JANUARY 10, 2020

INTEGRATION BY PARTS & TRIGONOMETRIC SUBSTITUTION.

(ILATE)
(LIATE)

Ex. $\int x^2 \ln(x)\,dx$

ALG. LOG

$u = \ln(x) \quad du = \frac{1}{x}$

$dv = x^2 dx \quad v = \frac{x^3}{3}$

$$\frac{x^3}{3}\ln(x) - \int \frac{1}{x}\cdot\frac{x^3}{3}dx \to \frac{x^3}{3}\ln(x) - \int \frac{x^2}{3}dx \to \frac{x^3}{3}\ln(x) - \frac{1}{3}\int x^2 dx \to \frac{x^3}{3}\ln(x) - \frac{1}{3}\left(\frac{x^3}{3}\right) \to \frac{x^3}{3}\ln(x) - \frac{x^3}{9} + C$$

ex. $\int \ln(x)dx = \int (1)\ln(x)dx$

$u = \ln(x) \quad du = \frac{1}{x}$

$dv = dx \quad v = x$

$$x\ln(x) - \int \frac{1}{x}\cdot\frac{x}{1}dx \to \boxed{x\ln(x) - x + C}$$

Ex. $\int e^x \cos(x)\,dx$

Exp TRIG

$u = \cos(x) \quad du = -\sin(x)dx$

$dv = e^x dx \quad v = e^x$

$$e^x\cos(x) + \int \sin(x)e^x dx$$

$u = \sin(x) \quad du = \cos(x)$

$dv = e^x dx \quad v = e^x$

$$e^x\sin(x) - \int \sin(x)e^x$$

$$\int e^x\cos(x) = e^x\cos(x) + e^x\sin(x) - \int e^x\cos(x)dx + C$$

$$2\int e^x\cos(x) = e^x\cos(x) + e^x\sin(x) + C$$

$$\boxed{\int e^x\cos(x) = \frac{e^x\cos(x) + e^x\sin(x)}{2} + C}$$

TRIG. INTEGRALS

ex:

$$\int \sin^m(x)dx,\ \int \cos^n(x)dx,\ \int \sin^m(x)\cos^n(x)$$

KEY: NOTICE IF M OR N POWER IS EVEN OR ODD.

IMPORTANT TO REMBER IDENTITIES:

$\sin^2(x) + \cos^2(x) = 1$

- $\cos^2(x) = 1 - \sin^2(x)$
- $\sin^2(x) = 1 - \cos^2(x)$
- $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$
- $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$
- $\tan^2(x) + 1 = \sec^2(x) \longrightarrow \frac{\sin^2 x}{\cos^2(x)} + 1 = \frac{1}{\cos^2(x)}$

$\tan^2(x) + 1 = \sec^2(x)$

IF M IS ODD: $\int \sin^m x\,dx = \int (\sin^{m-1}x)\sin(x)dx$

M-1 IS EVEN & WE CAN NOW USE $\sin^2 x$

Ex. $\int \sin^5(x)dx = \int \sin^4 x \sin x\,dx \to \int (\sin^2 x)^2 \sin(x)dx \to \int (1-\cos^2(x))\sin x\,dx \to \int (1-u^2)^2(-du) \to -\int (1-u^2)^2 du \to -\int (1-2u^2+u^4)du$

$u = \cos x$
$du = -\sin x\,dx$

$$-\left[u - \frac{2u^3}{3} + \frac{u^5}{5}\right] + C$$

$$-\left[\cos(x) - \frac{2}{3}\cos^3(x) + \frac{1}{5}\cos^5(x)\right] + C$$

REMEMBER
$(a+b)^2 = a^2 + 2ab + b^2$

Ex:

$$\int_0^{\pi/2} \cos^4 x \, dx \rightarrow \int_0^{\pi/2} (\cos^2(x))^2 dx \rightarrow \int_0^{\pi/2} \left(\frac{1+\cos(2x)}{2}\right)^2 dx \rightarrow \frac{1}{4}\int_0^{\pi/2} (1+2\cos(2x)+\cos^2(2x))dx \rightarrow \frac{1}{4}\left[\int dx + 2\int \cos(2x)dx + \int \cos^2(2x)dx\right]$$

ITS AWESOME

$$\rightarrow \frac{1}{4}\left[\frac{\pi}{2} + \frac{2\sin(2x)}{2}\Big|_0^{\pi/2} + \int_0^{\pi/2} \left(\frac{1+\cos^2(4x)}{2}\right)dx\right] \rightarrow \frac{1}{4}\left[\frac{\pi}{2} + 0 + \frac{1}{2}\int_0^{\pi/2} dx + \int_0^{\pi/2} \frac{\cos(4x)}{2}dx\right] \rightarrow \frac{\pi}{8} + \frac{\pi}{2}\left(\frac{1}{2}\right)\left(\frac{1}{4}\right) + \frac{1}{2}\left(\frac{\sin(4x)}{4}\right)\Big|_0^{\pi/2}$$