TEAM 3 KINETIC SCULPTURE ANALYSIS

1. **Transmission Element 1: Chain-Sprocket**
    a. **Speed:** The driving and driven sprockets are the same parts with equal circumference, pitch, and teeth count, they rotate in sync together at the same angular velocity. In a trial run, we measured 1.75 s for the time it takes for the sprocket to make a half rotation and lift the stairs. Since the rotation of the sprocket is governed by the rotation of the crank, a free input to the system, we can choose a constant average angular velocity from the input. Then the average angular velocity of the driving and driven sprocket is

    $$\omega\_in = \omega\_out = \pi/(1.75\text{ s}) = 1.8\text{ rad/s.}$$

    b. **Power:** First, we measured the average hand torque needed to rotate the sprocket to be 0.1 N m. (see **Figure 1**). Again, since hand torque is a free input, use the average velocity as the cam lifts the stairs (half rotation). Previously we found the angular velocity of the sprocket to be 1.8 rad/s. And so, the power going in the driving sprocket is

    P_in = Torque * Angular Velocity = 0.180 W.

    Meanwhile, for power out, we estimate that our roller chain has an efficiency of 98%. This value is based on the measured average efficiency of roller chains.[1] And so, our power out is

    P_out = efficiency * P_in = 0.176 W.

    c. **Loads (Torque):** We measured the input hand torque for our system to be

    torque_in = 0.1 N m

    We also previously found that P_out = 0.176 J/s. Since we know that P_out = torque_out * angular velocity. Rearranging this, torque_out = P_out/angular velocity. Plugging in values of P_out and angular velocity that we found previously, we get

    torque_out = 0.0978 N m.

[1] https://www.crossmorse.com/images/specifications/roller_chain.pdf

2. **Transmission Element 2: Cam-Stairs**
    a. **Speed**: We will again use a half rotation of the cam moving the stairs from bottom to top, starting and ending from rest. Then the velocity of the stairs is approximately sinusoidal. <u>To simplify time-varying calculations, we will report average</u> velocities. The cam rotates with the sprocket and shaft with no slipping, so both share the same angular velocity. The average angular velocity of the cam is then

    ω_cam = 1.8 rad/s.

    To find the linear velocity of the stairs, we can utilize the fact that the moving stairs rests on top of the cam. Assuming that the stairs are always in contact with the cam and moves in a purely vertical direction, then the total displacement of the stairs is given by the difference in the maximum and minimum radius of the cam.

    R_max - R_min = 2.6125 in - 0.7475 in = 1.865 in = 4.737 cm

    Also using the fact that the maximum and minimum radius of the cam is 180 degrees apart, we can find the time for the rotation. The average linear velocity of the stairs is then given by total displacement over time

    V_stairs = (R_max-R_min)/(π/ω_cam) = 2.7 cm/s

    This velocity is assuming that the stairs move at a constant velocity when traveling up and when traveling down.
    b. **Loads**: We previously found that the torque on the left (driven) and right (driving) sprockets and shaft, which rotates with the cams. Thus, the torque of the cams are equal to the torques of the sprockets shafts. Summing these two torques, we get

    torque_cam = torque_leftcam + torque_rightcam = 0.1978 N m.

    Meanwhile, for the cam to support and accelerate the stairs, it needs to exert a force that is greater than the weight of the stairs. The weight of the stairs is 0.128kg. Consider the free body diagram in **Figure 2**. The average load from the cam F_cam can be approximated by finding the acceleration to reach the average velocity over the time it takes to complete half the rotation. Then,

    F_cam $\cong$ m * v_stairs / (1.75 sec) + m*gravity_earth = 1.256 N

    c. **Power**: We previously found the power of each sprocket. Since the sprockets rotate with the shaft and the cam, then we know that the power on the cam would be the same as the power out from our previous calculation. Thus, the power going in from the cam to move the stairs is

    Power_cam = Power_out = 0.176 W

    Meanwhile, the average power coming out can be calculated as follows:

    Power_stairs = F_cam * V_stairs = 0.0339 W

    And so, we have an efficiency of:

    Efficiency = Power_stairs/Power_cam = 0.19

Figure 1 - Spring and hook scale measures 0.2 kgs required to lift the crank. The crank has a moment arm of 0.05 meters. Taking the cross product of force (using 0.2 kg * gravity_earth) and the moment arm, we calculate the torque to be 0.1 Nm. Note: the picture was taken slightly after the force measurement was done – Force measurement was taken when the force was perfectly perpendicular to the arm (when the handle was horizontal).

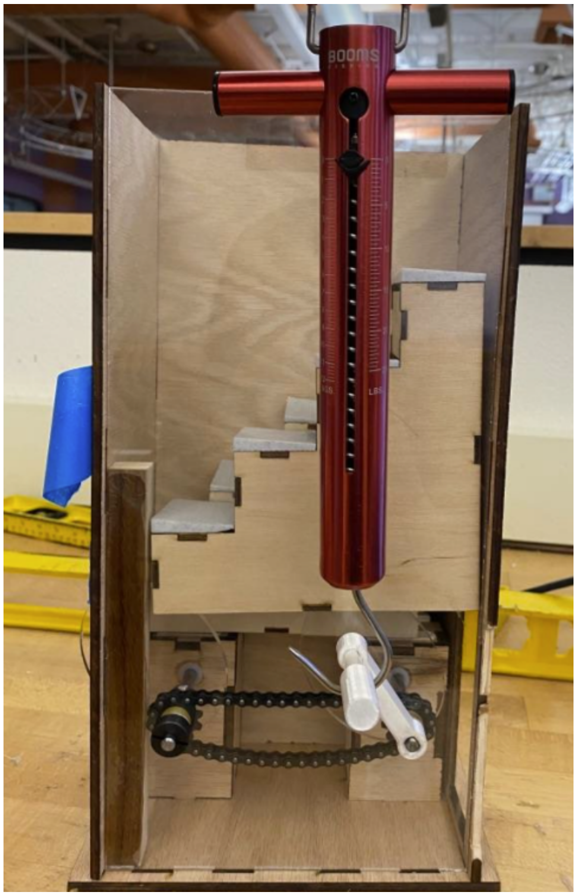
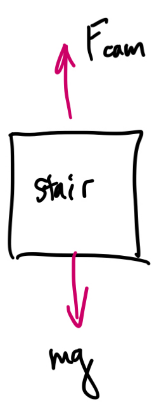
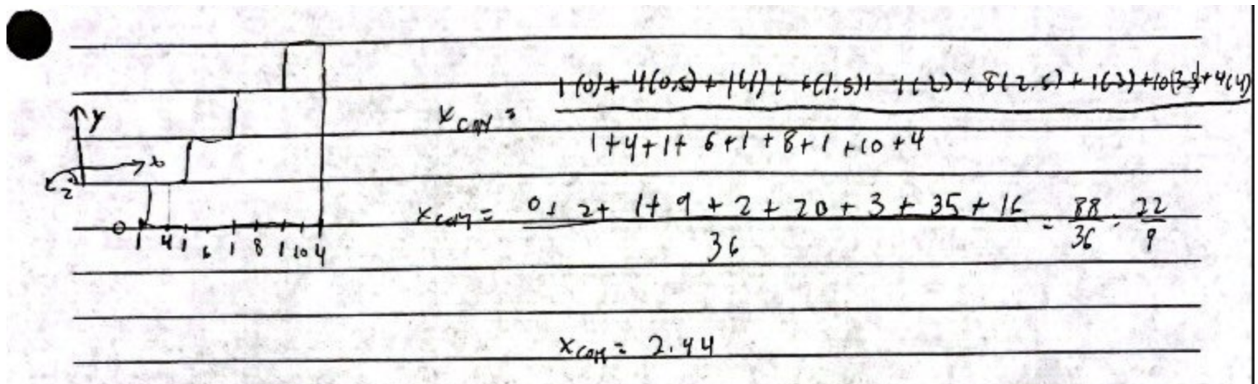


Figure 2 Free body diagram of the stairs

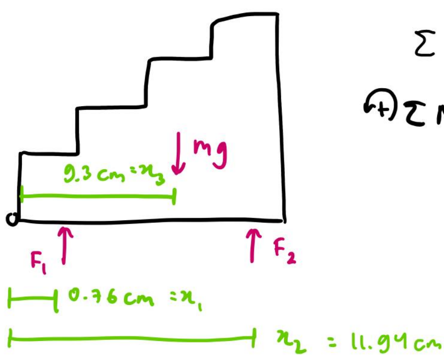


3. **Loads on Shaft:**

To find the loads on the shaft, we first find the center of mass of the stairs. Setting each face as one unit, we found the center of mass to be at 2.44 * the width of each step from the left end of the stairs. Since each step is 1.5in = 3.81cm wide, then the center of mass is located at 9.2964cm from the left end.



We then proceeded with our analysis by concentrating the force from the mass of the stairs at the center of mass and applied a force and moment balance on the stairs and the two shafts supporting the stairs.

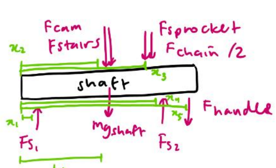


$$\Sigma F = 0 = F_1 + F_2 - mg$$

$$(+)\ \Sigma M_0 = F_1 x_1 + F_2 x_2 - mg\, x_3 = 0$$

Beyond supporting the mass of the stairs, the shafts are also supporting the mass of the sprocket, the cam, and the chains, while being supported by the frames of the sculpture. The mass of the cam is m_cam = 0.025kg, the mass of the sprocket is m_sprocket = 0.127kg, the mass of the chain is m_chain = 0.034kg. We can approximate that the mass of the chain is split up equally between the two shafts due to symmetry. The length of the shaft is L = 3inch = 0.076 m. the mass of the shaft is m_shaft = 0.019kg.

a. **Right Shaft**: From the force and moment balance, we get that F2 = Fstairs = 0.958N. The FBD of the right shaft is as follows:



$$\Sigma F = 0 = F_{s1} + F_{s2} - F_{cam} - F_{stairs} - F_{sprocket} - F_{chain}/2 - mg_{shaft} - F_{handle} = 0$$

$$\Sigma M_0 = x_1 F_{s1} - x_2 (F_{cam} + F_{stairs}) - x_3 (F_{sprocket} + F_{chain}/2) + x_4 F_{s2} - x_5 F_{handle} - \frac{L}{2} F_{shaft} = 0$$

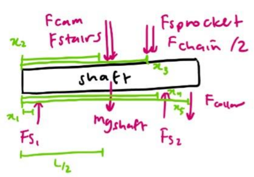
Where:

- Fcam = m_cam*g = 0.245N
- Fstairs = 0.958N
- Fsprocket = m_sprocket*g = 1.2446N
- Fchain/2 = m_chain*g/2 = 0.1666N
- Fhandle = m_handle*g = 0.01kg*g = 0.098N
- (mg)_shaft = m_shaft*g = 0.1862N
- L = 0.076m
- x1 = 0.004m
- x2 = 0.034m
- x3 = 0.048m
- x4 = 0.061m
- x5 = 0.07m

Solving for Fs1 and Fs2 using the force balance, we get:

- **Fs1 = 0.951N (back side)**
- **Fs2 = 1.947N (acrylic side, with handle)**

b. **Left Shaft**: From the force and moment balance, we get that F1 = Fstairs = 0.296N. The FBD of the left shaft is as follows:



$$\sum F = 0 = F_{s1} + F_{s2} - F_{cam} - F_{stairs} - F_{sprocket} - F_{chain}/2 - mg_{shaft} - F_{collar} = 0$$

$$\sum M_0 = x_1 F_{s1} - x_2 (F_{cam} + F_{stairs}) - x_3 (F_{sprocket} + F_{chain}/2) + x_4 F_{s2} - x_5 F_{collar} - \frac{L}{2} F_{shaft} = 0$$

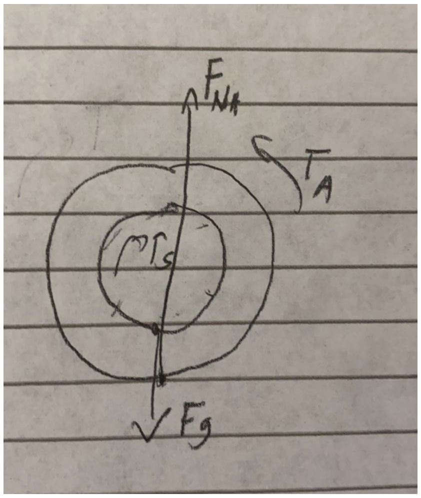
Where:

- Fcam = m_cam*g = 0.245N
- Fstairs = 0.296N
- Fsprocket = m_sprocket*g = 1.2446N
- Fchain/2 = m_chain*g/2 = 0.1666N
- Fcollar = m_collar*g = 0.07kg*g = 0.686N
- (mg)_shaft = m_shaft*g = 0.1862N
- L = 0.076m
- x1 = 0.004m
- x2 = 0.027m
- x3 = 0.048m
- x4 = 0.061m
- x5 = 0.07m

Solving for Fs1 and Fs2 using the force balance, we get:

- **Fs1 = 0.611N (back side)**
- **Fs2 = 2.213N (acrylic side with collar)**

4. **Loads on Bushings**
   The loads on the bushings can be described by the following FBD.



Where Fg = force on the bushing from the shaft and the bushing's weight (F_shaft + F_bushingweight), T_s = transmitted torque from the shaft, T_A = restoring torque from the acrylic, and F_NA = support force from the acrylic. For the bushings on the back side of the sculpture (the one connected to the wood), we have T_A = restoring torque from the wooden frame, and F_NA = support force from the wooden frame. Since the bushings are stationary, then T_A = T_s, and Fg = F_NA. F_shaft can be found in section 3 above.

5. **Bushing Evaluation**
   The shafts are supported by n=4 nylon bushings with a length of 5/16 inches and a diameter of 1/4 inch. Here, analysis will focus on PV calculations. This involves bearing pressure P and bearing surface speed V.
   Pressure equation (force over effective area):

   P = F_s / (D_bushing * L) = F_s / (0.2 in * 5/16 in) = F_s / 40.3 mm^2

   Linear velocity of shaft

   V = D_shaft / 2 * **ω = (0.25 in / 2) * 1.8 rad / sec = 0.00572 m / s**

   Choose PV analysis for the bushing with the largest support load, the **acrylic support on the acrylic load.**

   PV = (2.213 N / 40.3 mm^2)(0.00572 m / s) = 314.1 Pa m/s

Evaluate at least one bearing/bushing used at your application's loading and maximum speed
For ball bearings, this should be a bearing life calculation at standard reliability
For nylon bushings, this should be a PV calculation

For Nylon bushings:

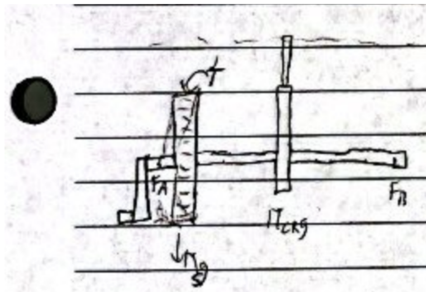
- Max V = 100 ft/min
- Max P = 400 psi
- Max PV = 5500 psi*ft/min

- https://advanced-emc.com/how-to-determine-the-pv-value-in-polymer-sleeve-bearings-and-bushings/

Our Bushing:

- V = 1.127 ft/min
- P = 7.962 psi
- PV = 8.973 psi*ft/min
- Our bushing is appropriate for use since none of the standards are exceeded.

6.

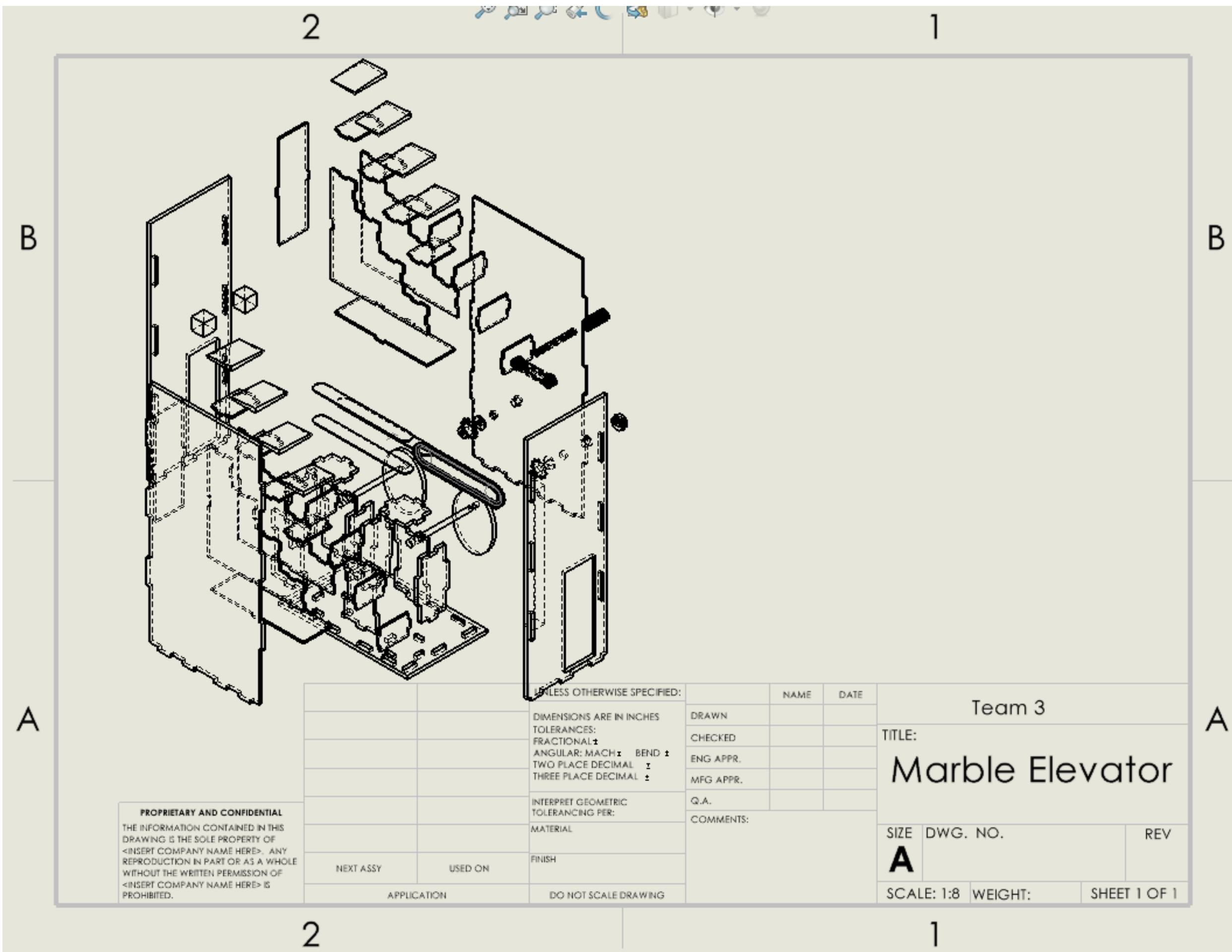


$$\Sigma T = T = F_{handle} r_{handle}$$

$$\Sigma F_y = 0 = F_A + F_B - M_s g - M_{cr} g$$

$$\Sigma M_0 = 0 = -x_s M_s g - x_{cr} M_{cr} g + x_B F_B$$

7.



8.