P 1. *Solve these three equations for y_1, y_2, y_3 in terms of c_1, c_2, c_3:*

$$Sy = \mathbf{c}, \quad \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}.$$

REPLACE y_1 WITH C_1

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \rightarrow \begin{bmatrix} 1(y_1) & 0(y_2) & 0(y_3) \\ 1(y_1) & 1(y_2) & 0(y_3) \\ 1(y_1) & 1(y_2) & 1(y_3) \end{bmatrix} \rightarrow \begin{matrix} y_1 = C_1 \\ y_1 + y_2 = C_2 \\ y_1 + y_2 + y_3 = C_3 \end{matrix} \rightarrow \begin{matrix} C_1 + y_2 = C_2 \\ C_1 + y_2 + y_3 = C_3 \end{matrix}$$

REPLACE $(C_1 + y_2)$ WITH C_2

$$\rightarrow C_2 + y_3 = C_3 \rightarrow \boxed{\begin{matrix} y_1 = C1 \\ y_2 = C_2 - C_1 \\ y_3 = C_3 - C_2 \end{matrix}}$$

SINCE WE HAVE C AND y, WE CAN FIND THE INVERSE MATRIX VALUES

$$S^{-1}c = y \rightarrow \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 - C_1 \\ C_3 - C_2 \end{bmatrix} \rightarrow \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}}_{S^{-1}} \underbrace{\begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}}_{C} = \underbrace{\begin{bmatrix} C_1 \\ C_2 - C_1 \\ C_3 - C_2 \end{bmatrix}}_{y}$$

P 2. *Give the three vectors*

$$\mathbf{r}_1 = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}, \mathbf{r}_2 = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}, \mathbf{r}_3 = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}.$$

Linear algebra says that these vectors must also lie in a plane. There must be many combinations with $y_1\mathbf{r}_1 + y_2\mathbf{r}_2 + y_3\mathbf{r}_3 = \mathbf{0}$. Find two set of y's.

RIGHT OFF THE BAT, WE CAN SEE THAT $y_1 r_1 + y_2 r_2 + y_3 r_3 = 0$ IF Y_1, y_2, y_3 ALL $= 0$.

$$1)\ y = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

TO FIND OTHER VALUES, WE CAN WRITE THIS AS A SYSTEM OF EQUATIONS:

MULTIPLY FIRST EQUATION BY 4 SO IT IS REDUCABLE

$$\begin{matrix} 1y_1 + 2y_2 + 3y_3 = 0 \\ 4y_1 + 5y_2 + 6y_3 = 0 \\ 7y_1 + 8y_2 + 9y_3 = 0 \end{matrix} \rightarrow 4y_1 + 5y_2 + 6y_3 - (4)(y_1 + 2y_2 + 3y_3 = 0) \rightarrow \begin{matrix} -3y_2 - 6y_3 = 0 \\ 4y_1 + 5y_2 + 6y_3 = 0 \\ 7y_1 + 8y_2 + 9y_3 = 0 \end{matrix} \rightarrow$$

WE COULD TRY TO REDUCE FURTHER, BUT BECAUSE WE KNOW THERE ARE MANY COMBINATIONS, LETS TRY PLUGGING IN VALUES.

TRYING $y_3 = 1$

PLUG-IN $y_2 = 2$, $y_3 = 1$

$$\rightarrow -3y_2 - 6(1) = 0 \rightarrow -3y_2 = 6 \rightarrow y_2 = 2 \rightarrow 4y_1 + 5(-2) + 6(1) = 0 \rightarrow 4y_1 = 10 - 6 \rightarrow y_1 = 1$$

$$\rightarrow \text{TESTING } y_1 = 1,\ y_2 = -2,\ y_3 = 1 \rightarrow \begin{matrix} 1(1) + 2(-2) + 3(1) = 0\ \checkmark \\ 4(1) + 5(-2) + 6(1) = 0\ \checkmark \\ 7(1) + 8(-2) + 9(1) = 0\ \checkmark \end{matrix} \rightarrow \boxed{1)\ y = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad 2)\ y = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}}$$

P 3. *The first of these equations plus the second equals the third:*

$$x + y + z = 2$$
$$x + 2y + z = 3$$
$$2x + 3y + 2z = 5.$$

(1) The first two planes meet along a line. The third plane contains that line. Why? (2) The equations have infinitely many solutions. Why? Find three specific solutions.

1) TO THINK OF IT IN SIMPLER TERMS IF YOU HAVE TWO LINES THAT INTERSECT, THEY INTERSECT AT A POINT. IF YOU ADD THE TWO LINES TOGETHER, YOU GET A THIRD LINE THAT GOES BETWEEN THE OTHER TWO THROUGH THAT POINT. SIMILARLY WITH PLANES. IF TWO PLANE INTERSECT, THE IS A LINE WHERE THE INTERSECT. ADDING THE TWO PLANES RESULTS IN A THIRD PLANE THAT GOES THROUGH THE MIDDLE OF THE TWO AND GOES THROUGH THE LINE.

2) FIRST, WE CAN TRY TO REDUCE OUR FIRST EQUATION:

$x+y+z=2$
$x+2y+z=3 \rightarrow ②-①\rightarrow$ $y=1$, $x+2y+z=3$ $\rightarrow$ $x+z=1$ $\rightarrow$ $z=0$
$2x+3y+2z=5$ $\quad$ $2x+3y+2z=5$ $\quad$ $2x+3y+2z=5$

TRY $x=1$

CHECK $2(1)+3(1)+2(0)=5$ ✓

→ TRYING $z=1$ & $x=0$ HAS SAME RESULT.

TRY $x=-1$, $y=1$, $z=2$ $\rightarrow$
$(-1)+(1)+(2)=2$ ✓
$(-1)+2(1)+(2)=3$ ✓
$2(-1)+3(1)+2(2)=5$ ✓

$\boxed{1)\ x=1,\ y=1,\ z=0 \quad 2)\ x=0,\ y=1,\ z=1 \quad 3)\ x=-1,\ y=1,\ z=2}$

P 4. *Give the matrix A and vector* **x** *as*

$$A = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 3 & 1 \\ -4 & 1 & 2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}.$$

*(1) Compute A***x** *by dot products of the rows with the column vector;*
*(2) Compute A***x** *as a combination of the columns.*

1) $\begin{bmatrix} 1(2) & 2(2) & 4(3) \\ -2(2) & 3(2) & 1(3) \\ -4(2) & 1(2) & 2(3) \end{bmatrix} \rightarrow \begin{matrix} 2+4+12= \\ -4+6+3= \\ -8+2+6= \end{matrix} \boxed{\begin{bmatrix} 18 \\ 5 \\ 0 \end{bmatrix}}$

2) $2\begin{bmatrix} 1 \\ -2 \\ -4 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} = \begin{matrix} 2+4+12= \\ -4+6+3= \\ -8+2+6= \end{matrix} \boxed{\begin{bmatrix} 18 \\ 5 \\ 0 \end{bmatrix}}$

$A = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 3 & 1 \\ -4 & 1 & 2 \end{bmatrix}, \quad x = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} \rightarrow$

$\begin{bmatrix} (2(1))+(2\cdot 2)+(3\times 4) \\ (-2(2))+(3\times 2)+(1\times 3) \\ (-4(2))+(1\times 2)+(2\times 3) \end{bmatrix} \quad \begin{matrix} 2+4+12= \\ -4+6+3 \\ -8+2+6 \end{matrix} \begin{bmatrix} 18 \\ 5 \\ 0 \end{bmatrix}$

$2\begin{bmatrix} 1 \\ -2 \\ -4 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} \rightarrow$

$2+4+12=18$
$-4+6+3=5$
$-8+2+6=0$

$x+y+z=2$ $\quad\rightarrow\quad$ $a\langle 1,1,1\rangle$
$x+2y+z=3$ $\quad\quad$ $b\langle 1,2,1\rangle$