Regression:

Regression is generally used for curve fitting task. Here we will demonstrate regression task for the following.

1) Fitting of line (one variable learning)

2) Fitting of line (two variable learning)

3) Fitting of a plane (two variable)

4) Fitting of M-dimentional hyperplane (M-dimention, both in matrix inversion and gradient descent)

5) Polynomial regression

6) Pratical example of regression task (salary prediction)

1) Fitting of line

a) Generation of line data ($y = w_1 x + w_0$)

i) Generate x, 1000 points from 0-1.

ii) Take $w_0 = 10$ and $w_1 = 1$ and generate y

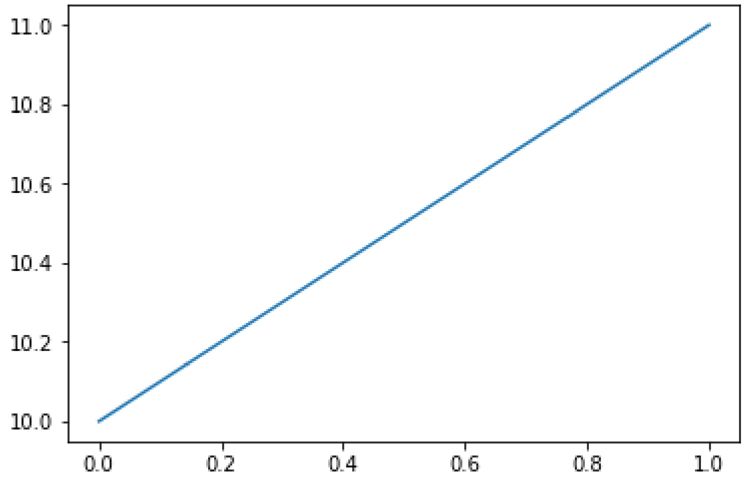
iii) Plot (x,y)

In [3]:

```
import numpy as np
import matplotlib.pyplot as plt

x=np.linspace(0,1,1000)
w1=1
w0=10
# write your equation here
y= w1*x+w0
%matplotlib inline
plt.plot(x,y)
```

Out[3]:

```
[<matplotlib.lines.Line2D at 0x7f6605b32e80>]
```



b) Corrupt the data using uniformly sampled random noise.

i) Generate random numbers uniformly from (0-1) with same size as y.

ii) Corrupt y and generate y_{cor} by adding the generated randomsamples with a weight of 0.1.

iii) Plot (x,y_{cor}) (use scatter plot)

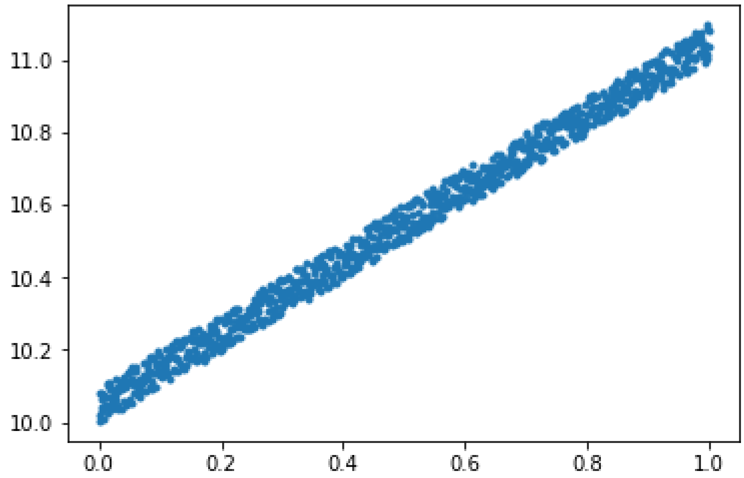
In [4]:

```
rnd_nos=np.random.random(y.shape)
y_cor=y+0.1*rnd_nos
print(rnd_nos.shape)
plt.plot(x,y_cor,'.')
```

```
(1000,)
```

Out[4]:

```
[<matplotlib.lines.Line2D at 0x7f6605ab3b00>]
```



c) Curve prediction using hurestic way.

i) Keep $w_0 = 10$ as constant and find w_1 ?

ii) Create a search space from -5 to 7 for w_1 , by generating 1000 numbers between that.

iii) Find y_{pred} using each value of w_1 .

iv) The y_{pred} that provide least norm error with y, will be decided as best y_{pred} .

$$error = \frac{1}{m}\sum_{i=1}^{M}(y_i - y_{pred_i})^2$$

v) Plot error vs srch_$w1$

vi) First plot the scatter plot (x,y_{cor}) , over that plot (x,$y_{bestpred}$).

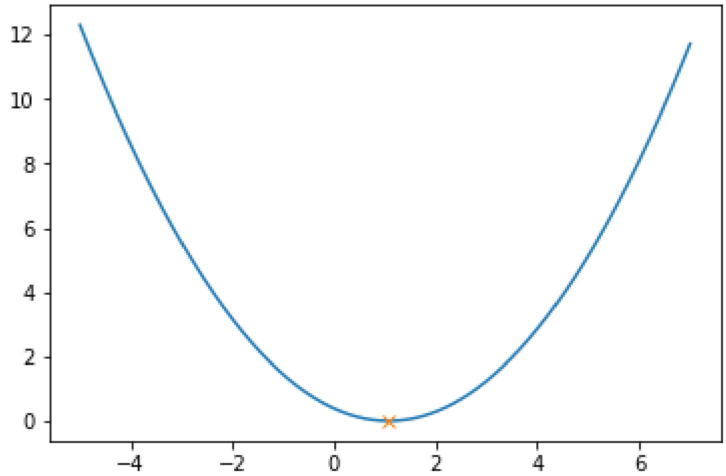
In [5]:

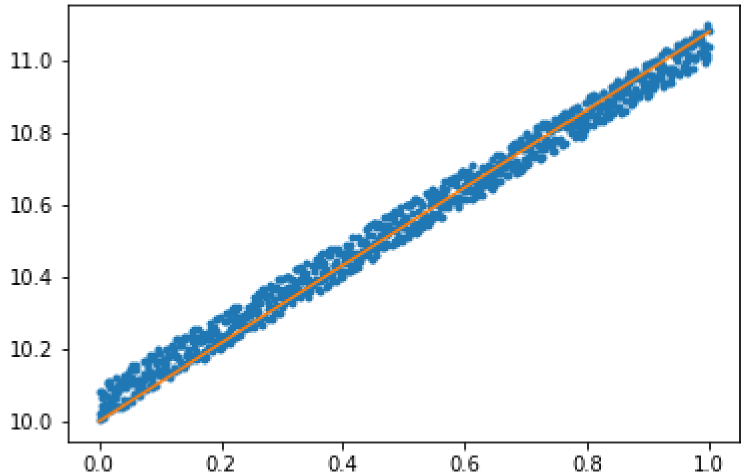
```
# implementation of heurastic search for 1 variable case
def hurestic_srch(x,y_cor):
  srch_w1=np.linspace(-5,7,1000)
  srch_w1=np.expand_dims(srch_w1,axis=1)
  x=np.expand_dims(x,axis=1)
  y_pred=srch_w1 @ x.T+w0    # @ used for matrix multiplication, */np.multiply p
oint wise multiplication,
  #print(x.shape)
  y_cor_rep=np.tile(y_cor,(x.shape[0],1))
  #print(y_cor_rep.shape)

  error=np.sum((np.power((y_cor_rep-y_pred),2)),axis=1)/(x.shape[0]) # row wise
 sum
  #print(error.shape)
  idx = np.where(error == np.min(error))
  w1_opt=srch_w1[idx]
  return w1_opt,error,srch_w1,idx

w1_opt,error,srch_w1,idx=hurestic_srch(x,y_cor)

print(w1_opt)

# error surface plot
plt.plot(srch_w1,error)
plt.plot(w1_opt,error[idx],'x')
plt.figure()
# ploting
#print(x.shape)
y_bestpred=w1_opt*x+w0
#print(y_bestpred.shape)
plt.plot(x,y_cor,'.')
plt.plot(x,y_bestpred.T)
```

```
[[1.07807808]]
```

Out[5]:

```
[<matplotlib.lines.Line2D at 0x7f6605a2e748>]
```





d) Gradient descent

i) $Error = \frac{1}{m}\sum_{i=1}^{M}(y_i - y_{pred_i})^2 = \frac{1}{m}\sum_{i=1}^{M}(y_i - (w_0 + w_1 x_i))^2$

ii) $\nabla Error|_{w1} = \frac{-2}{M}\sum_{i=1}^{M}(y_i - y_{pred_i}) \times x_i$

iii) $w_1|_{new} = w_1|_{old} - \lambda \nabla Error|_{w1} = w_1|_{old} + \frac{2\lambda}{M}\sum_{i=1}^{M}(y_i - y_{pred_i}) \times x_i$

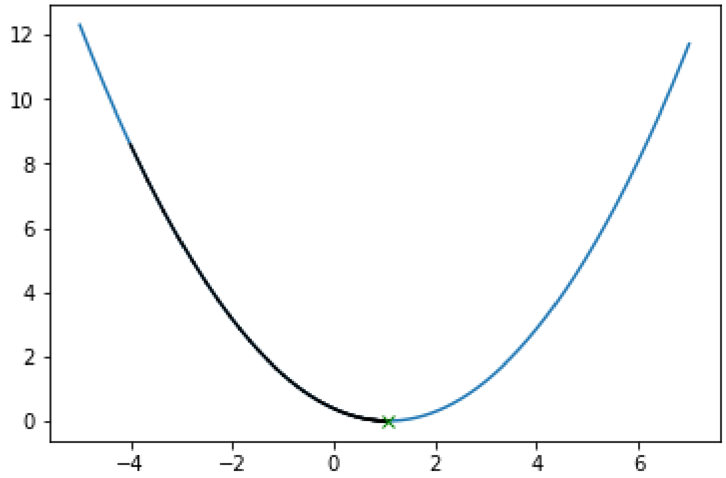
In [6]:

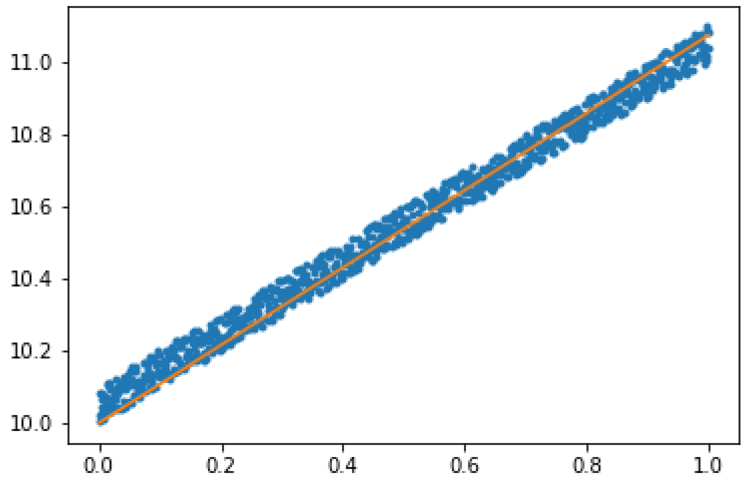
```
import matplotlib.pyplot as plt

def f(w1):
  return (w1*x+w0)

# Gradient computation
def grad_computation(y_actual, w1_old, lr, x):
    w1_new = w1_old + lr*np.average(2*(y_actual - f(w1_old))*x)
    return w1_new


def err(w1,y):
    return np.mean(np.power(y-f(w1),2))


# srch_w1=np.linspace(-10,10,1000)
#error=err(srch_w1,y_cor)
# print(error.shape)
plt.figure()
plt.plot(srch_w1,error)
# Gradient descent
w1_init = -4  # initialization
w0 = 10
lr = 0.1  # learning rate (0.1,2)
eps = 0.0000001

for i in list(range(1000)):
    if i == 0:
        w1_old = w1_init
        w1 = grad_computation(y_cor, w1_old, lr, x)
    else:
        w1_old = w1
        w1 = grad_computation( y_cor, w1_old, lr, x)

    dev = np.abs(err(w1,y_cor) - err(w1_old,y_cor))
    # print(dev)
    #plt.plot(w1,err(w1,y_cor),'x')
    plt.plot([w1_old,w1],[err(w1_old,y_cor),err(w1,y_cor)],color='k')


    if dev <= eps:
        break

print(w1)
plt.plot(w1,err(w1,y_cor),'x',color='g')

plt.figure()
# ploting
#print(x.shape)
y_bestpred=w1*x+w0
#print(y_bestpred.shape)
plt.plot(x,y_cor,'.')
plt.plot(x,y_bestpred)
```

```
1.0724476061926511
```

Out[6]:

```
[<matplotlib.lines.Line2D at 0x7f66058b02b0>]
```





2) Fitting line with two unknown variables

a) Generation of line data ($y = w_1 x + w_0$)

i) Generate x, 1000 points from 0-1.

ii) Take $w_0 = 5$ and $w_1 = 1.5$ and generate y

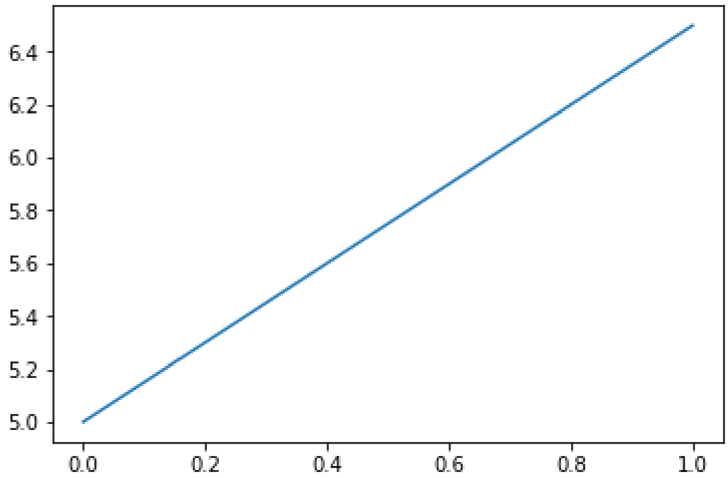
iii) Plot (x,y)

In [7]:

```
import numpy as np
import matplotlib.pyplot as plt

x = np.linspace(0,1,1000)
w0 = 5
w1 = 1.5
# write your equation here
y = w1*x + w0
plt.plot(x,y)
```

Out[7]:

```
[<matplotlib.lines.Line2D at 0x7f66057f1710>]
```



b) Corrupt the data using uniformly sampled random noise.

i) Generate random numbers uniformly from (0-1) with same size as y.

ii) Corrupt y and generate y_{cor} by adding the generated randomsamples with a weight of 0.1.

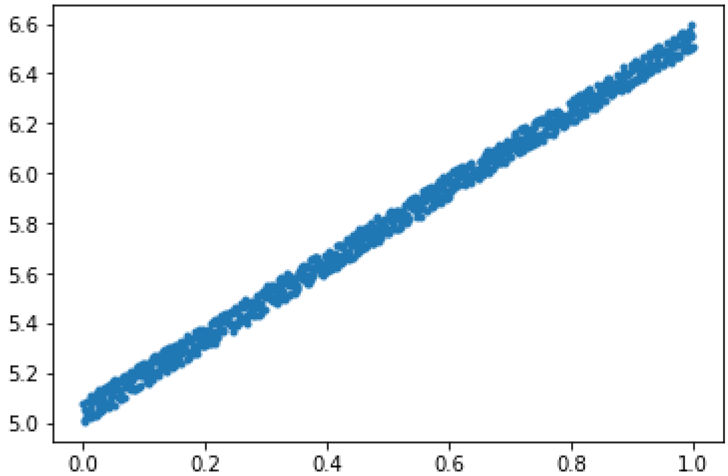
iii) Plot (x,y_{cor}) (use scatter plot)

In [8]:

```
rnd_nos = np.random.random(y.shape)
y_cor = y + 0.1*rnd_nos
# print(rnd_nos.shape)
plt.plot(x,y_cor,'.')
```

Out[8]:

```
[<matplotlib.lines.Line2D at 0x7f660574e710>]
```



c) Plot the error surface

we have all the data points available in y_{cor}, now we have to fit a line with it. (i.e from y_{cor} we have to predict the true value of w_1 and w_0)

i) take w_1 and w_0 from -10 to 10, to get the error surface.

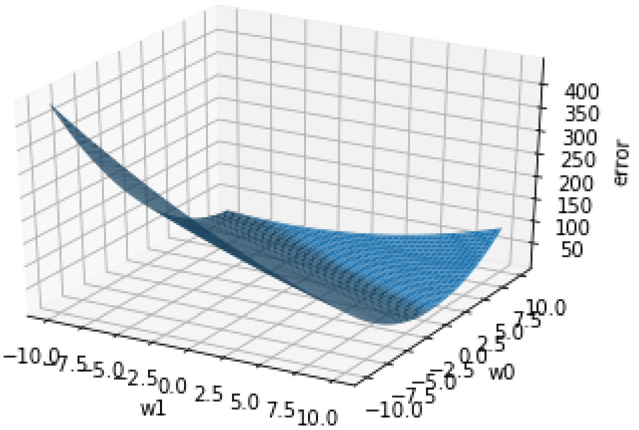
In [10]:

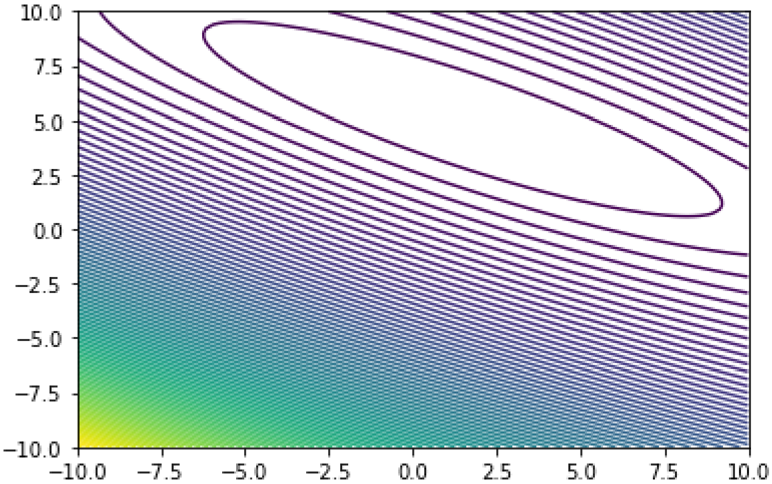
```
# c
from mpl_toolkits import mplot3d
def f(w1, w0,x):
  return (w1*x + w0)

srch_w1=np.linspace(-10,10,100)
srch_w0=np.linspace(-10,10,100)

S_w1,S_w0=np.meshgrid(srch_w1,srch_w0)
print(S_w1.shape)

def error(w1,w0,x,y):
  if len(w1.shape)==0:
    return np.mean(np.power(y-(f(w1,w0,x)),2))
  else:
    err=np.zeros(w1.shape)
    for x_i,y_i in zip(x,y):
      err1=np.power((np.tile(y_i,w1.shape)-(f(w1,w0,x_i))),2)
      err=err+err1
    return err/x.shape[0]
err=error(S_w1,S_w0,x,y_cor)
print(err.shape)

plt.figure()
ax = plt.axes(projection='3d')
ax.plot_surface(S_w1, S_w0, err)
ax.set_xlabel('w1')
ax.set_ylabel('w0')
ax.set_zlabel('error');

plt.figure()
plt.contour(S_w1, S_w0, err,100)
```

```
(100, 100)
(100, 100)
```

Out[10]:

```
<matplotlib.contour.QuadContourSet at 0x7f660542fcc0>
```





d) Gradient descent:

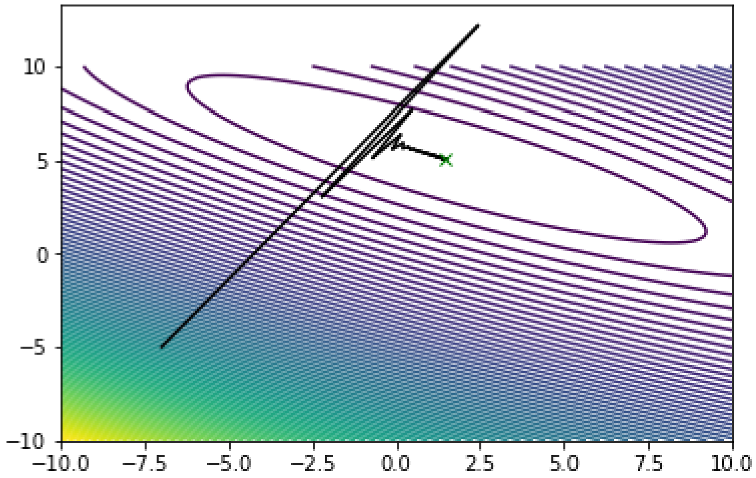
In [11]:

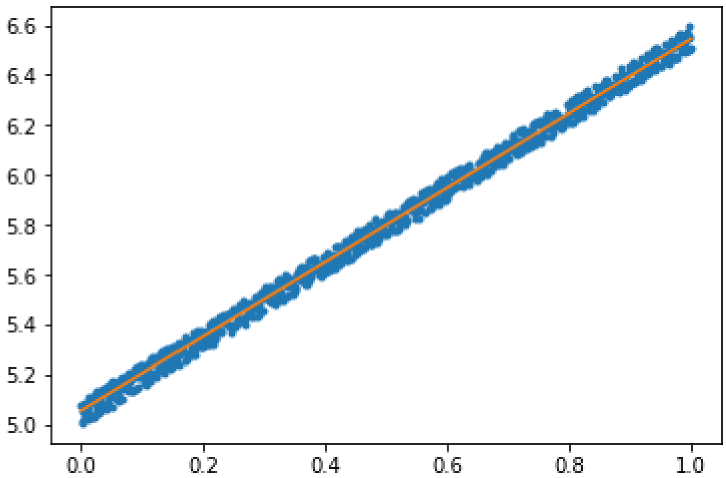
```
# Gradient descent
w1_init = -7 # initialization
w0_init = -5
lr = 0.6  # learning rate (0.9 diverges, 0.6 quite interesting)
eps = 0.000001

# Gradient computation
def grad_computation(y_actual, w0_old, w1_old, lr, x):
    wo_new = w0_old + lr*np.average(2*(y_actual - f(w1_old, w0_old,x)))
    w1_new = w1_old + lr*np.average(2*(y_actual - f(w1_old, w0_old,x))*x)
    return wo_new, w1_new








plt.figure()
plt.contour(S_w1, S_w0, err,100)



for i in list(range(1000)):
    if i == 0:
        w0_old = np.array([w0_init])
        w1_old = np.array([w1_init])
        y_pred = f(w1_old, w0_old,x)
        w0, w1 = grad_computation(y_cor, w0_old, w1_old, lr, x)
    else:
        w0_old = w0
        w1_old = w1
        y_pred = f(w1_old, w0_old,x)
        w0, w1 = grad_computation(y_cor, w0_old, w1_old, lr, x)

    dev = np.abs(error(w1,w0,x,y_cor) - error(w1_old,w0_old,x,y_cor))
    # # print(dev)
    plt.plot([w1_old,w1],[w0_old,w0],color='k')

    if dev <= eps:
        break

print(w0, w1)
plt.plot(w1,w0,'x',color='g')



plt.figure()
# ploting
#print(x.shape)
y_bestpred=w1*x+w0
#print(y_bestpred.shape)
plt.plot(x,y_cor,'.')
plt.plot(x,y_bestpred)
```

```
[5.05757316] [1.48667622]
```

Out[11]:

```
[<matplotlib.lines.Line2D at 0x7f660550aa58>]
```





3. Fitting of a plane (two variables)

Here, we will try to fit plane using multiveriate regression

i) Generate x1 and x2 from range -1 to 1, (30 samples)

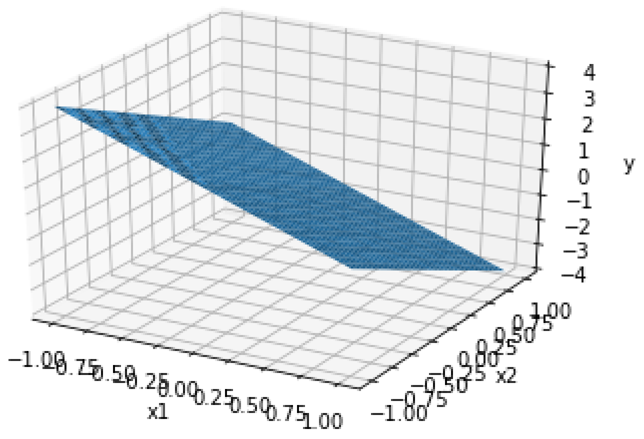
ii) Equation of plane $y = w0 + w1x1 + w2x2$

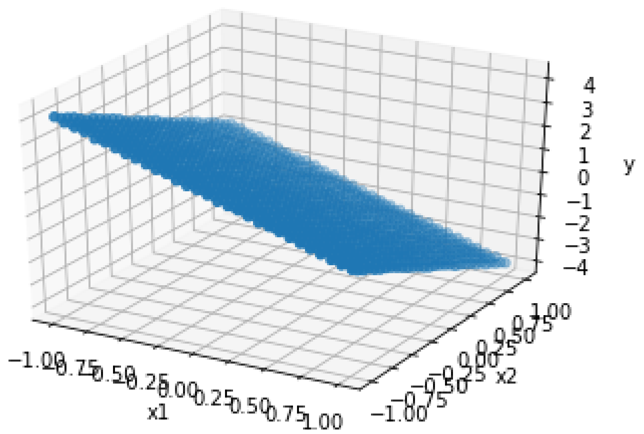
iii) Here we will fix w0 and will learn w1 and w2

In [12]:

```
# data generation
x1=np.linspace(-1,1,30)
x2=np.linspace(-1,1,30)

# equation of plane

w0=0
w1=-2
w2=-2

y= w0+w1*x1+w2*x2

# plot of plane
X1,X2=np.meshgrid(x1,x2)
Y=w0+w1*X1+w2*X2
plt.figure()
ax = plt.axes(projection='3d')
ax.plot_surface(X1, X2, Y)
ax.set_xlabel('x1')
ax.set_ylabel('x2')
ax.set_zlabel('y');

# corupt the data using random noise

rand=np.random.uniform(0,1,Y.shape)
Y_cor=Y+0.1*rand

plt.figure()
ax = plt.axes(projection='3d')
ax.scatter3D(X1, X2, Y_cor,'.')
ax.set_xlabel('x1')
ax.set_ylabel('x2')
ax.set_zlabel('y');

# generated corrupted data points
x1=X1.flatten()
x2=X2.flatten()
y_cor=Y_cor.flatten()

print(x1.shape)
```

```
(900,)
```





b) Error surface

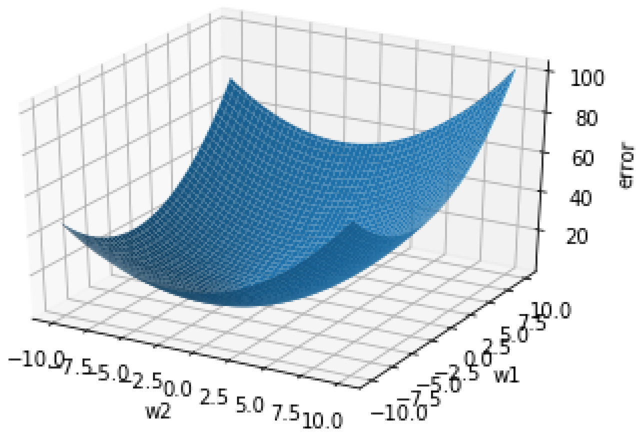
In [13]:

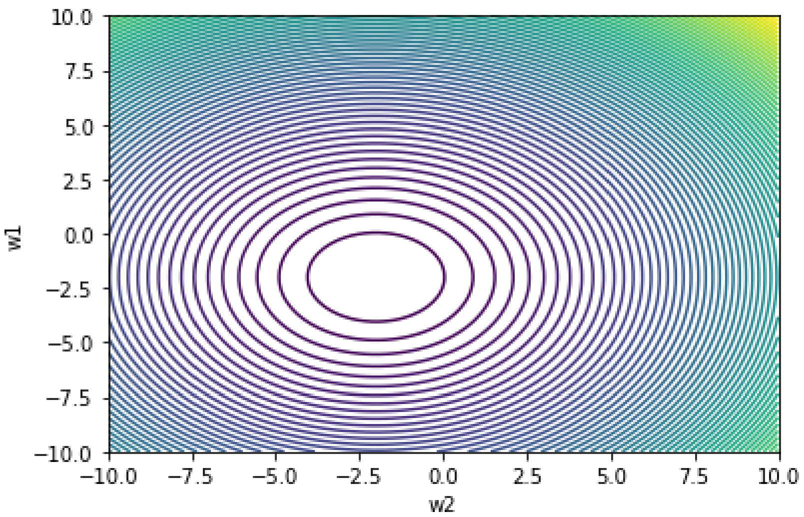
```
def f(w2,w1,w0,x1,x2):
  return (w0+w1*x1+w2*x2)

srch_w2=np.linspace(-10,10,100)
srch_w1=np.linspace(-10,10,100)

S_w2,S_w1=np.meshgrid(srch_w2,srch_w1)
print(S_w1.shape)

def error(w2,w1,w0,x1,x2,y):
  if len(w1.shape)==0:
    return np.mean(np.power(y-(f(w2,w1,w0,x1,x2)),2))
  else:
    err=np.zeros(w1.shape)
    for x1_i,x2_i,y_i in zip(x1,x2,y):
      #print(w1.shape)
      err1=np.power((np.tile(y_i,w1.shape)-(f(w2,w1,w0,x1_i,x2_i))),2)
      err=err+err1
    return err/x1.shape[0]
err=error(S_w2,S_w1,w0,x1,x2,y_cor)
print(err.shape)

plt.figure()
ax = plt.axes(projection='3d')
ax.plot_surface(S_w2,S_w1,err)
ax.set_xlabel('w2')
ax.set_ylabel('w1')
ax.set_zlabel('error');

plt.figure()
plt.contour(S_w2, S_w1, err,100)
plt.xlabel('w2')
plt.ylabel('w1')
```

```
(100, 100)
(100, 100)
```

Out[13]:

```
Text(0,0.5,'w1')
```





c) Gradient descent:

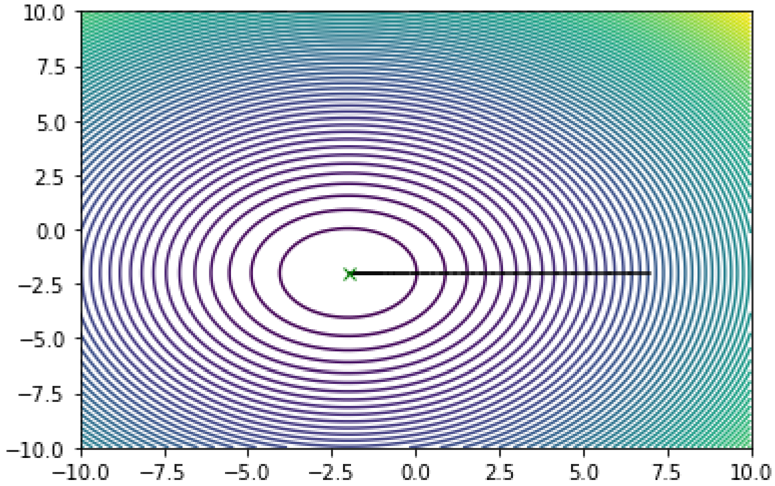
In [14]:

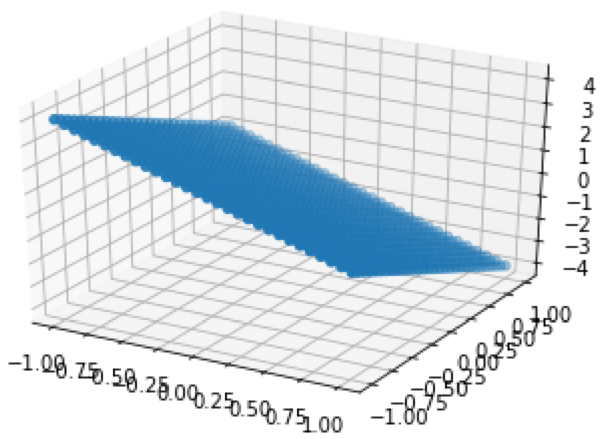
```
# Gradient descent
w2_init = 7 # initialization
w1_init = -2
lr = 0.1  # learning rate (0.9 diverges, 0.6 quite interesting)
eps = 0.0000001

# Gradient computation
def grad_computation(y_actual, w2_old, w1_old,w0, lr, x1,x2):
    w2_new = w2_old + lr*np.average(2*(y_actual - f(w2_old,w1_old, w0_old,x1,x2
))*x2)
    w1_new = w1_old + lr*np.average(2*(y_actual - f(w2_old,w1_old, w0_old,x1,x2
))*x1)
    return w2_new, w1_new









plt.figure()
plt.contour(S_w2, S_w1, err,100)




for i in list(range(10000)):
    if i == 0:
        w2_old = np.array([w2_init])
        w1_old = np.array([w1_init])
        w2, w1 = grad_computation(y_cor, w2_old, w1_old,w0, lr, x1,x2)
    else:
        w2_old = w2
        w1_old = w1
        w2, w1 = grad_computation(y_cor, w2_old, w1_old,w0, lr, x1,x2)

    dev = np.abs(error(w2,w1,w0,x1,x2,y_cor) - error(w2_old,w1_old,w0,x1,x2,y_co
r))
    # # print(dev)
    plt.plot([w2_old,w2],[w1_old,w1],color='k')

    if dev <= eps:
        break

print(w2, w1)
plt.plot(w2,w1,'x',color='g')


# final surface plot
plt.figure()
ax = plt.axes(projection='3d')
ax.scatter3D(X1, X2, Y_cor,'.')
ax.set_xlabel('x1')
ax.set_ylabel('x2')
ax.set_zlabel('y');

y_bestpred=w0+w1*X1+w2*X2

ax = plt.axes(projection='3d')
ax.scatter3D(X1, X2, y_bestpred,'.')
```

```
[-1.99799849] [-1.9999355]
```

Out[14]:

```
<mpl_toolkits.mplot3d.art3d.Path3DCollection at 0x7f66050a9f28>
```





4. Fitting of M-dimentional hyperplane (M-dimention, both in matrix inversion and gradient descent)

Here we will vectorize the input and will use matrix method to solve the regression problem.

let we have M- dimensional hyperplane we have to fit using regression, the inputs are $x1, x2, x3, \ldots, x_M$. in vector form we can write $[x1, x2, \ldots, x_M]^T$, and similarly the weights are $w1, w2, \ldots w_M$ can be written as a vector $[w1, w2, \ldots w_M]^T$, Then the equation of the plane can be written as:

$$y = w1x1 + w2x2 + \ldots + w_M x_M$$

$w1, w2, \ldots., wM$ are the scalling parameters in M different direction, and we also need a offset parameter w0, to capture the offset variation while fitting.

The final input vector (generally known as augmented feature vector) is represented as $[1, x1, x2, \ldots, x_M]^T$ and the weight matrix is $[w0, w1, w2, \ldots w_M]^T$, now the equation of the plane can be written as:

$$y = w0 + w1x1 + w2x2 + \ldots + w_M x_M$$

In matrix notation: $y = x^T w$ (for a single data point), but in general we are dealing with N- data points, so in matrix notation

$$Y = X^T W$$

where Y is a $N \times 1$ vector, X is a $M \times N$ matrix and W is a $M \times 1$ vector.

$$Error = \frac{1}{N} ||Y - X^T W||^2$$

it looks like a optimization problem, where we have to find W, which will give minimum error.

1. **By computation:**

$\nabla Error = 0$ will give us W_{opt}, then W_{opt} can be written as:

$$W_{opt} = (XX^T)^{-1} XY$$

1. **By gradient descent:**

$$W_{new} = W_{old} + \frac{2\lambda}{N} X(Y - X^T W_{old})$$

In [15]:

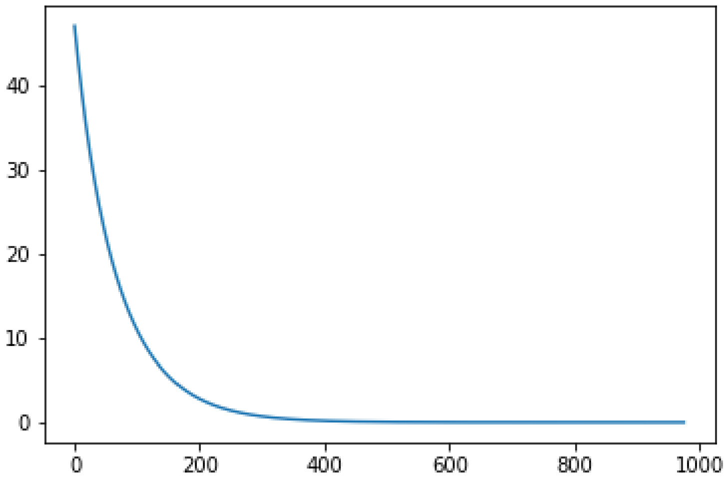
```
import numpy as np
import matplotlib.pyplot as plt

class regression:
  # Constructor
  def __init__(self, name='reg'):
    self.name = name  # Create an instance variable

  def grad_update(self,w_old,lr,y,x):
    w=w_old+(2/x.shape[1])*lr*(x @ (y-(x.T @ w_old)))
    return w

  def error(self,w,y,x):
    return np.mean(np.power((y-x.T @ w),2))

  def mat_inv(self,y,x_aug):
    return np.linalg.pinv((x_aug @ x_aug.T)) @ x_aug @ y
    # by Gradien descent
  def Regression_grad_des(self,x,y,lr):
    err=[]
    for i in range(1000):
      if i==0:
        w_init=np.random.uniform(-1,1,(x_aug.shape[0],1))
        w_old=w_init
        w_pred=self.grad_update(w_old,lr,y,x_aug)
      else:
        w_old=w_pred
        w_pred=self.grad_update(w_old,lr,y,x_aug)

      err.append(self.error(w_pred,y,x_aug))
      dev=np.abs(self.error(w_pred,y,x_aug)-self.error(w_old,y,x_aug))
          # print(i)
      if dev<=0.000001:
        break

    return w_pred,err


################################################################################
############################
# Generation of data
sim_dim=5
sim_no_data=1000
x=np.random.uniform(-1,1,(sim_dim,sim_no_data))
print(x.shape)

w=np.array([[1],[2],[3],[5],[9],[3]]) # W=[w0,w1,.....,wM]'
print(w.shape)

# # augment feat

x_aug=np.concatenate((np.ones((1,x.shape[1])), x),axis=0)
print(x_aug.shape)


y=x_aug.T @ w  # vector multiplication
print(y.shape)
```

```
## corrupted by noise
nois=np.random.uniform(0,1,y.shape)
y=y+0.1*nois

### the data (x_aug and y is generated)#####
################################################################################
###############################
# by computation (Normal equation)
reg=regression()
w_opt=reg.mat_inv(y,x_aug)
print(w_opt)

# by Gradien descent
lr=0.01
w_pred,err=reg.Regression_grad_des(x_aug,y,lr)
print(w_pred)

plt.plot(err)
```

```
(5, 1000)
(6, 1)
(6, 1000)
(1000, 1)
[[1.04867794]
 [1.99721559]
 [2.99922308]
 [4.99971614]
 [8.99812062]
 [3.00162203]]
[[1.04926043]
 [1.99242029]
 [2.99426209]
 [4.99306726]
 [8.98765103]
 [2.99821402]]
```

Out[15]:

```
[<matplotlib.lines.Line2D at 0x7f6604309828>]
```



5. Polynomial regression:

1. Generate data using relation $y = 0.25x^3 + 1.25x^2 - 3x - 3$
2. Corrupt y by adding random noise (uniformly sampled)
3. fit the generated curve using different polynomial order. (Using matrix inversion, and Home work using gradient descent)

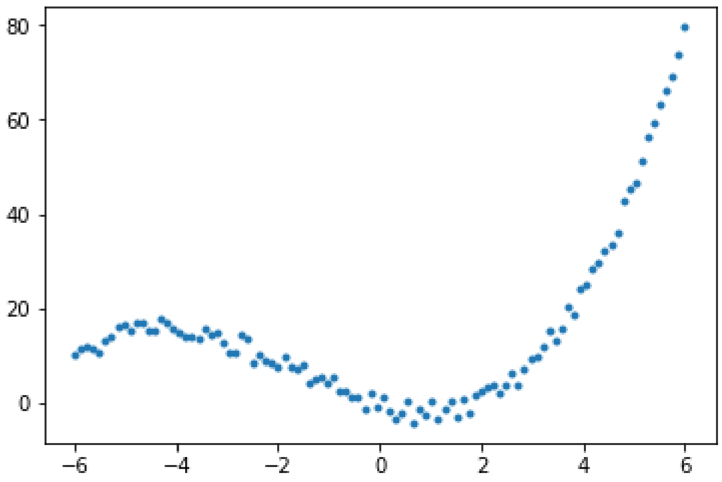
In [16]:

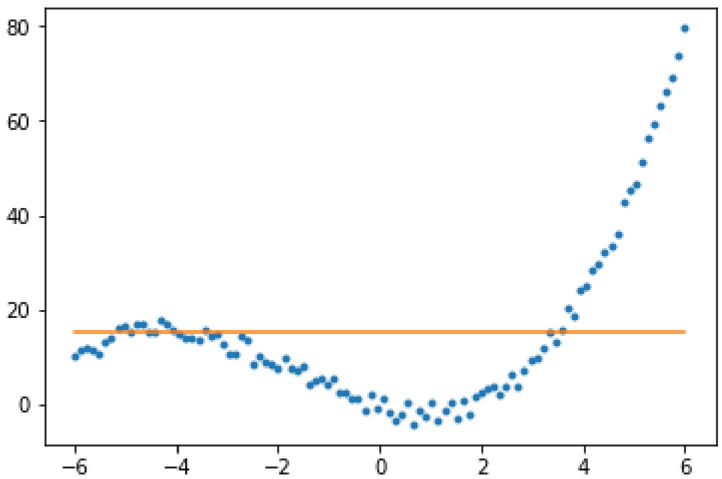
```
## data generation

x=np.linspace(-6,6,100)
# print(x.shape)
x=x[np.newaxis,:]
# print(x.shape)


w=np.array([[-3],[-3],[1.25],[0.25]])
# print(w.shape)

def data_transform(X,degree):
  X_new=[]
  for i in range(degree +1):
    X_new.append(X**i)
  X_new = np.concatenate(X_new)
  return X_new


X=data_transform(x,3)

y=X.T @ w

y=y+5*np.random.uniform(0,1,y.shape)

plt.plot(x.T,y,'.')

reg=regression()

# by computation

# for degree 0 polynomial fitting
degree=0
X_1=data_transform(x,degree)
# print(X_1.shape)
w_mat=reg.mat_inv(y,X_1)
# print(y.shape)
# print(w_mat.shape)
y_pred=X_1.T @ w_mat
# print(y_pred.shape)
plt.figure()
plt.plot(x.T,y,'.')
plt.plot(x.T,y_pred)


# for degree 1 polynomial fitting
degree=1
X_1=data_transform(x,degree)
# print(X_1.shape)
w_mat=reg.mat_inv(y,X_1)
# print(y.shape)
# print(w_mat.shape)
y_pred=X_1.T @ w_mat
# print(y_pred.shape)
plt.figure()
plt.plot(x.T,y,'.')
plt.plot(x.T,y_pred)

# for degree 2 polynomial fitting
```

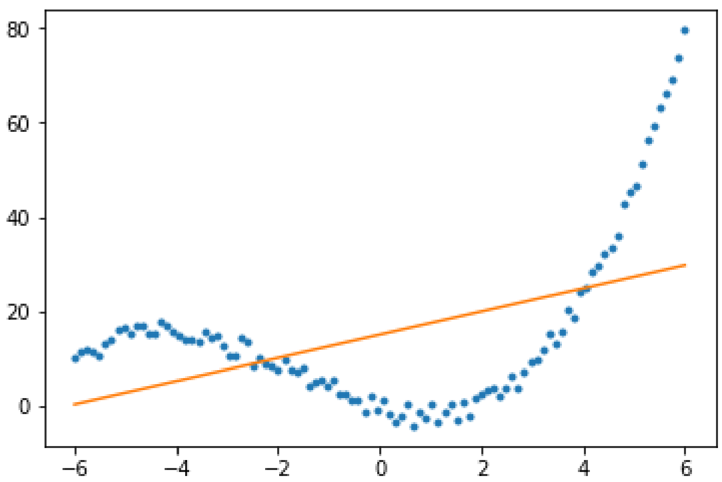
```
degree=2
X_1=data_transform(x,degree)
# print(X_1.shape)
w_mat=reg.mat_inv(y,X_1)
# print(y.shape)
# print(w_mat.shape)
y_pred=X_1.T @ w_mat
# print(y_pred.shape)
plt.figure()
plt.plot(x.T,y,'.')
plt.plot(x.T,y_pred)


# for degree 3 polynomial fitting
degree=3
X_1=data_transform(x,degree)
# print(X_1.shape)
w_mat=reg.mat_inv(y,X_1)
# print(y.shape)
# print(w_mat.shape)
y_pred=X_1.T @ w_mat
# print(y_pred.shape)
plt.figure()
plt.plot(x.T,y,'.')
plt.plot(x.T,y_pred)


# for degree 4 polynomial fitting
degree=4
X_1=data_transform(x,degree)
# print(X_1.shape)
w_mat=reg.mat_inv(y,X_1)
# print(y.shape)
# print(w_mat.shape)
y_pred=X_1.T @ w_mat
# print(y_pred.shape)
plt.figure()
plt.plot(x.T,y,'.')
plt.plot(x.T,y_pred)

# xx=np.linalg.pinv((X_1 @ X_1.T)) @ X_1 @ y
# print(xx.shape)
```

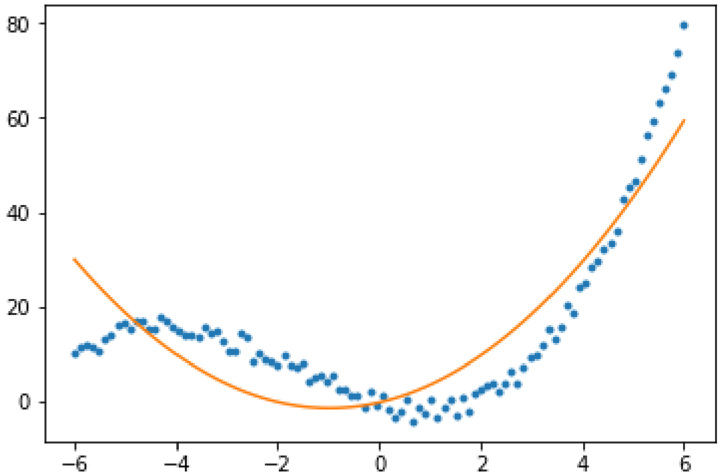
Out[16]:

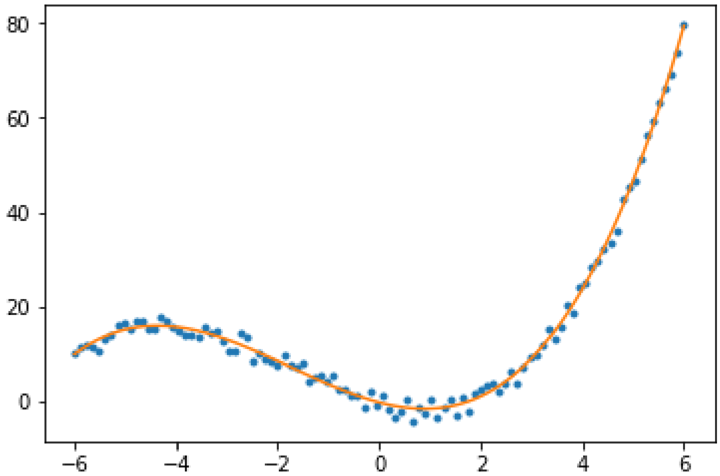
```
[<matplotlib.lines.Line2D at 0x7f6604227630>]
```

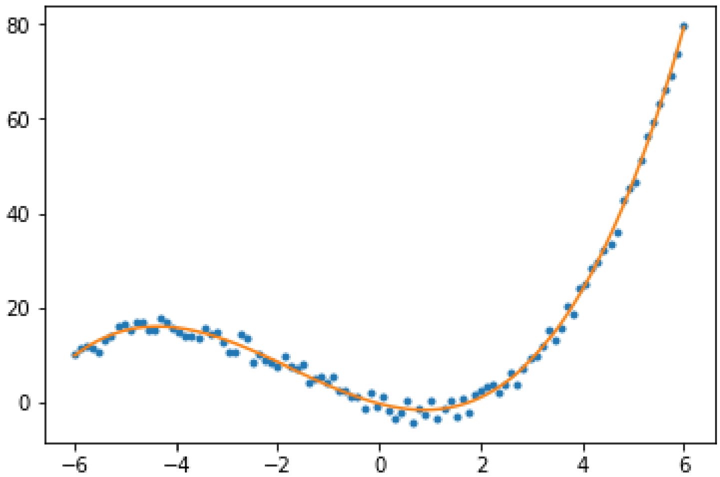







80
60
40
20
0
−6
−4
−2
0
2
4
6


80
60
40
20
0
−6
−4
−2
0
2
4
6


80
60
40
20
0
−6
−4
−2
0
2
4
6

6: Practical example (salary prediction)

1. Read data from csv file
2. Do train test split (90% and 10%)
3. Perform using matrix inversion method (Gradiant descent homework)
4. find the mean square error in test.

In [17]:

```
import numpy as np
# from google.colab import drive
# drive.mount('/gdrive')
###### Csv data read
import csv
with open('/home/jagabandhu/Documents/Machine_learning_workshop_blr/Workshop2/co
des_to_share/salary_pred_data1.csv','rt')as f:
  data = csv.reader(f)
  row1=[]
  for row in data:
        row1.append(row)


X=row1[1:]
#print(len(X))
XX=np.zeros((len(X),len(X[0])))

for i in range(len(X)):
    XX[i,:]=X[i]


X=XX.T
# print(X.shape)
#########   train test separation ################

train_data=X[:,0:900]
test_data=X[:,900:]

x_train=train_data[0:5,:]
y_train=train_data[5,:]
y_train=y_train.T
y_train=y_train[:,np.newaxis]
# print(x_train.shape)

x_test=test_data[0:5,:]
y_test=test_data[5,:]
y_test=y_test.T
y_test=y_test[:,np.newaxis]
# print(x_test.shape)


# augment data ###########

x_train=np.concatenate((np.ones((1,x_train.shape[1])), x_train),axis=0)
# print(x_train.shape)

reg=regression()

# by computation ##############

w_pred=reg.mat_inv(y_train,x_train)

# print(w_pred)

error=reg.error(w_pred,y_train,x_train)/((np.max(y_train)-np.mean(y_train))**2)

print('Normalized training error=',error,'\n')

# testing  ###################
```

```
def aug(x):
  return np.concatenate((np.ones((1,x.shape[1])), x),axis=0)

y_pred=(aug(x_test)).T @ w_pred

# mean square error (testing) (normalized) ############

error=reg.error(w_pred,y_test,aug(x_test))/((np.max(y_test)-np.mean(y_test))**2)

print('Normalized testing error=',error,'\n')

print('predicted salary=',y_pred[0:3],'\n')
print('actual salary=',y_test[0:3])
```

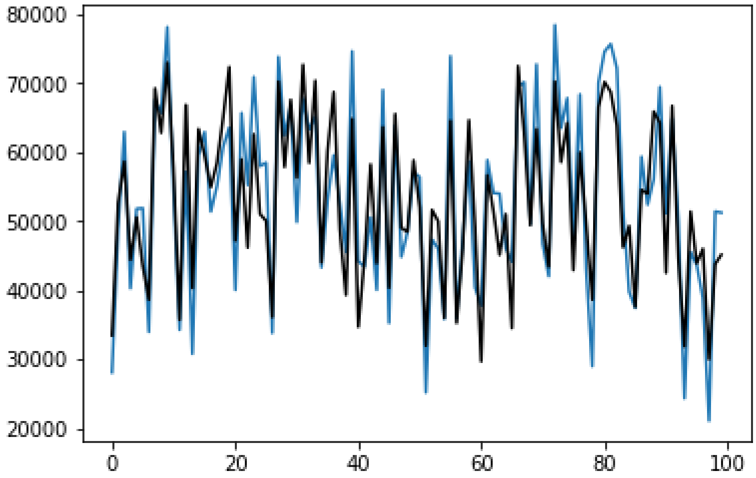
```
Normalized training error= 0.02827224237168212

Normalized testing error= 0.05534340421775587

predicted salary= [[33469.35497582]
 [52694.83918006]
 [58642.13537189]]

actual salary= [[28084.]
 [48940.]
 [62952.]]
```

In [18]:

```
# Showing the testing prediction

plt.plot(y_test) # using actual data
plt.plot(y_pred,'k') # using predicted data
```

Out[18]:

```
[<matplotlib.lines.Line2D at 0x7f6605a76358>]
```



Use standard scikit tool to perform the same.

1. Reference: https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LinearRegression.html (https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LinearRegression.html)

In [19]:

```
import numpy as np
from sklearn.linear_model import LinearRegression
```

In [20]:

```
print(x_train)
print(y_train.shape)
```

```
[[ 1.  1.  1. ...  1.  1.  1.]
 [ 2.  4.  1. ...  2.  2.  3.]
 [11. 14. 13. ...  3.  3.  9.]
 [34. 28. 55. ... 56. 57. 59.]
 [ 4.  1.  3. ...  2.  2.  1.]
 [ 3.  4.  2. ...  2.  6.  3.]]
(900, 1)
```

In [21]:

```
reg_scikit = LinearRegression()
```

In [22]:

```
reg_scikit.fit(x_train.T,y_train)
w_opt=reg_scikit.coef_
print(w_opt.T)
```

```
[[0.00000000e+00]
 [1.81398462e+03]
 [5.83067154e+01]
 [1.64280940e+00]
 [3.42742579e+02]
 [4.95762443e+03]]
```

In [23]:

```
# testing  ###################

def aug(x):
  return np.concatenate((np.ones((1,x.shape[1])), x),axis=0)

y_pred=reg_scikit.predict(aug(x_test).T)

plt.plot(y_test) # using actual data
plt.plot(y_pred,'k') # using predicted data
```

Out[23]:

```
[<matplotlib.lines.Line2D at 0x7f660554bef0>]
```