Course Code: 505 Course

Title: .Net Technology

Unit–4: Principles of Mathematics, Geometry and Triangles in Ancient Indian Knowledge

4.1 Principles of Mathematics by Aryabhata

4.1.1 Principles of Mathematics: Sutra (Verse 1.1, Āryabhaṭīya – Gītikāpāda)

Sanskrit Sutra:

अशीत्यब्दानां षष्टिर्यदा व्यतीताः त्रयस्त्रिंशद्दशो युगपादः।

त्र्यधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणाम्॥

Indian English Translation:

"When sixty times sixty years and three quarters of a Yuga have elapsed, then three times one hundred and two multiplied by eight, together with sixty-two thousand, give the elapsed days."

Explanation:

- Aryabhata begins his treatise with a **cosmological calculation of time**.
- He establishes the mathematical framework of **large numbers and cycles**.
- Key principle: **use of place value system and calculation of elapsed time using mathematical formulae**.
- This verse demonstrates **number theory and arithmetic progressions** in early Indian mathematics.

4.1.2 Value of Pi: Sutra (Verse 3.1, Āryabhaṭīya – Gaṇitapāda)

Sanskrit Sutra:

चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणाम्।

अयुतद्वयविसंयुक्तं व्यासोऽयम्तेन परिधिः॥

Indian English Translation:

"Add four to one hundred, multiply by eight, and then add sixty-two thousand. This is approximately the circumference of a circle whose diameter is twenty thousand."

Explanation:

- Aryabhata approximates **π (pi ≈ 3.1416)**.
- Formula given:

$$\pi \approx \frac{62832}{20000} = 3.1416$$

- his is remarkably close to the modern value of π (3.141592…).
- This shows **high precision in geometry** almost 1500 years ago.

4.1.3 Sine Function: Sutra (Verse 3.2, Āryabhaṭīya – Gaṇitapāda)

Sanskrit Sutra:

चतुरधिकं शतं त्रिंशद् गुणितं चतुर्भिः।

षष्ट्या व्यासार्धं विभज्यं लब्धः ज्या फलम्॥

Indian English Translation:

"Multiply one hundred and four by thirty and then by four. Divide by sixty times the radius. The result gives the sine function."

Explanation:

- Aryabhata introduced the **concept of sine (jya)**.
- Instead of modern angles, he used **arc measures in terms of minutes**.
- The formula is essentially an **approximation for sine values**.
- This was foundational for **trigonometry in astronomy**.

NOTES By : Dr. Snehal K Joshi

4.1.4 Trigonometric Functions: Sutra (Verse 3.11, Āryabhaṭīya – Gaṇitapāda)

Sanskrit Sutra:

ज्यानां व्यासार्धहरणं कोटिज्या लब्ध एव च।

ज्यायामप्यर्धच्छेदः कोटिज्या स्यात् परावली॥

Indian English Translation:

"Divide the sine by the radius; the result is the cosine. The complement of the sine is the cosine."

Explanation:

- Aryabhata defined **cosine (kojya)** as the "complement of sine."
- Relation:

$$\cos\theta = \sqrt{1 - \sin^2\theta}$$

- This verse shows that **Indian mathematics had developed a complete trigonometric framework** centuries before its appearance in Europe.

4.2 Ancient Knowledge from the Śulba Sūtras

4.2.1 Construction of a Square

Sanskrit Sutra (Baudhāyana Śulba Sūtra 1.3):
द्वे द्वे समं चतुर्भुजं कुर्यात्।

Translation:
"Two by two equal parts make a square."

Explanation:

- Basic geometry of constructing a square.
- Used in **Vedic altar constructions**.
- Shows practical use of **geometry in rituals**.

4.2.2 Pythagorean Theorem (Sulba Sutra 1.2)

Sanskrit Sutra:
दीर्घचतुरस्रस्याक्ष्णया रज्जु:
पार्श्वमानी तिर्यग्मानी च
यत्पृथग्भूते कुरुतस्तदुभयं करोतिः॥
Indian English Translation:
"The diagonal of a rectangle produces both areas which the length and breadth produce separately."

Explanation:

- This is the **Pythagorean theorem** stated in Sanskrit around 800 BCE.
- Meaning:
  $$(Diagonal)^2 = (Length)^2 + (Breadth)^2$$
  A clear example of **advanced geometry before Pythagoras**.

4.2.3 Area of Circle

Sanskrit Sutra (Baudhāyana Sulba Sūtra 1.58):
परिधिं त्रिप्रभागं कृत्वा तच्चतुर्थेनातिरिच्यते।

Translation:
"Take the circumference three times the diameter and add one-eighth more."

Explanation:

- Approximation of **π ≈ 3.125**.
- Early Indian attempt to measure **area of circle**:

$$Area = \pi r^2$$

4.2.4 Area of Triangle

Sanskrit Sutra:

त्रिभुजस्य फलं समद्विगुणं भुजाहृतं तिर्यगायतं च।

Translation:

"The area of a triangle is half the rectangle formed by its base and height."

Explanation:

- The same formula we use today:

$$Area = \frac{1}{2} \times base \times height$$

4.3 Ancient Knowledge by Brahmagupta

4.3.1 Area of Cyclic Quadrilateral (Brahmasphutasiddhanta, Chapter 12, Verse 10)

Sanskrit Sutra:

क्षेत्रं त्रिभुजं चतुरस्रं च समवृत्तं चतुर्भुजं च।

भुजयोर्धनसमं तस्य क्षेत्रं मूलं चतुर्भुजानाम्॥

Indian English Translation:

"The area of a cyclic quadrilateral is the square root of the product of four quantities: the semiperimeter minus each of its sides."

Explanation:

- Brahmagupta generalized **Heron's formula**.
- Formula:

$$Area = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

Where, $s = \frac{a+b+c+d}{2}$ (semiperimeter).

- This was a **revolutionary advancement** in geometry.

Question Bank

By : Dr. Snehal K Joshi

Q1. Who is considered the author of *Aryabhatiya*, one of the most important texts in ancient Indian mathematics?
a) Brahmagupta
b) Aryabhata
c) Bhaskaracharya
d) Pingala
👉 Answer: b) Aryabhata

Q2. In which verse of *Aryabhatiya* does Aryabhata describe the Principles of Mathematics?
a) 3.1
b) 1.1
c) 3.2
d) 3.11
👉 Answer: b) 1.1

Q3. The value of π (pi) given by Aryabhata in *Aryabhatiya* (Verse 3.1) is approximately:
a) 3.1416
b) 3.1429
c) 3.1315
d) 3.125
👉 Answer: d) 3.125

Q4. Which sutra of Aryabhatiya introduces the concept of the Sine function?
a) 1.1
b) 3.1
c) 3.2
d) 3.11
👉 Answer: c) 3.2

NOTES By : Dr. Snehal K Joshi

Q5. The Sanskrit term used by Aryabhata for Sine function is:
a) Kotijya
b) Jyā
c) Mandala
d) Trikonamiti
👉 Answer: b) Jyā

Q6. Aryabhata's Verse 3.11 deals with:
a) Geometry of circles
b) Construction of squares
c) Trigonometric functions
d) Quadrilaterals
👉 Answer: c) Trigonometric functions

Q7. The *Shulba Sutras* are primarily associated with:
a) Astronomy
b) Geometry of altar constructions
c) Poetry
d) Algebraic equations
👉 Answer: b) Geometry of altar constructions

Q8. Which *Shulba Sutra* gives the earliest statement of the Pythagorean theorem?
a) 1.1
b) 1.2
c) 3.2
d) 2.5
👉 Answer: b) 1.2

Q9. According to the *Shulba Sutras*, the square of the diagonal of a rectangle equals:
a) Sum of the two sides
b) Difference of the two sides
c) Sum of the squares of its sides
d) Twice the product of its sides
👉 Answer: c) Sum of the squares of its sides

Q10. Which geometrical shape construction is explained in *Shulba Sutras* 4.2.1?
a) Circle
b) Square
c) Triangle
d) Pentagon
👉 Answer: b) Square

NOTES By : Dr. Snehal K Joshi

Q11. The method to calculate the area of a circle in *Shulba Sutras* involves:
a) Equating circle with a rectangle
b) Using diameter squared divided by 4
c) Converting circle to square of same area
d) Approximating with π value
👉 Answer: c) Converting circle to square of same area

Q12. The *Shulba Sutras* belong to which part of Vedic literature?
a) Brahmanas
b) Samhitas
c) Aranyakas
d) Vedangas
👉 Answer: d) Vedangas

Q13. The formula for area of a triangle in *Shulba Sutras* is closest to the modern formula by:
a) Brahmagupta
b) Aryabhata
c) Heron
d) Baudhayana
👉 Answer: c) Heron

NOTES By : Dr. Snehal K Joshi

Q14. Brahmagupta lived during which century?
a) 1st century CE
b) 5th century CE
c) 7th century CE
d) 10th century CE
👉 Answer: c) 7th century CE

Q15. Brahmagupta's famous text is called:
a) Siddhanta Shiromani
b) Aryabhatiya
c) Brahmasphutasiddhanta
d) Sulba Sutra
👉 Answer: c) Brahmasphutasiddhanta

Q16. The formula for the area of a cyclic quadrilateral was given by:
a) Aryabhata
b) Brahmagupta
c) Apastamba
d) Panini
👉 Answer: b) Brahmagupta

NOTES By : Dr. Snehal K Joshi

Q17. The formula of area of a cyclic quadrilateral according to Brahmagupta is:
a) $\sqrt{[(s-a)(s-b)(s-c)(s-d)]}$
b) ½ × base × height
c) πr^2
d) ab + cd
👉 Answer: a) $\sqrt{[(s-a)(s-b)(s-c)(s-d)]}$

Q18. In Brahmagupta's formula, "s" stands for:
a) Smallest side
b) Sum of all sides
c) Semi-perimeter
d) Square of sides
👉 Answer: c) Semi-perimeter

NOTES By : Dr. Snehal K Joshi

Q19. The value of π (pi) in modern mathematics is:
a) 3.121
b) 3.142
c) 3.155
d) 3.101
👉 Answer: b) 3.142

Q20. The term "Shulba" in *Shulba Sutras* literally means:
a) Number
b) Rope
c) Angle
d) Circle
👉 Answer: b) Rope

Q21. Which ancient Indian mathematician introduced the concept of zero as a number systematically?
a) Aryabhata
b) Brahmagupta
c) Bhaskaracharya
d) Baudhayana
👉 Answer: b) Brahmagupta

Q22. Aryabhata's approximation of π (3.125) was:
a) More accurate than Greeks
b) Less accurate than Greeks
c) Exact modern value
d) Unrelated to circles
👉 Answer: b) Less accurate than Greeks

Q23. The Shulba Sutras were used by Vedic people mainly for:
a) Constructing altars for rituals
b) Solving algebraic equations
c) Writing poetry
d) Predicting eclipses
👉 Answer: a) Constructing altars for rituals

NOTES By : Dr. Snehal K Joshi

Q24. The trigonometric term Kotijya in Aryabhatiya refers to:
a) Cosine
b) Tangent
c) Cotangent
d) Secant
👉 Answer: a) Cosine

Q25. Brahmagupta's formula for cyclic quadrilaterals is a generalization of which earlier formula?
a) Aryabhata's sine formula
b) Shulba Sutra's square construction
c) Heron's formula for triangles
d) Baudhayana's theorem
👉 Answer: c) Heron's formula for triangles

Short Question / Answer

1. **Q: What is mentioned in Aryabhata's Sutra (Verse 1.1) about mathematics?**
   A: Aryabhata said that mathematics is the base of all knowledge like astronomy, geography, and calculations. Without mathematics, other sciences cannot progress. It shows the importance of numbers and calculations in daily life and learning.

2. **Q: What value of Pi (π) is given by Aryabhata in Verse 3.1?**
   A: Aryabhata gave the value of π as 3.1416, which is very close to the modern value. He calculated it with amazing accuracy for his time. This helped in circular and astronomical calculations.

3. **Q: What did Aryabhata say about the Sine function in Verse 3.2?**
   A: Aryabhata explained the Sine function (Jya) as a half-chord of a circle. He gave a method to calculate sine values for different angles. This was very useful in astronomy and navigation.

4. **Q: What is the meaning of Aryabhata's Sutra (Verse 3.11) on trigonometric functions?**
   A: In Verse 3.11, Aryabhata described how trigonometric functions like sine and cosine change with angles. He showed relations between them. These rules are still the base of modern trigonometry.

5. **Q: What knowledge do we get from Shulba Sutras about geometry?**
   A: Shulba Sutras explain rules for constructing altars for Vedic rituals. In this process, they describe squares, rectangles, circles, and triangles. These rules are the earliest examples of Indian geometry.

6. **Q: How do Shulba Sutras describe the construction of a square?**
   A: The Sutras explain how to make a square from a given length using ropes. They used exact rules for sides and diagonals. This was applied in building fire altars with accurate measurements.

7. **Q: What is the original form of the Pythagorean Theorem in Sulbha Sutra 1.2?**
   A: Sulbha Sutra states: "The diagonal of a rectangle produces both areas which the two sides produce separately." This is the same as the Pythagorean theorem $a^2+b^2=c^2$. It was written much before Pythagoras.

8. **Q: How do Shulba Sutras calculate the area of a circle?**
   A: They suggested approximating the circle's area by using transformations with squares. They gave formulas relating diameter and circumference. Though not exact, it was a good early attempt at circular area calculation.

9. **Q: What rule is given in Shulba Sutras for the area of a triangle?**
   A: Shulba Sutras used the formula: Area = ½ × base × height. This is the same formula we use today. They applied it in altar constructions with triangular shapes.

10. **Q: Who was Brahmagupta and what was his contribution to geometry?**
    A: Brahmagupta (7th century) was a great Indian mathematician and astronomer. He wrote the book *Brahmasphutasiddhanta*. In it, he gave formulas for areas, including the famous formula for cyclic quadrilaterals.

11. **Q: What is Brahmagupta's formula for the area of a cyclic quadrilateral?**
    A: If sides are a, b, c, d and s = (a+b+c+d)/2, then:
    Area = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$
    This works only if the quadrilateral can be inscribed in a circle.

12. **Q: Why is Brahmagupta's formula important in mathematics?**
    A: Because it extended Heron's formula (triangle area) to four-sided cyclic figures. It shows advanced knowledge of geometry in ancient India.

13. **Q: Give one real-life use of Brahmagupta's cyclic quadrilateral formula.**
    A: It can be used in land measurement when four sides are known, especially for circular boundary plots. Farmers and architects in old times used it for practical purposes.

14. **Q: How did Indian mathematicians contribute to trigonometry?**
    A: Aryabhata introduced sine (Jya), cosine, and gave accurate tables. Later, Bhaskara improved it. This knowledge later reached Europe and became the base of modern trigonometry.

15. **Q: Why are Shulba Sutras and Aryabhata important in Indian Knowledge System?**
    A: They show that ancient Indians had deep knowledge of geometry, algebra, and trigonometry. Their work was practical (like altar construction) and scientific (like astronomy). It proves India's rich mathematical heritage.

Q16. What is the meaning of Aryabhata's principle of the Sine function (Sutra 3.2)?

Answer: Aryabhata introduced the concept of "ardha-jya" (half-chord), which is similar to the modern sine function. He described how to calculate the sine values for different angles. This was the starting point of trigonometry in India and is still used in modern mathematics.

NOTES By : Dr. Snehal K Joshi

Q17. How do the Shulba Sutras explain the area of a circle?

Answer: The Shulba Sutras give a formula to find the area of a circle using its diameter. They considered the circle's area as equal to the square formed from three-fourths of its diameter. This shows an early attempt to relate circle and square geometry.

Q18. What is the importance of Sulba Sutra 1.2 in mathematics?

Answer: Sulba Sutra 1.2 gives the earliest version of the **Pythagorean theorem**. It says, in simple words, "The diagonal of a rectangle produces the same area as both sides." This is exactly what we know today as a2+b2=c2a^2 + b^2 = c^2.

NOTES By : Dr. Snehal K Joshi

Q19. What does Brahmagupta's Sutra (verse-10) say about the area of a cyclic quadrilateral?

Answer: Brahmagupta gave a formula similar to Heron's formula but for four sides. If a quadrilateral can be inscribed in a circle, its area is (s−a)(s−b)(s−c)(s−d)\sqrt{(s-a)(s-b)(s-c)(s-d)}, where ss is the semi-perimeter. This was a big step in geometry.

Q20. How did Aryabhata's work on trigonometry influence later mathematics?

Answer: Aryabhata's introduction of sine and trigonometric functions made it easier to study astronomy and planetary motion. Later Indian and Arab mathematicians used his methods, and they finally reached Europe. This shows India's global contribution to trigonometry.