ARRAYS - LINEAR ALGEBRA

Lecture Date: 19 SEP 16

Array Multiplication (Dot Product)

Recall definition of dot product

$$\vec{a} \cdot \vec{b} = \sum_{k=1}^{n} a_k b_k \quad \rightarrow \text{ output is a scalar}$$

MATLAB will automagically compute dot products

Matrix multiplication is just an extension of the dot product.

A * B can only be carried out if the # columns in A is equal to the number of rows in B

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ B_{31} & B_{32} \end{bmatrix}$$

$$A * B = \cancel{\{(A_{11} * B_{11})}$$

$$\cancel{[(A_{11}B_{11} + A_{21}B_{12} + A_{13}B_{13}) \ (A B_{12} +}$$

$$\begin{bmatrix} (A_{11}B_{11} + A_{12}B_{21} + A_{13}B_{31}) & (A_{11}B_{12} + A_{12}B_{22} + A_{13}B_{32}) \\ (A_{21}B_{11} + A_{22}B_{21} + A_{23}B_{31}) & (A_{21}B_{12} + A_{22}B_{22} + A_{23}B_{32}) \\ (A_{31}B_{11} + A_{32}B_{21} + A_{33}B_{31}) & (A_{31}B_{12} + A_{32}B_{22} + A_{33}B_{32}) \end{bmatrix}$$

WHY? Convienent for systems of linear equations

Consider

$$A_{11}x_1 + A_{12}x_2 + A_{13}x_3 = B_1$$
$$A_{21}x_1 + A_{22}x_2 + A_{23}x_3 = B_2$$
$$A_{31}x_1 + A_{32}x_2 + A_{33}x_3 = B_3$$

can be written

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}$$

because of linear algebra rules

in matrix form $AX = B$ ... now we are getting somewhere!

<u>Array Division</u>

But we want to solve for X. This will be the solution to our 3 equations.

Briefly ...

Identity matrix - square in which diagonal elements are 1's and rest is 0's

ASK: how do you make it

When identity matrix multiplies another matrix, the matrix is unchanged (following rules of matrix mult)

ex) $\begin{bmatrix} 7 & 3 & 8 \\ 4 & 11 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 3 & 8 \\ 4 & 11 & 5 \end{bmatrix}$

OR

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 2 \\ 15 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \\ 15 \end{bmatrix}$$

$$\Rightarrow \quad AI = IA = A$$

Recall $A*B \neq B*A$ for Matrices

The matrix B is the inverse of A if

$BA = AB = I$ both matrices MUST be square

Show them

```
>> A = [2 1 4; 4 1 8; 2 -1 3]
>> B = inv(A)
>> A * B
>> B*A
```

and

```
>> A*A^-1
```

so A^{-1} is inverse A

Left Division, \

Can be used to solve $AX = B$!

recall

X and B are column vectors.

multiply each side of eqn by A^{-1} $\quad A^{-1}AX = A^{-1}B$

left side is X since

$$A^{-1}AX = IX = X$$

So the solution of $AX = B$ is

$$\begin{cases} X = A^{-1}B & \text{OR} \\ X = A\backslash B \end{cases}$$

Note: even though these look equivalent the way MATLAB calculates makes

a much better choice

Right Division, /

Use right division to solve $XC = D$

$$X \cdot C \cdot C^{-1} = D \cdot C^{-1}$$

$$X = D \cdot C^{-1}$$

$$X = D/C$$

SOLVE SYSTEMS!

Consider

$$4x - 2y + 6z = 8$$
$$2x + 8y + 2z = 4$$
$$6x + 10y + 3z = 0$$

$$\begin{bmatrix} 4 & -2 & 6 \\ 2 & 8 & 2 \\ 6 & 10 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$$

```
>> A = [4 -2 6; 2 8 2; 6 10 3]
>> B = [8; 4; 0]
>> X = A\B
```

OR

```
>> Xb = inv(A) * B
```

OR

```
>> C = [4 2 6; -2 8 10; 6 2 3]
>> D = [8 4 0]
>> Xc = D/C
```

OR

```
>> Xd = D * inv(c)
```