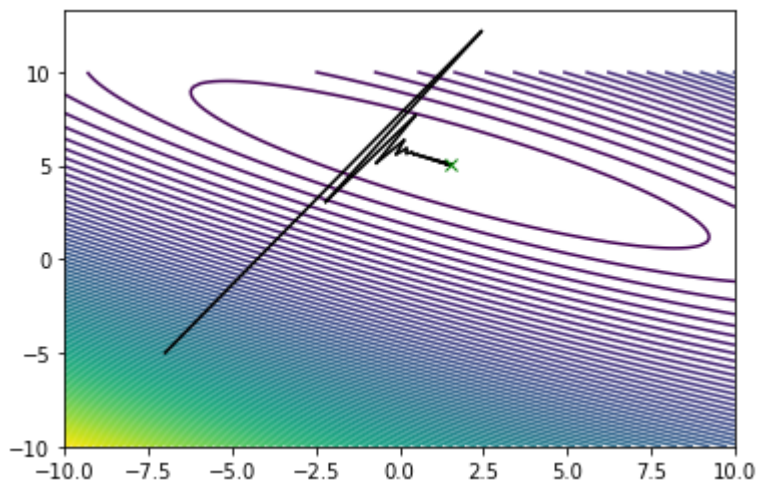
print(w0, w1)
plt.plot(w1,w0,'x',color='g')

plt.figure()
# plotting
#print(x.shape)
y_bestpred=w1*x+w0
#print(y_bestpred.shape)
plt.plot(x,y_cor, '.')
plt.plot(x,y_bestpred)
```

```
[5.0547047] [1.48986653]
```

```
Out[ ]:
```

```
[<matplotlib.lines.Line2D at 0x7f177c679048>]
```



3. Fitting of a plane (two variables)

Here, we will try to fit plane using multivariate regression

i) Generate x_1 and x_2 from range -1 to 1, (30 samples)

ii) Equation of plane $y = w_0 + w_1x_1 + w_2x_2$

iii) Here we will fix w_0 and will learn w_1 and w_2

In []:

```
# data generation
x1=np.linspace(-1,1,30)
x2=np.linspace(-1,1,30)

# equation of plane

w0=0
w1=-2
w2=-2

y= # write the equation of plane here

# plot of plane
X1,X2=np.meshgrid(x1,x2)

Y=#write the equation of plane here


plt.figure()
ax = plt.axes(projection='3d')
ax.plot_surface(X1, X2, Y)
ax.set_xlabel('x1')
ax.set_ylabel('x2')
ax.set_zlabel('y');

# corrupt the data using random noise

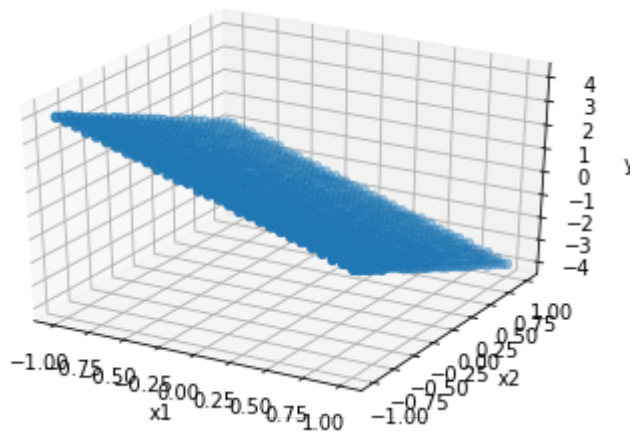
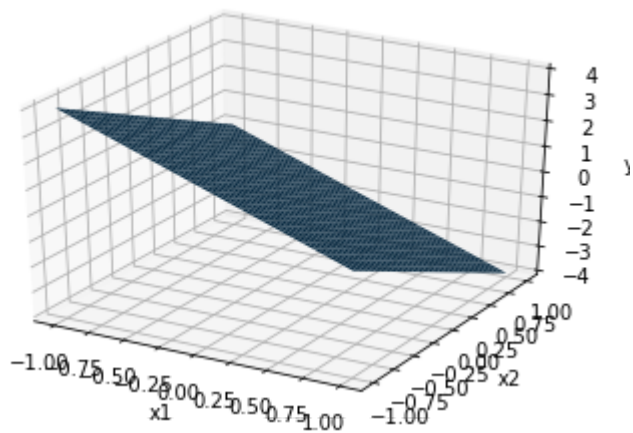
rand=np.random.uniform(0,1,Y.shape)
Y_cor=Y+0.1*rand

plt.figure()
ax = plt.axes(projection='3d')
ax.scatter3D(X1, X2, Y_cor, '.')
ax.set_xlabel('x1')
ax.set_ylabel('x2')
ax.set_zlabel('y');

# generated corrupted data points
x1=X1.flatten()
x2=X2.flatten()
y_cor=Y_cor.flatten()

print(x1.shape)
```

(900,)



b) Error surface

In []:

```
def f(w2,w1,w0,x1,x2):
    return (w0+w1*x1+w2*x2)

srch_w2=np.linspace(-10,10,100)
srch_w1=np.linspace(-10,10,100)

S_w2,S_w1=np.meshgrid(srch_w2,srch_w1)
print(S_w1.shape)

def error(w2,w1,w0,x1,x2,y):
    if len(w1.shape)==0:
        return np.mean(np.power(y-(f(w2,w1,w0,x1,x2)),2))
    else:
        err=np.zeros(w1.shape)
        for x1_i,x2_i,y_i in zip(x1,x2,y):
            #print(w1.shape)
            err1=np.power((np.tile(y_i,w1.shape)-(f(w2,w1,w0,x1_i,x2_i))),2)
            err=err+err1
        return err/x1.shape[0]
err=error(S_w2,S_w1,w0,x1,x2,y_cor)
print(err.shape)

plt.figure()
ax = plt.axes(projection='3d')
ax.plot_surface(S_w2,S_w1,err)
ax.set_xlabel('w2')
ax.set_ylabel('w1')
ax.set_zlabel('error');

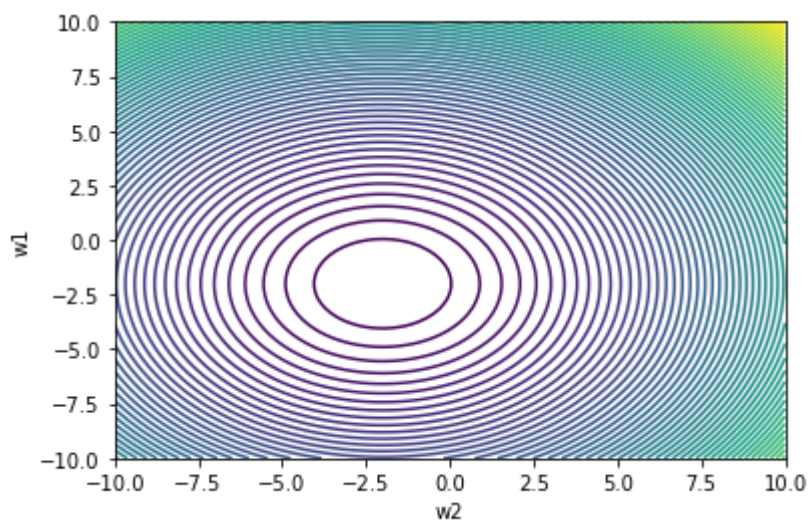
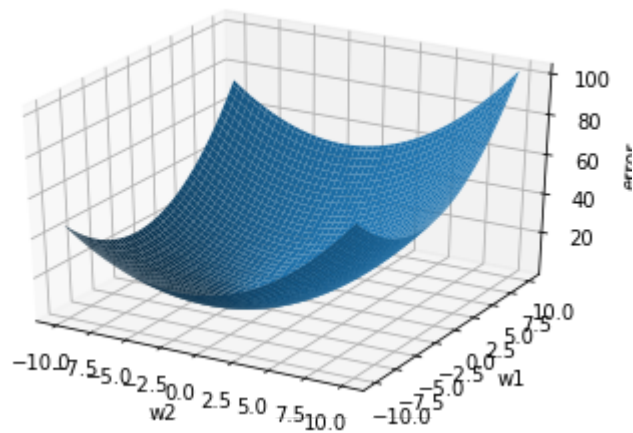
plt.figure()
plt.contour(S_w2, S_w1, err,100)
plt.xlabel('w2')
plt.ylabel('w1')
```

```
(100, 100)
```

```
(100, 100)
```

```
Out[ ]:
```

```
Text(0, 0.5, 'w1')
```



c) Gradient descent:

In []:

```
# Gradient descent
w2_init = 7 # initialization
w1_init = -2
lr = 0.1 # learning rate (0.9 diverges, 0.6 quite interesting)
eps = 0.0000001

# Gradient computation
def grad_computation(y_actual, w2_old, w1_old, w0, lr, x1, x2):
    w2_new = # write your update equation
    w1_new = # write your update equation
    return w2_new, w1_new

plt.figure()
plt.contour(S_w2, S_w1, err, 100)

for i in list(range(10000)):
    if i == 0:
        w2_old = np.array([w2_init])
        w1_old = np.array([w1_init])
        w2, w1 = grad_computation(y_cor, w2_old, w1_old, w0, lr, x1, x2)
    else:
        w2_old = w2
        w1_old = w1
        w2, w1 = grad_computation(y_cor, w2_old, w1_old, w0, lr, x1, x2)

    dev = np.abs(error(w2, w1, w0, x1, x2, y_cor) - error(w2_old, w1_old, w0, x1, x2, y_cor))
    # # print(dev)
    plt.plot([w2_old, w2], [w1_old, w1], color='k')

    if dev <= eps:
        break

print(w2, w1)
plt.plot(w2, w1, 'x', color='g')

# final surface plot
plt.figure()
ax = plt.axes(projection='3d')
ax.scatter3D(X1, X2, Y_cor, '.')
ax.set_xlabel('x1')
ax.set_ylabel('x2')
ax.set_zlabel('y');

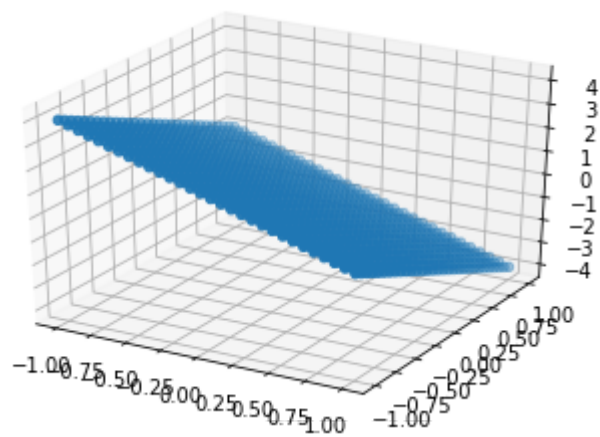
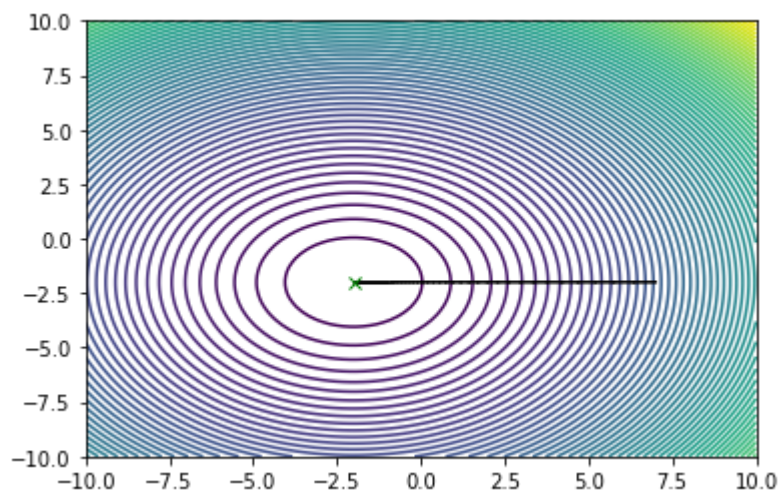
y_bestpred = w0 + w1 * X1 + w2 * X2

ax = plt.axes(projection='3d')
ax.scatter3D(X1, X2, y_bestpred, '.')
```

```
[-2.00083583] [-1.99902039]
```

```
Out[ ]:
```

```
<mpl_toolkits.mplot3d.art3d.Path3DCollection at 0x7f177c993dd8>
```



4. Fitting of M-dimensional hyperplane (M-dimension, both in matrix inversion and gradient descent)

Here we will vectorize the input and will use matrix method to solve the regression problem.

let we have M- dimensional hyperplane we have to fit using regression, the inputs are $x_1, x_2, x_3, \dots, x_M$. in vector form we can write $[x_1, x_2, \dots, x_M]^T$, and similarly the weights are w_1, w_2, \dots, w_M can be written as a vector $[w_1, w_2, \dots, w_M]^T$, Then the equation of the plane can be written as:

$$y = w_1x_1 + w_2x_2 + \dots + w_Mx_M$$

w_1, w_2, \dots, w_M are the scaling parameters in M different direction, and we also need a offset parameter w_0 , to capture the offset variation while fitting.

The final input vector (generally known as augmented feature vector) is represented as $[1, x_1, x_2, \dots, x_M]^T$ and the weight matrix is $[w_0, w_1, w_2, \dots, w_M]^T$, now the equation of the plane can be written as:

$$y = w_0 + w_1x_1 + w_2x_2 + \dots + w_Mx_M$$

In matrix notation: $y = x^T w$ (for a single data point), but in general we are dealing with N- data points, so in matrix notation

$$Y = X^T W$$

where Y is a $N \times 1$ vector, X is a $M \times N$ matrix and W is a $M \times 1$ vector.

$$Error = \frac{1}{N} ||Y - X^T W||^2$$

it looks like a optimization problem, where we have to find W, which will give minimum error.

1. By computation:

$\nabla Error = 0$ will give us W_{opt} , then W_{opt} can be written as:

$$W_{opt} = (XX^T)^{-1}XY$$

1. By gradient descent:

$$W_{new} = W_{old} + \frac{2\lambda}{N} X(Y - X^T W_{old})$$

In []:

```
import numpy as np
import matplotlib.pyplot as plt

class regression:
    # Constructor
    def __init__(self, name='reg'):
        self.name = name # Create an instance variable

    def grad_update(self,w_old,lr,y,x):
        w= # write your update equation here
        return w

    def error(self,w,y,x):
        return np.mean(np.power((y-x.T @ w),2))

    def mat_inv(self,y,x_aug):
        return # write your equation here
        # by Gradient descent
    def Regression_grad_des(self,x,y,lr):
        err=[]
        for i in range(1000):
            if i==0:
                w_init=np.random.uniform(-1,1,(x_aug.shape[0],1))
                w_old=w_init
                w_pred=self.grad_update(w_old,lr,y,x_aug)
            else:
                w_old=w_pred
                w_pred=self.grad_update(w_old,lr,y,x_aug)

        err.append(self.error(w_pred,y,x_aug))
        dev=np.abs(self.error(w_pred,y,x_aug)-self.error(w_old,y,x_aug))
        # print(i)
        if dev<=0.000001:
            break

        return w_pred,err

#####
#####
# Generation of data
sim_dim=5
sim_no_data=1000
x=np.random.uniform(-1,1,(sim_dim,sim_no_data))
print(x.shape)

w=np.array([[1],[2],[3],[5],[9],[3]]) # W=[w0,w1,...,wM] '
print(w.shape)

# # augment feat

x_aug=np.concatenate((np.ones((1,x.shape[1])), x),axis=0)
print(x_aug.shape)

y=x_aug.T @ w # vector multiplication
print(y.shape)
```

```

## corrupted by noise
nois=np.random.uniform(0,1,y.shape)
y=y+0.1*nois

### the data (x_aug and y is generated)#####
#####
#####
# by computation (Normal equation)
reg=regression()
w_opt=reg.mat_inv(y,x_aug)
print(w_opt)

# by Gradient descent
lr=0.01
w_pred,err=reg.Regression_grad_des(x_aug,y,lr)
print(w_pred)

plt.plot(err)

```

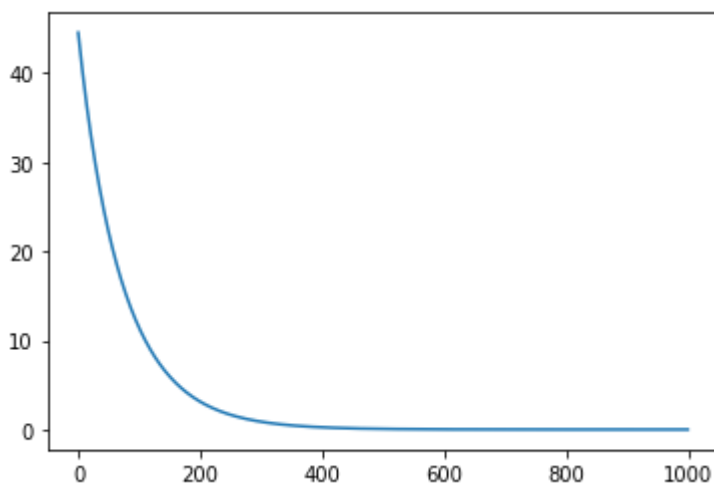
```

(5, 1000)
(6, 1)
(6, 1000)
(1000, 1)
[[1.0490242 ]
 [1.9998412 ]
 [2.99827382]
 [5.00088607]
 [9.0012772 ]
 [2.99881619]]
[[1.0489947 ]
 [1.98690233]
 [2.99364713]
 [4.99021553]
 [8.98736351]
 [2.99235975]]

```

Out[]:

[<matplotlib.lines.Line2D at 0x7f177ca3ba58>]



5. Polynomial regression:

1. Generate data using relation $y = 0.25x^3 + 1.25x^2 - 3x - 3$
2. Corrupt y by adding random noise (uniformly sampled)
3. fit the generated curve using different polynomial order. (Using matrix inversion, and Home work using gradient descent)

In []:

```
## data generation

x=np.linspace(-6,6,100)
# print(x.shape)
x=x[np.newaxis,:]
# print(x.shape)

w=np.array([[ -3],[ -3],[1.25],[0.25]])
# print(w.shape)

def data_transform(X,degree):
    X_new=[]
    for i in range(degree +1):
        X_new.append(X**i)
    X_new = np.concatenate(X_new)
    return X_new

X=data_transform(x,3)

y=X.T @ w

y=y+5*np.random.uniform(0,1,y.shape)

plt.plot(x.T,y,'.')

reg=regression()

# by computation

# for degree 0 polynomial fitting
degree=0
X_1=data_transform(x,degree)
# print(X_1.shape)
w_mat=reg.mat_inv(y,X_1)
# print(y.shape)
# print(w_mat.shape)
y_pred=# insert your code here  $X^T x W$ 
# print(y_pred.shape)
plt.figure()
plt.plot(x.T,y,'.')
plt.plot(x.T,y_pred)

# for degree 1 polynomial fitting
degree=1
X_1=data_transform(x,degree)
# print(X_1.shape)
w_mat=reg.mat_inv(y,X_1)
# print(y.shape)
# print(w_mat.shape)
y_pred=# insert your code here  $X^T x W$ 
# print(y_pred.shape)
plt.figure()
plt.plot(x.T,y,'.')
plt.plot(x.T,y_pred)

# for degree 2 polynomial fitting
```

```
degree=2
X_1=data_transform(x,degree)
# print(X_1.shape)
w_mat=reg.mat_inv(y,X_1)
# print(y.shape)
# print(w_mat.shape)
y_pred=# insert your code here  $X^T \times W$ 
# print(y_pred.shape)
plt.figure()
plt.plot(x.T,y,'.')
plt.plot(x.T,y_pred)

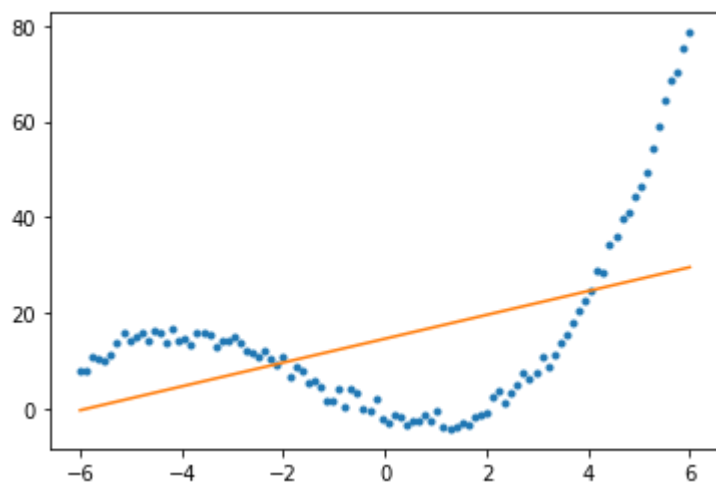
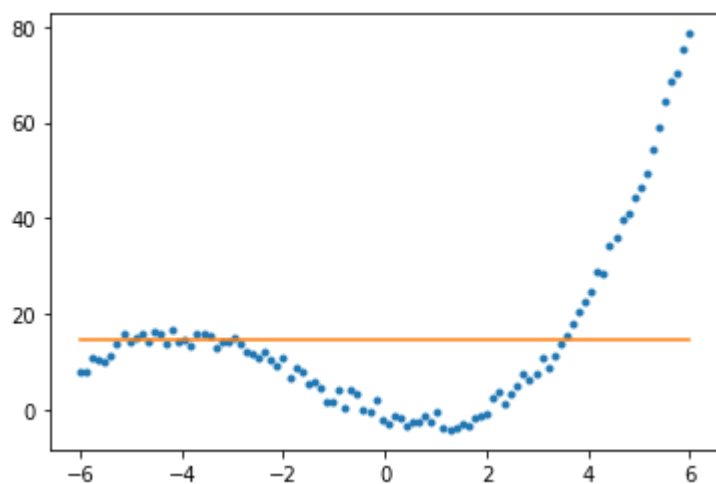
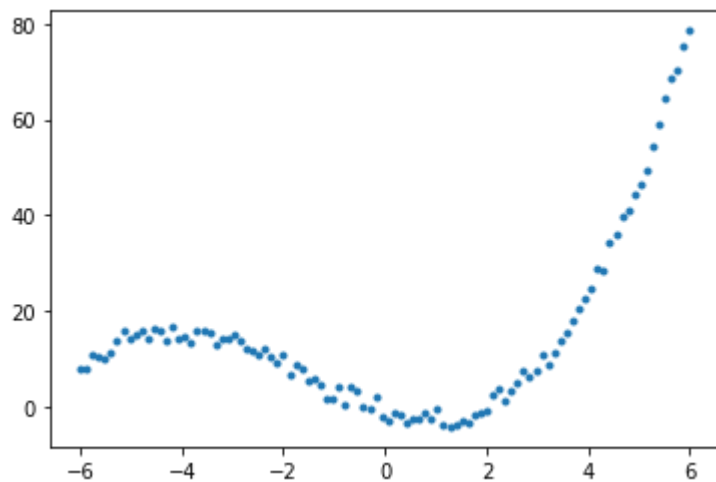
# for degree 3 polynomial fitting
degree=3
X_1=data_transform(x,degree)
# print(X_1.shape)
w_mat=reg.mat_inv(y,X_1)
# print(y.shape)
# print(w_mat.shape)
y_pred=# insert your code here  $X^T \times W$ 
# print(y_pred.shape)
plt.figure()
plt.plot(x.T,y,'.')
plt.plot(x.T,y_pred)

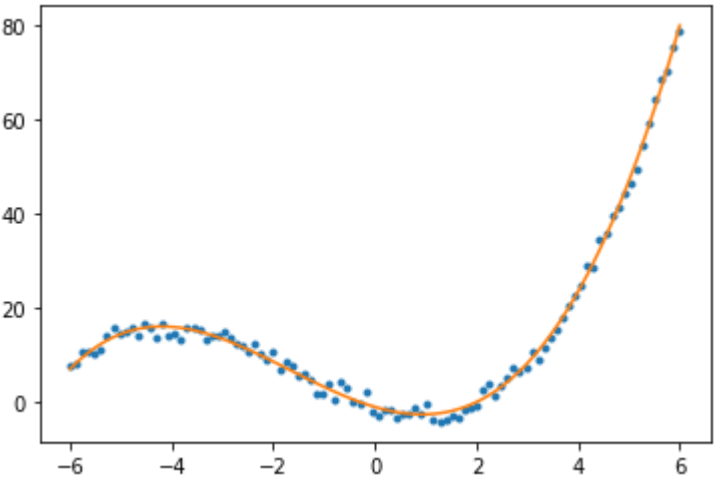
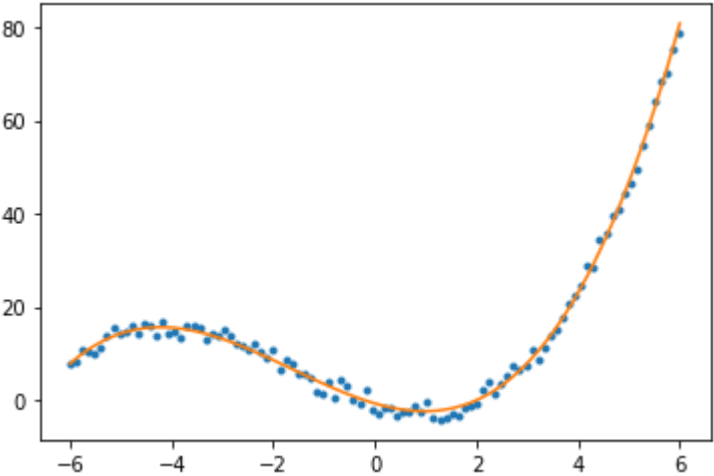
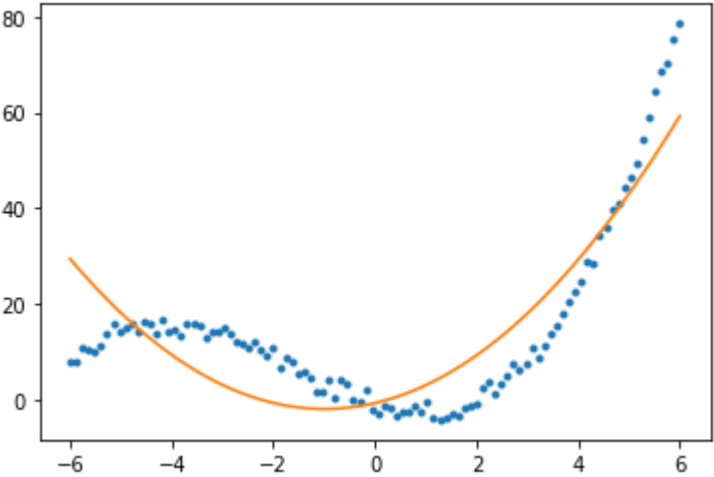
# for degree 4 polynomial fitting
degree=4
X_1=data_transform(x,degree)
# print(X_1.shape)
w_mat=reg.mat_inv(y,X_1)
# print(y.shape)
# print(w_mat.shape)
y_pred=# insert your code here  $X^T \times W$ 
# print(y_pred.shape)
plt.figure()
plt.plot(x.T,y,'.')
plt.plot(x.T,y_pred)

#  $xx=np.linalg.pinv((X_1 @ X_1.T)) @ X_1 @ y$ 
# print(xx.shape)
```


Out[]:

[<matplotlib.lines.Line2D at 0x7f177ad4c3c8>]





6: Practical example (salary prediction)

1. Read data from csv file
2. Do train test split (90% and 10%)
3. Perform using matrix inversion method (Gradient descent homework)
4. find the mean square error in test.

In []:

```

import numpy as np
from google.colab import drive
drive.mount('/gdrive')
##### Csv data read
import csv
with open('/gdrive/My Drive/Machine learning workshop blr/Colab_notebooks/salary_pred_data1.csv','rt') as f:
    data = csv.reader(f)
    row1=[]
    for row in data:
        row1.append(row)

X=row1[1:]
#print(len(X))
XX=np.zeros((len(X),len(X[0])))

for i in range(len(X)):
    XX[i,:]=X[i]

X=XX.T
# print(X.shape)
##### train test separation #####

train_data=X[:,0:900]
test_data=X[:,900:]

x_train=train_data[0:5,:]
y_train=train_data[5,:]
y_train=y_train.T
y_train=y_train[:,np.newaxis]
# print(x_train.shape)

x_test=test_data[0:5,:]
y_test=test_data[5,:]
y_test=y_test.T
y_test=y_test[:,np.newaxis]
# print(x_test.shape)

# augment data #####

x_train=np.concatenate((np.ones((1,x_train.shape[1])), x_train),axis=0)
# print(x_train.shape)

reg=regression()

# by computation #####

w_pred=# insert your code here (training, call reg function)

# print(w_pred)

error=reg.error(w_pred,y_train,x_train)/((np.max(y_train)-np.mean(y_train))**2)

print('Normalized training error=',error,'\n')

# testing #####

```

```
def aug(x):
    return np.concatenate((np.ones((1,x.shape[1])), x),axis=0)

y_pred= #insert your code here (aug(x)^T x w_pred)

# mean square error (testing) (normalized) #####

error=reg.error(w_pred,y_test,aug(x_test))/((np.max(y_test)-np.mean(y_test))**2)

print('Normalized testing error=',error,'\n')

print('predicted salary=',y_pred[0:3],'\n')
print('actual salary=',y_test[0:3])
```

Go to this URL in a browser: https://accounts.google.com/o/oauth2/auth?client_id=947318989803-6bn6qk8qdgf4n4g3pfee6491hc0brc4i.apps.googleusercontent.com&redirect_uri=urn%3aietf%3awg%3aoauth%3a2.0%3aob&scope=email%20https%3a%2f%2fwww.googleapis.com%2fauth%2fdocs.test%20https%3a%2f%2fwww.googleapis.com%2fauth%2fdrive%20https%3a%2f%2fwww.googleapis.com%2fauth%2fdrive.photos.readonly%20https%3a%2f%2fwww.googleapis.com%2fauth%2fpeopleapi.readonly&response_type=code

Enter your authorization code:

.....

Mounted at /gdrive

Normalized training error= 0.02827224237168212

Normalized testing error= 0.05534340421775587

predicted salary= [[33469.35497582]
[52694.83918006]
[58642.13537189]]

actual salary= [[28084.]
[48940.]
[62952.]]

In []:

```
# Showing the testing prediction

plt.plot(y_test) # using actual data
plt.plot(y_pred,'k') # using predicted data
```

Use standard scikit tool to perform linear regression.

1. Reference: https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LinearRegression.html (https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LinearRegression.html).

In []:

```
import numpy as np
from sklearn.linear_model import LinearRegression
```

In []:

```
print(x_train)
print(y_train.shape)
```

In []:

```
reg_scikit = LinearRegression()
```

In []:

```
reg_scikit.fit(x_train.T,y_train)
w_opt=reg_scikit.coef_
print(w_opt.T)
```

In []:

```
# testing #####

def aug(x):
    return np.concatenate((np.ones((1,x.shape[1])), x),axis=0)

y_pred=reg_scikit.predict(aug(x_test).T)

plt.plot(y_test) # using actual data
plt.plot(y_pred,'k') # using predicted data
```