$\int u\,dv = uv - \int v\,du$

$fg - \int g f'$

CHAIN RULE: $f'(g(x))g'(x)$

$\int 2x\cos(3x)dx \rightarrow 2\int x\cos(3x)dx$

$f = x$
$g' = \cos 3x$
$f' = dx$
$g = \int \cos(3x) \rightarrow \frac{1}{3}\int \cos(u) \rightarrow \frac{\sin(u)}{3} \rightarrow \frac{\sin(3x)}{3}$

$\frac{x\sin(3x)}{3} - \int \frac{\sin(3x)}{3}dx \rightarrow \frac{1}{9}\int \sin(u)du \rightarrow \frac{1}{9} - \cos(u) \rightarrow \frac{-\cos(u)}{9} \rightarrow -\frac{\cos(3x)}{9}$

$u = 3x$
$du = 3dx$
$\frac{1}{3}du = dx$

$\frac{x\sin(3x)}{3} - \left(-\frac{\cos(3x)}{9}\right) \rightarrow 2\left(\frac{x\sin(3x)}{3} + \frac{\cos(3x)}{9}\right) \rightarrow \frac{2}{3}x\sin(3x) + \frac{2}{9}\cos(3x) + 9$

1b $\int x^2 e^x dx$

$f = x^2$
$g' = e^x$
$f' = 2x\,dx \rightarrow \frac{1}{2}f' = x\,dx$
$g = \int e^x dx = e^x$

$\int x^2 e^x = x^2e^x - 2\int xe^x dx \rightarrow x^2e^x - 2(xe^x + e^x) \rightarrow \boxed{x^2e^x - 2xe^x + 2e^x + C}$

$u = x$
$du = dx$

$2\int ue^x du \rightarrow 2\int xe^x du \rightarrow xe^x + e^x$

1c $\int \frac{xe^{4x}}{2}dx \rightarrow \frac{1}{2}\int xe^{4x}$

$f = x$
$g' = e^{4x}$

$f' = dx$
$g = \int e^{4x} \rightarrow \frac{1}{4}\int e^u \rightarrow \frac{e^{4x}}{4}$

$u = 4x$
$\frac{1}{4}du = dx$

$\frac{xe^{4x}}{4} - \frac{1}{4}\int e^{4x}dx \rightarrow \frac{xe^{4x}}{4} - \frac{1}{4}\left(\frac{e^{4x}}{4}\right) \rightarrow \frac{1}{2}\left(\frac{xe^{4x}}{4} - \frac{e^{4x}}{16}\right) \rightarrow \boxed{\frac{xe^{4x}}{8} - \frac{e^{4x}}{32} + C}$

1d $\int \ln(2x+1)dx$

$u = 2x+1$
$du = 2dx \rightarrow \frac{1}{2}du = dx \quad \rightarrow \frac{1}{2}\int \ln(u)du$

$f = \ln(u)$
$g' = du$

$f' = \frac{1}{u}$
$g = \int du \rightarrow u$

$\frac{1}{2}\left[u\ln u - \int 1\,du\right] \rightarrow \frac{1}{2}\left[(2x+1)(\ln(2x+1)) - (2x+1)\right]$

$\frac{1}{2}(2x+1)\ln(2x+1) - \frac{1}{2}(2x+1) \rightarrow \boxed{\frac{1}{2}(2x+1)\ln(2x+1) - x + C}$

1e $\int (\ln x)^2 dx \to \int u^2 u^{-1} du \to \int u\, du \to \int \ln(x) dx \to$

$f = \ln(x)^2$
$g' = dx$

$f' = 2\ln(x) \cdot \frac{1}{x} \to \frac{2\ln(x)}{x}$ $\quad x\ln(x)^2 - \int \frac{2\ln(x)}{x}(x) \to x\ln(x)^2 - 2\int \frac{\cancel{x}\ln(x)}{\cancel{x}} \to x\ln(x)^2 - 2\int \ln(x)dx$

$g = \int dx \to x$ $\quad \to x\ln(x)^2 - 2(x\ln(x) - x) + C$

$\boxed{x\ln(x)^2 - 2x\ln(x) + 2x + C}$

1f $\int x^2 \cos(x^3) dx \to \frac{1}{3}\int \cos(u) dx \to \frac{1}{3}\sin(u) \to \boxed{\frac{1}{3}\sin(x^3) + C}$

$u = x^3$
$\frac{1}{3}du = x^2 dx$

1g $\int x \sec^2 x\, dx \to$

$f = x$
$g' = \sec^2 x\, dx$

$f' = 1$
$g = \int \sec^2 x \to \tan(x) + c$

$x\tan(x) - \int 1(\tan(x))dx \to \boxed{x\tan(x) + \ln(\tan(x)) + C}$

1h $\int_0^1 x e^{-2x} dx$

$f = x$
$g' = e^{-2x} dx$

$f' = 1$
$g = \int e^{-2x} dx \to -\frac{1}{2}\int e^u du \to -\frac{1}{2}e^{-2x}$

$u = -2x$
$du = -2dx \to -\frac{1}{2}du = dx$

$\frac{-xe^{-2x}}{2} + \frac{1}{2}\int e^{-2x} dx \to -\frac{1}{4}\int e^u du \to -\frac{1}{4}e^u \to -\frac{1}{4}e^{-2x}$

$u = -2x$
$du = -2dx \to -\frac{1}{2}du = dx$

$\frac{-xe^{-2x}}{2} - \frac{e^{-2x}}{4} \to \frac{-2xe^{-2x} - e^{-2x}}{4}$

$\frac{-2(1)e^{-2(1)} - e^{-2(1)}}{4} - \frac{-2(0)e^{-2(0)} - e^{-2(0)}}{4}$

$\frac{-2e^{-2} - e^{-2}}{4} - \frac{(0-1)}{4} \to -\frac{2}{4e^2} - \frac{1}{4e^2} + \frac{1}{4} \to \frac{1}{4} - \frac{3}{4e^2} \to \boxed{\frac{e^2 - 3}{4e^2}}$

1.

$$\int_1^e \ln(x^2)\,dx$$

$f = \ln(x^2)$
$g' = dx$

$f' = \frac{2x}{x^2}$
$g = x$

$$x\ln(x^2) - \int \frac{2x^3}{x^2}\,dx$$

$u = 2x^3$
$du = 6x^2 \rightarrow \frac{1}{6}du = x^2\,dx$

$$\rightarrow \frac{1}{6}\int \frac{u}{x^2}x^2 \rightarrow \frac{1}{6}\int u\,dx \rightarrow 1$$

$$x\ln(x^2) - 1$$

$$\left(x\ln(e^2) - 1\right) - \left(x\ln(1) - 1\right) \rightarrow 1 - (-1) = \boxed{2}$$

2. A PARTICAL MOVES ALONG A STRAIGHT LINE WITH A VELOCITY OF $v(t) = t^2 e^{-t}$ METERS/SEC AFTER T SECONDS. HOW FAR DOES IS TRAVEL IN THE FIRST 3 SECONDS?

$$\int_0^3 t^2 e^{-t}\,dt$$

$f = t^2$
$g' = e^{-t}dt$

$f' = 2t$
$g = \int e^{-t}dt \rightarrow -e^{-t}$

$$-t^2e^{-t} - \int 2t - e^{-t} \rightarrow -t^2e^{-t} - \left(-2e^{-t} - (-e^{-t})2t\right)$$

$$-t^2e^{-t} + 2e^{-t} + 2te^{-t}$$

$$-(t^2 + 2t + 2)e^{-t} + C$$

$$-(9 + 6 + 2)e^{-3} - (-(0 + 0 + 2)1$$

$$\boxed{2 - 17e^{-3}}$$