# Introduction to Probability & Statistics

Assignment 2, 2024/25

## Assignment Questions – answers to be uploaded

Make sure to justify your reasoning by making reference to any of the properties (P1)–(P7) covered in lectures, where you are using them in your answers.

**AQ1.** When $\mathbb{P}(A) = 1/2, \mathbb{P}(B) = 1/3$, and $\mathbb{P}(A \cup B) = 3/4$, calculate $\mathbb{P}(A \cap B)$ and $\mathbb{P}(A^c \cup B^c)$.

Answer

Solving the relation $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$ (property (P6)) for $\mathbb{P}(A \cap B)$ gives

$$\mathbb{P}(A \cap B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cup B) = 1/2 + 1/3 - 3/4 = 1/12$$

**[2]**. Using De Morgan's law, and then (P4), we get

$$\mathbb{P}(A^c \cup B^c) = \mathbb{P}((A \cap B)^c) = 1 - \mathbb{P}(A \cap B) = 1 - 1/12 = 11/12.$$

**[3]**

**AQ2.** Let $A$ and $B$ be two events. Prove that if $A \subseteq B$ then

$$\mathbb{P}(B \cap A^c) = \mathbb{P}(B) - \mathbb{P}(A).$$

Answer

Since $A \subseteq B$, we can write $B$ as the *disjoint* union $B = (B \cap A^c) \cup A$. **[2]** This gives

$$\mathbb{P}(B) = \mathbb{P}((B \cap A^c) \cup A) \overset{(P3)}{=} \mathbb{P}(B \cap A^c) + \mathbb{P}(A).$$

Rearranging gives the required result. **[3]**

**AQ3.** Calculate $\mathbb{P}(A \cup B)$ if it is given that $\mathbb{P}(A) = 1/3$ and $\mathbb{P}(B \mid A^c) = 1/2$.

Answer

We use our usual trick of rewriting a union in terms of the union of disjoint events. In this case the best is to write $A \cup B = A \cup (B \cap A^c)$ and thus, by (P3), $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B \cap A^c)$. We can then use that by the multiplication rule

$$\mathbb{P}(B \cap A^c) = \mathbb{P}(B|A^c)\,\mathbb{P}(A^c)$$

**[2]**. Furthermore $\mathbb{P}(A^c) = 1 - \mathbb{P}(A)$, by (P4). Putting everything together gives

$$\begin{aligned}\mathbb{P}(A \cup B) &= \mathbb{P}(A) + \mathbb{P}(B \mid A^c)(1 - \mathbb{P}(A)) \\ &= \frac{1}{3} + \frac{1}{2}(1 - 1/3) = \frac{2}{3}.\end{aligned}$$

**[3]**

**AQ4.** Prove that, for any events $A, B, C$ for which the following probabilities are all defined,

$$\mathbb{P}(A \cap B \cap C) = \mathbb{P}(A)\,\mathbb{P}(B \mid A)\,\mathbb{P}(C \mid A \cap B) \, .$$

Answer

Expanding the RHS using the definition of conditional probability, we obtain

$$\mathbb{P}(A)\,\mathbb{P}(B \mid A)\,\mathbb{P}(C \mid A \cap B) = \mathbb{P}(A)\,\frac{\mathbb{P}(A \cap B)}{\mathbb{P}(A)}\,\frac{\mathbb{P}(A \cap B \cap C)}{\mathbb{P}(A \cap B)} = \mathbb{P}(A \cap B \cap C) \, .$$

**[5]**