1:

a: $\lim_{n\to\infty}\sum_{i=1}^{n}(5(x_i)^2-3(x_i)^3)\Delta x$ OVER $[0,2]$

$\int_0^2 5x^2-3x^3\,dx=$ DEFINITE INTEGRAL

$F=\frac{5}{3}x^3-\frac{3}{4}x^4$

$a=0$

$b=2$

$F(b)=\frac{5}{3}2^3-\frac{3}{4}2^4 \to \frac{5}{3}(8)-\frac{3}{4}(16)\to\frac{40}{3}-\frac{48}{4}\to\frac{160}{12}-\frac{144}{12}\to\frac{16}{12}\to\frac{4}{3}$

$F(a)=0$

$F(b)-F(a)=4/3-0=4/3$

b. $\lim_{n\to\infty}\sum_{i=1}^{n}\sin(2x_i)\cos(2x_i)\Delta x$ OVER $[0,\pi]$

$\int_0^{\pi}\sin(2x)\cos(2x)\,dx$

$F(x)=-\cos(2x)\sin(2x)$

$F(b)=-\cos(2\pi)\sin(2\pi)\to -1(0)=0$

$F(a)=-\cos(0)\sin(0)\to 1(0)=0$

$F(b)-F(a)=0-0=0$

2:

a: $\int_0^3 x^2\,dx=\lim_{n\to\infty}\sum_{i=1}^{n}(x_i^2)\Delta x$ OVER $[0,3]$

$\Delta x=3/n$

$x_i=0+i\Delta x\to 3i/n$

$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{9i^2}{n^2}\cdot\frac{3}{n}\to\boxed{\lim_{n\to\infty}\sum_{i=1}^{n}\frac{27i^2}{n^3}}$

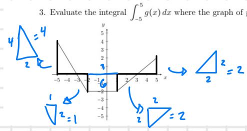
b: $\int_{\pi}^{\frac{3\pi}{2}}\sin(x)\cos(x)=\lim_{n\to\infty}\sum_{i=1}^{n}\sin(x_i)\cos(x_i)\Delta x$

$\Delta x=\frac{\frac{3\pi}{2}-\pi}{n}\to\frac{\frac{3\pi}{2}-\frac{2\pi}{2}}{n}\to\frac{\frac{\pi}{2}}{\frac{n}{1}}\to\frac{\pi}{2}\cdot\frac{1}{n}\to\frac{\pi}{2n}$

$x_i=\pi+i\Delta x\to\pi+\frac{\pi i}{2n}$

$\boxed{\lim_{n\to\infty}\sum_{i=1}^{n}\sin\left(\pi+\frac{\pi i}{2n}\right)\cos\left(\pi+\frac{\pi i}{2n}\right)\frac{\pi}{2n}}$

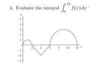
3. Evaluate the integral $\int_{-5}^{5} g(x)\,dx$ where the graph of $y=g(x)$ is shown below.



$4+2=6$

$6-(6+1+2)=-3$

4. Evaluate the integral $\int_0^{12} f(x)\,dx$ where the graph of $y=f(x)$ is shown below.



$2\times2=4$

$\cap_{0\ 2}=1.5707963267 9\to\frac{\pi}{2}$

$\cap_{6\ 12}=14.1371669411\to\frac{9}{2}\pi$

$\frac{9\pi}{2}+\frac{\pi}{2}\quad\frac{10\pi}{2}-4$

$(5\pi-4)=11.7079632679$

5.

a $\int_0^1\frac{1}{(11x+1)^3}dx\to$ $u=11x+1$, $du=11dx\to\frac{1}{11}du=dx$

$\int\frac{1}{u^3}\cdot\frac{1}{11}du\to\frac{1}{11}\int_1^{12}\frac{1}{u^3}du\to\frac{1}{11}\Big|_1^{12}-\frac{1}{2u^2}\to\frac{1}{11}\Big|_1^{12}\left(-\frac{1}{2(12)^2}-\left(-\frac{1}{2(1)^2}\right)\right)\to\frac{1}{11}\left(\frac{143}{288}\right)\to\boxed{\frac{143}{3168}}$

b $\int_0^1\frac{1}{11x+1}dx$ $u=11x+1$, $du=11dx\to\frac{1}{11}du=dx$

$\int\frac{1}{u}\cdot\frac{1}{11}du\to\frac{1}{11}\int\frac{1}{u}du\to\frac{1}{11}\int\frac{1}{11x+1}du\to\frac{1}{11}\Big|_1^{12}\ln(u)\to\frac{\ln(u)}{11}\to\frac{\ln(11x+1)}{11}\to\frac{\ln(12)}{11}-\frac{\ln(1)}{11}\to\frac{\ln(12)}{11}-\frac{0}{11}\to\boxed{\frac{\ln(12)}{11}}$

c $\int_0^{\pi/2}\cos^3\theta\sin\theta\,d\theta$ $u=\cos\theta$, $du=-\sin\theta\,d\theta\to d\theta=-\frac{1}{\sin\theta}du$

$-\int\frac{u^3\sin\theta}{1}\frac{1}{\sin\theta}du\to-\int u^3du\to-\frac{u^4}{4}\to-\frac{\cos^4\theta}{4}\to-\frac{\cos^4(\frac{\pi}{2})}{4}-\left(-\frac{\cos^4(0)}{4}\right)\to-\frac{0}{4}+\frac{1}{4}\to\boxed{\frac{1}{4}}$

d

$\int e^t \sin(e^t) \cos(e^t) dt$ $\quad u = \sin(e^t)$, $du = e^t \cos(e^t) dt \rightarrow dt = \frac{e^{-t}}{\cos(e^t)}$

$\int u \, du \rightarrow \frac{u^2}{2} \rightarrow \boxed{\frac{\sin^2(e^t)}{2} + C}$

e

$\int \frac{e^x}{e^x + 5}$ $\quad u = e^x + 5$, $du = e^x dx \rightarrow du \frac{1}{e^x} = dx$

$\int \frac{e^x}{u} \frac{1}{e^x} du \rightarrow \int \frac{1}{u} du \rightarrow \ln(u) \rightarrow \boxed{\ln(e^x + 5) + C}$

f

$\int \frac{e^x + 5}{e^x} \rightarrow \int \frac{e^x}{e^x} dx + \int \frac{5}{e^x} dx \rightarrow x + \int \frac{5}{e^x} dx \rightarrow x + \int 5e^{-x} dx \rightarrow$ $\quad u = -x$, $du = -1dx \rightarrow dx = -du$

$-5 \int e^u du \rightarrow -5e^u \rightarrow -5e^{-x} + C \rightarrow \boxed{x - 5e^{-x} + C}$

g. $\int_0^{\pi/4} \sec^2\theta \tan^2\theta \, d\theta$

$u = \tan\theta$

$du = \sec^2\theta \, d\theta \rightarrow d\theta = \frac{du}{\sec^2\theta}$ $\quad$ SIN

$\int u^2 du \rightarrow \frac{u^3}{3} \rightarrow \frac{\tan^3\theta}{3} + C \rightarrow \frac{\tan^3(\frac{\pi}{4})}{3} - \frac{\tan^3(0)}{3} \rightarrow \frac{1}{3} - 0 = \boxed{\frac{1}{3}}$

h. $\int_{1/e}^{e} \frac{3}{x\sqrt[3]{\ln x + 7}} dx$ $\quad u = \ln x + 7$, $du = \frac{1}{x} dx$, $dx = du\,x$

POWER RULE: $\frac{u^{n+1}}{n+1}$

$\int \frac{3}{x\sqrt[3]{u}} \frac{x}{1} du \rightarrow \int \frac{3}{\sqrt[3]{u}} \rightarrow \int \frac{3}{u^{1/3}} \rightarrow 3 \int \frac{1}{u^{1/3}} \rightarrow 3\left(\frac{3u^{2/3}}{2}\right) \rightarrow \frac{9u^{2/3}}{2} \rightarrow \frac{9(\ln(e)+7)^{2/3}}{2} - \frac{9(\ln(\frac{1}{e})+7)^{2/3}}{2}$

$\rightarrow \frac{9(1+7)^{2/3}}{2} \rightarrow \frac{9(8)^{2/3}}{2} - \frac{9(4)}{2}$

$\rightarrow 18 - \frac{9(6)^{2/3}}{2} \rightarrow \boxed{\frac{36 - 9\sqrt[3]{36}}{2}}$