Introduction to Probability & Statistics

Assignment 1, 2024/25

Assignment Questions – answers to be uploaded

AQ1. Let E, F and G be three events. Find expressions for these events:

a. Only F occurs;
b. F and G occur;
c. E or F occurs, but not G;
d. At least two of the events occur.

Answer

a. $F \cap E^c \cap G^c$; **[1]**
b. $F \cap G$; **[1]**
c. $(E \cup F) \cap G^c$; **[1]**
d. $(E \cap F) \cup (E \cap G) \cup (F \cap G)$. **[2]**

(You may have slightly different answers to the above, but if so then you should be able to prove that your versions are equivalent to mine!)

AQ2. In how many ways can three men and four women be seated in a row if

a. there are no restrictions on the order;
b. no two members of the same sex can sit next to one another?

Answer

a. With no restrictions there are $7! = 5040$ ways. **[2]**
b. Any arrangement satisfying this constraint must take the form $WMWMWMW$ (where W is a woman, and M a man). There are $3! = 6$ ways of arranging the men, and $4! = 24$ ways of arranging the women: this gives 144 arrangements. **[3]**

AQ3. A student has 1 bookshelf, 9 novels (among which 2 titles appear twice each), 4 textbooks on probability theory and 6 knitting books. In how many ways can he put the books on the shelf such that books in the same category (novels, textbooks, knitting books) are next to each other? Possibilities where the doubles are switched are considered to be the same and are not counted separately. (Hint: approx. 9.4×10^9.)

Answer

The 3 categories can be placed in $3! = 6$ ways. The 4 textbooks can be placed in $4! = 24$ ways, and the 6 knitting books in $6! = 720$. **[2]** For the novels, by Proposition 1.4 there are $9!/(2!2!)$ ways of arranging them. **[2]** This gives a total of $3!4!6!\frac{9!}{2!2!} = 9,405,849,600$ ways to order all books. **[1]**

AQ4. Three friends want to place a pizza order (one pizza each), from a restaurant which offers three types of pizza (A, B and C). How many different possible pizza orders are there? (E.g. one possible order is that two people choose A, and one person B.)

Answer

(I realised after setting this question that there was the possibility of confusion here, caused by my use of the word "order". When referring to a "pizza order" I was meaning the "request" that was sent to the restaurant, as indicated in the final line of the question. So an order was meant to be of the sort "We'd like two of pizza A and one of pizza B", and the restaurant wouldn't care which person wanted which pizza. The solution below uses that assumption. ***However****, if you thought that the exact permutation of pizzas (corresponding to which person wanted which pizza) had to be taken into account, then you will still have received marks for this provided that (as always) you clearly explained what you were doing.)*

A general pizza order can be written as a vector (n_A, n_B, n_C), where n_A is the number of people ordering pizza A, etc. We require the number of such vectors for which

- each entry is a non-negative integer
- the entries sum to three. **[3]**

From Proposition 1.10 we know that this number is given by $C^{3+3-1}_{3-1} = C^5_2 = 10$. **[2]**

Another possible answer: n_A people choose pizza A, for some $0 \le n_A \le 3$. Once these n_A people are sorted out, we have $3 - n_A$ people left, so n_B can take the values $0, \ldots, 3 - n_A$ (i.e. there are $4 - n_A$ possible values for n_B). So we have to sum

$$\sum_{n_A=0}^{3} (4 - n_A) = 4 + 3 + 2 + 1 = 10\,.$$

Another method (though less elegant) is to simply list all of the possibilities:

$$(3,0,0),\ (2,1,0),\ (2,0,1),\ (1,2,0),\ (1,0,2),\ (1,1,1),\ (0,3,0),\ (0,0,3),\ (0,1,2),\ (0,2,1)\,.$$

(This would not be a good idea if the numbers were much bigger, of course!)