2) a) $n^3 \in O(2^n)$

$n^3 < c \cdot 2^n \qquad n > n_0 \qquad c > 1$

$$\lim_{n\to\infty} \frac{n^3}{2^n} = \frac{3n^2}{n.2^n.\ln 2} = \frac{3n}{2^n.\ln 2} = \lim_{n\to\infty} \frac{3}{n \ln 2} = 0$$

b) $2^n \in o(3^n)$

$2^n < c.3^n \qquad n > n_0 \qquad c > 1$

$$\lim_{n\to\infty} \frac{2^n}{3^n} = \frac{n.2^n.\ln 2}{n.3^n.\ln 3} = \lim_{n\to\infty} \frac{2^n.\ln 2}{\underbrace{3^n.\ln 3}_{\text{faster}}} = 0$$

c) $n! \in o(100^n)$

according to stirling $n! \cong \sqrt{2\pi n} \cdot \left(\frac{n}{e}\right)^n$

$$\lim_{n\to\infty} \frac{n!}{100^n} = \lim_{n\to\infty} \frac{\sqrt{2\pi n}.\left(\frac{n}{e}\right)^n}{100^n} = \lim_{n\to\infty} \sqrt{2\pi n} \left(\frac{n}{100e}\right)^n = \infty$$

5) a) Input size is size of list which is n

c) if size is 0 $T(0) = c$

other size is n $T(n) = T(n/3) + T(n/3)$

d) $T(n/3) + T(n/3) = T(n)$

according to substitution I guess it should be $T(n) = O(n\log n)$

$T(n) <= cn\log n \rightarrow T(n) = 2T(n/3)$

$<= 2.(cn.\log(n/3))$

$= 2(cn\log n - cn\underbrace{\log 3}_{\sim 1})$

$= cn\log n - cn$

$\underline{<= cn\log n}$

b)

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4) b) Main operation is Comparison of two matrix elements.

c) It calculates if the matrix is symetric.

d) - Worst case $= T(n) \in \Theta(n^2)$ | $T(n) \in O(n^2)$

- Best $= O(1)$

- Average $= \sum_{n=0}^{n-1} i = \frac{(n-1)n}{2n} = \frac{n-1}{2} = \Theta(n)$

a) $(n-1)(n-1) = \underline{\underline{n^2 - 2n + 1}}$

3) for i=0 last index of inputArray

    if D[i] equals key
        return i
    return -1

Worst case : $O(n)$ → if there is not the key or end of the list.

Average Case : $\frac{\sum_{i=1}^{n+1} \Theta(i)}{n+1} = \frac{\Theta((n+1) \times (n+2)/2)}{n+1} = \Theta(n)$

Best case : $O(1)$ key equal D[i]

10) a) According to Master Method $T(n) = \Theta(n^2)$

b) According to " " $T(n) = \Theta(n^2 \log^2 n)$

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Suppose

1) a) $f(n) = O(g(n))$ if there exist positive constants c and n_0

$f(n) \leq c.g(n)$. Let m is any odd integer $> c, n_0$

$g(m) = 1$, So $f(m) = m$

↳ $f(m) > c$

↳ $f(m) > c.g(m)$

if $m > n_0$ there is a contradiction.

So initial assumption is wrong.

f is not $O(g)$ also g is not $O(f)$

c) $O(g(n)) = f(n)$ if there exist positive constant c and n_0 such that $0 \leq f(n) \leq c_1 g(n)$ $n \geq n_0$

$O(g(n)^k) = f(n)^k$ $0 \leq f(n)^k \leq c_2\ g(n)^k$

k is integer c_2 constant and $f(n) \in O(g(n))$ then $f(n)^k \in O(g(n)^k)$

7) func findZero (A[n])

```
if(n==-1)
  {if(A[0]==0)
      return 1;
   else
      return 0;
  }
else
   return findZero(A[0:(n/2)-1]) + findZero(A[n/2 : n-1]);
```

$T(n) = 2.T(n/2) + 1$

$a=2$ $b=2$ $a > b^d$ According to master Theorem

$f(n) = \Theta(1)$ $2 > 2^0$ $T(n) \in \Theta(n^{\log_2 2}) = \Theta(n)$

$n^d = 1$

$d = 0$

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9) a) $T(n) = T(n-1) + n^2 + 1 \qquad T(0) = 3$

$n=1 \quad T(1) = T(0) + 1^2 + 1$

$n=2 \quad T(2) = T(1) + 2^2 + 1$

$T(3) = T(2) + 3^2 + 1$

$T(4) = T(3) + 4^2 + 1$

$\vdots$

$T(n) = T(n-1) + n^2 + 1$

$T(n) = T(0) + \sum_{i=1}^{n} i^2 + i \;=\; 3 + \sum_{i=1}^{n} i^2 + i$

$= 3 + \frac{n.(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$= 3 + \frac{2n^3 + 3n^2 + n}{6} + \frac{3n^2 + 3n}{6}$

$= \in \Theta(n^3)$

c) $T(n) = 2T(n-1) - T(n-2) + n \qquad T(0) = 0 \quad T(1) = 0$

$n=2 \quad T(2) = 2T(1) - T(0) + 2$

$n=3 \quad T(3) = 2T(2) - T(1) + 3$

$T(4) = 2T(3) - T(2) + 4$

$T(5) = 2T(4) - T(3) + 5$

$\cancel{T(n-1) = 2T(n-2) - T(n-3) + (n-1)}$

$T(n) = 2T(n-1) - T(n-2) + n$

$T(n) = T(n-1) + \cancel{2T(1) - T(0)} + 2 + 3 + \ldots + n$

$T(n) = T(n-1) + \sum_{i=2}^{n} i$

$n=1 \quad T(1) = T(0).$

$n=2 \quad T(2) = T(1) + 2$

$n=3 \quad T(3) = T(2) + 2 + 3$

$n=4 \quad T(4) = T(3) + 2 + 3 + 4$

$\vdots$

$T \qquad T(n) = T(n-1) + 2 + 3 + 4 + \ldots + n$

$T(n) = T(0) + 2(n-1) + 3(n-2) + 4.(n-3)$

$T(n) = (n-1)\,i$

$\sum_{i=2}^{n} (n+1-i).i \qquad \Theta(n)$

8) a) $T(n) = -4\underset{x}{T(n-1)} - \underset{b}{4}\underset{1}{T(n-2)}$ $\quad T(0)=0 \quad T(1)=1$

$x^2 = ax+b$

$x^2 = -4x-4$

$x^2+4x+4=0 \quad (x+2)^2=0 \quad \underline{\underline{x=-2}}$

$T(n) = c_1.(-2)^n + c_2.n.(-2)^n$

$T(0) = c_1.1 + c_2.0.1 = 0 \quad \rightarrow \underline{c_1=0}$

$T(1) = c_1.-2 + c_2.1.-2 = 1 \quad \rightarrow \underset{0}{-2c_1} - 2c_2 = 1$

$c_2 = -1/2$

$T(n) = 0.(-2)^n + \frac{-1}{2}.n.(-2)^n$

$= \frac{-1}{2}n.(-2)^n \in \Theta(n)$

b) $T(n) = \underset{x}{T(n-1)} + \overset{b}{\underset{b}{6}}\underset{1}{T(n-2)}$ $\quad T(0)=0 \quad T(1)=6$

$x^2 = ax+b$

$x^2 = x+6$

$x^2-x-6=0$

$(x-3)(x+2)=0 \quad x=3 \quad x=-2$

$T(n) = c_1(x^+)^n + c_2(x^-)^n$

$T(n) = c_1.3^n + c_2.(-2)^n$

$T(0) = c_1+c_2 \Rightarrow 3 = c_1+c_2 \Rightarrow 6 = 2c_1+2c_2$

$T(1) = c_1.3 + c_2(-2) \Rightarrow \underline{6 = 3c_1 - 2c_2}$

$12 = 5c_1 \quad c_1 = \frac{12}{5} = 3{,}4 \quad c_2 = 0{,}6$

$T(n) = \frac{12}{5}.3^n + \frac{3}{5}(-2)^n \in \Theta(3^n)$