01/08/20

U SUBSTITUTION

A METHOD TO FIND AN ANTIDERIVATIVE.

GOALS:
GET RID OF ALL X

INTEGRATION BY PARTS:

PRODUCT RULE: $(fg)' = f'(x)g(x) + f(x)g'(x)$

IF WE INTEGRATE BOTH SIDES:

$$\int (fg)'dx = \int f'(x)g(x) + f(x)g'(x)$$
$$\downarrow$$
$$fg = \int f'(x)g(x) + \int f(x)g'(x)$$
$$\downarrow$$
$$\int f(x)g'(x) = fg - \int f'(x)g(x)$$

$u = f(x)$ $\quad du = f'(x)dx$
$v = g(x)$ $\quad dv = g'(x)dx$

FORMULA:

$$\int u\,dv = uv - \int v\,du$$

FIND u & dv & COMPUTE v & du

$$\int fg = fg - \int f'g$$

RIGHT POINT INTEGRAL METHOD IS NOT DESIREABLE FOR COMPUTATIONS

WE HAVE THE EVELUATION THEOREM, BUT IT REQUIRES THE FUNCTION TO BE CONTINUOUS.

TO FIND F (U-SUBSTITUTION): "REVERSE" TO CHAIN RULE.

Ex: $\int_0^3 x^2(x^3+1)dx$

$u = x^3+1$ $\quad x=0,\ u=1$
$du = 3x^2$ $\quad x=3,\ u=28$
$3du = x^2$

$$\int_1^{28} u\frac{du}{3} \to \frac{1}{3}\int_1^{28} u\,du = \frac{1}{3}\frac{u^2}{2}\Big|_1^{28} = \frac{1}{3}\left(\frac{28^2}{2} - \frac{1^2}{2}\right)$$

Ex $\int_0^2 x\cos\left(\frac{x^2}{4}\right)dx$

$u = \frac{x^2}{4}$

$du = \frac{x}{2}dx$

$x=0,\ u=0$
$x=2,\ u=1$

$$\to \int_0^2 x\cos\left(\frac{x^2}{4}\right)dx = \int_0^1 \cos(u)2du \to 2\int_0^1 \cos(u)du = 2\sin(u)\Big|_0^1 = 2\sin(1) - 2\sin(0)$$
$$= 2\sin(1)$$

Ex: $\int \frac{x^3}{\sqrt{9+x^2}}dx = \int x^2(9+x^2)^{-\frac{1}{2}} = \int \frac{1}{2}x^2(9+x^2)^{-\frac{1}{2}}2x\,dx$

$u = 9+x^2$
$du = 2x\,dx$

$x^2 = u-9$

$$= \int \frac{1}{2}(u-9)u^{-\frac{1}{2}}du$$
$$= \frac{1}{2}\int u^{\frac{1}{2}} - 9u^{-\frac{1}{2}}du$$
$$= \frac{1}{2}\int u^{\frac{1}{2}}du - \frac{9}{2}\int u^{-\frac{1}{2}}du$$
$$= \frac{1}{2}\left(\frac{2}{3}u^{\frac{3}{2}}\right) - \frac{9}{2}\left(2u^{\frac{1}{2}}\right) + C$$
$$= \frac{1}{3}(9+x^2)^{\frac{3}{2}} - 9(9+x^2)^{\frac{1}{2}} + C$$

INTEGRATION BY PARTS: ("REVERSE" PROCESS TO PRODUCT RULE)

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

INTEGRATE IN BOTH SIDES:

$$\int \frac{d}{dx}(f(x)g(x)) = \int f'(x)g(x)dx + \int f(x)g'(x)dx$$
$$\Rightarrow f(x)g(x) = \int f'(x)g(x)dx + \int f(x)g'(x)dx$$
$$\Rightarrow \int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx$$

IF $u = f(x)$
$dv = g'(x)dx$

$$\boxed{\text{FORMULA: } \int u\,dv = uv - \int v\,du}$$

YOUR JOB? CORRECTLY SELECT u

HOW DO YOU CHOOSE?

USUALLY! CHOOSE u USING

LOGRITHMIC f(x)
TRIG f(x)
ILATE - EXPONENTIALS
ALGEBRAIC EXPRESSION
INVERSE f(x)

Ex:

$$\int \underset{\text{ALG}}{x}\underset{\text{EXP.}}{e^{2x}}dx$$

$u = x$
$du = dx$
$dv = e^{2x}dx$
$v = \frac{1}{2}e^{2x}$

$$uv - \int v\,du = \frac{x}{2}e^{2x} - \int \frac{1}{2}e^{2x}dx \to \frac{x}{2}e^{2x} - \frac{1}{4}e^{4x} + C$$