

## Heuristic Analysis

### 0. Optimal Plan for Problem 1, 2, and 3

We define the optimal plan as the algorithm that arrives at the goal with the shortest path length.

|                     | Problem 1  | Problem 2   | Problem 3  |
|---------------------|--|---|--|
| <b>Path Length</b>  | 6  | 9   | 12   |
| <b>Time Elapsed</b> | 0.0464265490   | 16.643273759  | 117.58682158   |
| <b>Plan</b>         | Load(C1, P1, SFO)<br>Load(C2, P2, JFK)<br>Fly(P1, SFO, JFK)<br>Fly(P2, JFK, SFO)<br>Unload(C1, P1, JFK)<br>Unload(C2, P2, SFO) | Load(C1, P1, SFO)<br>Load(C2, P2, JFK)<br>Load(C3, P3, ATL)<br>Fly(P1, SFO, JFK)<br>Fly(P2, JFK, SFO)<br>Fly(P3, ATL, SFO)<br>Unload(C3, P3, SFO)<br>Unload(C2, P2, SFO)<br>Unload(C1, P1, JFK) | Load(C1, P1, SFO)<br>Load(C2, P2, JFK)<br>Fly(P1, SFO, ATL)<br>Load(C3, P1, ATL)<br>Fly(P2, JFK, ORD)<br>Load(C4, P2, ORD)<br>Fly(P2, ORD, SFO)<br>Fly(P1, ATL, JFK)<br>Unload(C4, P2, SFO)<br>Unload(C3, P1, JFK)<br>Unload(C2, P2, SFO)<br>Unload(C1, P1, JFK) |

### 1. Non Heuristic Search Metrics

#### Breadth First Search Algorithm Analysis

|                     | Problem 1            | Problem 2          | Problem 3        |
|---------------------|----------------------|--------------------|------------------|
| <b>Path Length</b>  | 6                    | 9                  | 12               |
| <b>Time Elapsed</b> | 0.046426549000898376 | 16.643273759000294 | 117.586821582001 |
| <b>Expansions</b>   | 43                   | 3346               | 14120            |
| <b>Goal Tests</b>   | 56                   | 4612               | 17673            |
| <b>New Nodes</b>    | 180                  | 30534              | 124926           |

## Depth First Search Algorithm Analysis

|              | Problem 1            | Problem 2         | Problem 3         |
|--------------|----------------------|-------------------|-------------------|
| Path Length  | 12                   | 1085              | 660               |
| Time Elapsed | 0.010793465000460856 | 9.589888878996135 | 4.247559849995014 |
| Expansions   | 12                   | 1124              | 677               |
| Goal Tests   | 13                   | 1125              | 678               |
| New Nodes    | 48                   | 10017             | 5608              |

## Uniform Cost (Non-Heuristic) Search Algorithm Analysis

|              | Problem 1           | Problem 2          | Problem 3         |
|--------------|---------------------|--------------------|-------------------|
| Path Length  | 6                   | 9                  | 12                |
| Time Elapsed | 0.04852645599748939 | 15.063412668008823 | 62.33303530601552 |
| Expansions   | 55                  | 4853               | 18223             |
| Goal Tests   | 57                  | 4855               | 18225             |
| New Nodes    | 24                  | 44041              | 159618            |

The algorithm trades off optimality for speed. The trade-off between the suggested path length and the time elapsed is consistent with our understanding the breadth first search and the depth first search algorithm. While the depth first search algorithm searches the tree of possibilities rapidly, it does not arrive at an optimal solution, suggesting path lengths of 12, 1085, and 660 for problems 1,2, and 3 respectively. This is a consequence of the algorithm traversing to the deepest layer of the tree, and only exploring a singular subtree in the process. In contrast, although the breadth first search algorithm takes longer, it reports an optimal path length of 6, 9 and 12 for problems 1,2, and 3 respectively. This is also aligned with the nature of the breadth first search algorithm which essentially explores the entire frontier of all subtrees starting from the root . While run time may suffer from the drastic increase in the number of expansions and and new nodes explored, the algorithm returns an optimal path length. A small improvement to the breadth first search algorithm, the uniform cost search algorithm, produces the best non-heuristic search performance by yielding the optimal path length with a short time elapsed. The simple fix is that instead of expanding the shallowest node, the node with the lowest *path cost*  $g(n)$  is expanded. Therefore, this ensures that uniform cost search

expands nodes in their optimal path cost, explaining the resulting optimal path length and reduction in time elapsed.

## 2. Heuristic Search Metrics

### A\* Search Algorithm Analysis

|                     | Problem 1           | Problem 2          | Problem 3         |
|---------------------|---------------------|--------------------|-------------------|
| <b>Path Length</b>  | 6                   | 9                  | 12                |
| <b>Time Elapsed</b> | 0.05020688200602308 | 14.830767720006406 | 64.43145233998075 |
| <b>Expansions</b>   | 55                  | 4853               | 18223             |
| <b>Goal Tests</b>   | 57                  | 4855               | 18225             |
| <b>New Nodes</b>    | 224                 | 44041              | 159618            |

### A\* Ignore Preconditions Search Algorithm Analysis

|                     | Problem 1           | Problem 2         | Problem 3         |
|---------------------|---------------------|-------------------|-------------------|
| <b>Path Length</b>  | 6                   | 9                 | 12                |
| <b>Time Elapsed</b> | 0.04958194699429441 | 5.136973121989286 | 21.63100447699253 |
| <b>Expansions</b>   | 41                  | 1450              | 5040              |
| <b>Goal Tests</b>   | 43                  | 1452              | 5042              |
| <b>New Nodes</b>    | 170                 | 13303             | 44944             |

### A\* Level Sum Search Algorithm Analysis

|                     | Problem 1         | Problem 2          | Problem 3          |
|---------------------|-------------------|--------------------|--------------------|
| <b>Path Length</b>  | 6                 | 9                  | 12                 |
| <b>Time Elapsed</b> | 1.376313813001616 | 221.07149807699898 | 1429.6793429190002 |
| <b>Expansions</b>   | 11                | 86                 | 316                |
| <b>Goal Tests</b>   | 13                | 88                 | 318                |
| <b>New Nodes</b>    | 50                | 841                | 2912               |

### 3. Compare and Contrast

We observe that the **ignore preconditions** heuristic resulted in the fastest run-time among all the A\* heuristics, with a range of 0.0496 - 21.6310 seconds. However, the “**level sum**” heuristic resulted in the least number of expansions, goal tests, and new nodes, in contrast to both the ignore preconditions heuristic and the baseline A\* search algorithm. All three A\* heuristic searches resulted in a path length of 6 for Problem 1, 9 for Problem 2, and 12 for Problem 3.

The “best heuristic” used in these problems can be best described by what the user’s definition of “best” entails. If “best” is defined as a fast execution time, then the clearest candidate is **A\* ignore preconditions**. However, if the “best heuristic” is defined as minimizing the number of expansions, goal tests, and total new nodes, then the clear winner is the A\* level sum search algorithm.

If we consider individual nodes as states, and all possible edges as actions between those states, then the ignore preconditions heuristic would function to increase the number of possible edges and therefore connectivity across all possible states. As a result, the elimination of preconditions and enhanced connectivity between nodes could explain the drastic increase in run time performance to get from the start state to the goal state.

The performance of the **level sum heuristic** is also expected, due to the heuristic’s traversal of the planning graph which, although temporally expensive, is ultimately accurate in returning the optimal path. As a brief overview, according to Norvig et al., in Artificial Intelligence, 3rd edition, the level sum heuristic exists as an extension of the subgoal independence assumption. By assuming that individual subgoals are decomposable, and the cost to achieve the optimal goal may be approximated by the sum of all subgoals required to achieve the end goal state. In the context of air cargo loading, the problem space is sufficiently decomposable for the level sum heuristic to return an optimal path.