$$\int \frac{x^2 - 2x - 1}{(x-1)^2(x^2+1)}dx \rightarrow \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1}$$

$$x^2 - 2x - 1 = A(x-1)(x^2+1) + B(x^2+1) + (Cx+D)(x-1)^2$$

IF $x = 0 \rightarrow -1 = -A + B + D \rightarrow -1 = -A - 1 + D \quad \boxed{D - A = 0}$

IF $x = 1 \rightarrow -2 = 0 + 2B \rightarrow \boxed{B = -1}$

IF $x = -1 \rightarrow 2 = -4A - 2 \quad -4C + 4D \rightarrow$

$$4 = -4A - 4C + 4D \rightarrow 1 = -A - C + D \quad \overset{0}{\overbrace{D - A}} - C = 1 \rightarrow \boxed{C = -1}$$

IF $x = 2 \rightarrow 4 - 4 - 1 = A(1)(5) + (-1)(5) + (2C+D)(1)$

$$-1 = 5A - 5 - 2 + D$$

$$6 = 5A + D \rightarrow \boxed{A = 1, \ D = 1}$$

Ex: $\int \frac{x^3}{x^2+1}dx$

EUCLIDEAN ALGORITHM

$$\frac{DIVIDEND}{DIVISOR} = QUOTIENT + \frac{REMAINDER}{DIVISOR}$$

1. POWER IS HIGHER ON TOP, SO DO LONG DIVISION

$$\begin{array}{r} x \leftarrow \text{QUOTIENT} \\ x^2+1 \overline{\smash{)}\, x^3} \leftarrow \text{DIVIDEND} \\ \underline{-\ x^3 + x} \\ 0 - x = \boxed{-x} \leftarrow \text{REMAINDER} \end{array}$$

DIVISOR

$\rightarrow \quad x - \frac{x}{x^2+1} \quad \rightarrow \quad \int x\,dx - \int \frac{x}{x^2+1}dx$

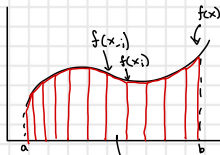$$\boxed{\frac{1}{2}x^2 - \frac{1}{2}\ln|x^2+1| + C}$$

NUMERICAL INTEGRATION (TRAPEIZODAL RULE)

$$\int e^{x^2}dx = ??? \quad \text{HARD TO FIND } F(x)$$

REMEMBER:

$$A = \frac{B+b}{2}h \quad \text{"AREA OF TRAPIZOID"}$$

$f(x)$
$f(x_{i-1})$
$f(x_i)$

a ... b

$$\Delta x = \frac{b-a}{n} = X_i - X_{i-1}$$

$f(x_{i-1})$ $f(x_i) \rightarrow A = \frac{1}{2}\Delta x(f(x_i) + f(x_{i-1}))$

Δx

$$\int_a^b f(x)dx = \sum_{i=1}^n A = \sum_{b=1}^a \frac{1}{2}\Delta x(f(x_i) + f(x_{i-1}))$$

$$\int_a^b f(x)dx = \frac{1}{2}\Delta x \left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n) \right]$$

Ex use TRAPEZOIDAL RULE: $n=4$, ESTIMATE $\int_1^2 x^2 dx$

$$\Delta x = \frac{B-A}{n} \rightarrow \frac{2-1}{4} \rightarrow \boxed{\frac{1}{4}} \qquad\qquad \frac{1}{3}x^3 \rightarrow \frac{8}{3} - \frac{1}{3} \rightarrow \boxed{\frac{7}{3}}$$

$x_0 = 1$

$x_1 = 1 + \frac{1}{4} \rightarrow \frac{5}{4}$

$x_2 = 1 + \frac{1}{2} \rightarrow \frac{3}{2}$

$x_3 = 1 + \frac{3}{4} \rightarrow \frac{3}{4}$

$x_4 = 1 + 1 \rightarrow 2$

$$\frac{1}{2}\Delta x \left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$$

$$\int_1^2 x^2 dx = \frac{1}{2}\left(\frac{1}{4}\right)\left[f(1) + 2f\left(\frac{5}{4}\right) + 2f\left(\frac{3}{2}\right) + 2f\left(\frac{3}{4}\right) + f(2)\right]$$

$$\frac{1}{2}\left(\frac{1}{4}\right)\left[1 + 2\left(\frac{25}{16}\right) + 2\left(\frac{9}{4}\right) + 2\left(\frac{49}{16}\right) + 4\right]$$

$$1 + \frac{50}{16} + \frac{18}{4} + \frac{98}{16} + 4$$

$$\frac{16}{16} + \frac{50}{16} + \frac{72}{16} + \frac{98}{16} + \frac{64}{16}$$

$$\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\frac{300}{16} \rightarrow \frac{1}{4}\;\frac{150}{16}$$

$$\frac{150}{64} \qquad \frac{75}{32}$$