3. a) For c to be approximately equal to v_j,

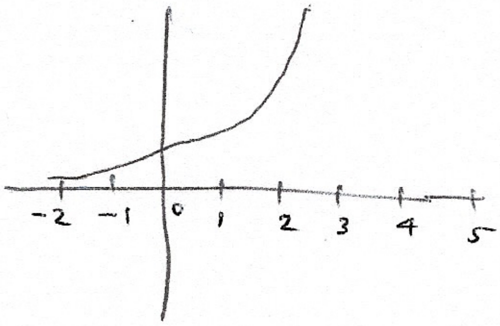
given, $c = \sum_{i=1}^{n} v_i \sigma_i$

σ_i should be approx. be $= 0$ for all $i \neq j$

&, σ_j should be approx. be $= 1$

Now, $$\sigma_i = \frac{\exp(k_i^T q)}{\sum_{j=1}^{n} \exp(k_j^T q)}$$

&, we know, $\exp(x) < 1$, when $x < 0$,



so, if the condition satisfies,

$$\underline{(k_i^T q) < 0 < (k_j^T q)}$$

<u>for all $i \neq j$</u>

then, c will be approximately equal to v_j

3.i) Given the equation,

$$c = \sum_{i=1}^{n} v_i \alpha_i$$

we can only get,

$$c = \tfrac{1}{2}(v_a + v_b) \text{ where, } v_a, v_b \in \{v_1, v_2, \ldots, v_n\}$$

if, $\alpha_a = \alpha_b = \frac{1}{2}$

and, $\alpha_i = 0$, for all $i \neq a, b$

Now, if we look at, α_i equation,

$$\alpha_i = \frac{\exp(k_i^T q)}{\sum_{j=1}^{n} \exp(k_j^T q)}$$

which means, for, $\alpha_a = \alpha_b = \frac{1}{2}$,

$$k_a^T q = k_b^T q$$

& all $k_i^T q = 0$, for all $i \neq a, b$

Now, if take the value of q, as,

$q = m(k_a + k_b)$ where m is a scalar with value greater than 0,

then, $k_a^T q = k_b^T q = m$, as, dot product of 2 perpendicular vector is 0

& $k_i^T q = 0$ for all $i \neq 0$,

Hence, $\underline{q = m(k_a + k_b)}$ should, satisfy criteria,

$$c = \tfrac{1}{2}(v_a + v_b)$$

3.c) i) As, the covariance co-efficient α, in,

$$\Sigma_i = \alpha I, \text{ is vanishingly small,}$$

we can assume, that,

each, k_i is approximately equal to it's mean vector, μ_i,

$$k_i \approx \mu_i$$

Now, in order to get,

$$c = \frac{1}{2}(v_a + v_b)$$

the problem is exactly similar to previous problem,

so, $\underline{q = m(\mu_a + \mu_b) \text{ where } m \gg 0}$

3.c) ii) As the covariance co-efficient α, in,

$$\Sigma_i = \alpha I \text{, is vanishingly small,}$$

for all $i \neq a$,

we can assume, that,

all, $k_i \simeq \mu_i$ for all $i \neq a$

Now, the co-variance for item a, is given by,

$$\Sigma_a = \alpha I + \tfrac{1}{2}(\mu_a \mu_a^T) \text{, and, } \alpha \text{ is vanishingly small,}$$

so, we can assume,

$$k_a \simeq \sigma \mu_a \text{ where, } \sigma \sim N(1, 0.5)$$

Also, we assume here, $\underline{q = m(\mu_a + \mu_b)}$

so, $k_a^T q = \sigma m$

& $k_b^T q = m$ as, $k_b \simeq \mu_b$

Now, $c = \sum_{i=1}^{n} v_i \alpha_i$

$$= \frac{\exp(k_a^T q)}{\exp(k_a^T q) + \exp(k_i^T q)} v_a + \frac{\exp(k_i^T q)}{\exp(k_a^T q) + \exp(k_b^T q)} v_b$$

$$= \frac{\exp(\sigma m)}{\exp(\sigma m) + \exp(m)} v_a + \frac{\exp(m)}{\exp(\sigma m) + \exp(m)} v_b$$

$$= \frac{1}{\exp((1-\sigma)m) + 1} v_a + \frac{1}{\exp((\sigma - 1)m) + 1} v_b$$

Now, as, $m \gg 0$, $\exp(m) \gg 0$ &, $\frac{1}{\exp(m)}$ is almost 0

So, when, $\sigma = 0$, $c = v_b$

and, when, $\sigma = 2$, $c = v_a$

So, vector c will oscillate between v_a & v_b.

3.d) i) We already know, from, 3.(b) that,

$$q = m(k_a + k_o) \text{ makes, } c = \tfrac{1}{2}(V_a + V_o), \text{ where, } m \gg 0$$

& from, 3(c i),

$$q = m(\mu_a + \mu_o) \text{ makes, } c = \tfrac{1}{2}(V_a + V_o) \text{ where, } m \gg 0$$

So, here, $\underline{q_1 = m(\mu_a + \mu_i)}$ will give, $c_1 = \frac{1}{2}(V_a + V_i)$ where $m \gg 0$,

and, $\underline{q_2 = m(\mu_a + \mu_i)}$ will give $c_2 = \frac{1}{2}(V_a + V_i)$ where $m \gg 0$,

and, as, $c = \frac{1}{2}(c_1 + c_2)$

$$= \tfrac{1}{2}(V_a + V_i)$$