Homework Problems

HOMEWORK 3

1. Evaluate the following integrals.

   (a) $\int \sin^3 x \ \cos x \, dx$

   Solution: $\int \sin^3 x \ \cos x \, dx \ = \ \frac{1}{4} \sin^4 x + C$

   (b) $\int \tan^5(2x) \ \sec^2(2x) \, dx$

   Solution: $\int \tan^5(2x) \ \sec^2(2x) \, dx \ = \ \frac{1}{12} \tan^6(2x) + C$

   (c) $\int \sin^3 x \, dx$

   Solution: $\int \sin^3 x \, dx \ = \ \frac{1}{3} \cos^3 x - \cos x + C$

   (d) $\int \cos^5 x \, dx$

   Solution: $\int \cos^5 x \, dx \ = \ \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + C$

   (e) $\int \sin^3 x \ \cos^3 x \, dx$

   Solution: $\int \sin^3 x \ \cos^3 x \, dx \ = \ \frac{1}{4} \sin^4 x - \frac{1}{6} \sin^6 x + C$

   (f) $\int \tan x \ \sec^3 x \, dx$

   Solution: $\int \tan x \ \sec^3 x \, dx \ = \ \frac{1}{3} \sec^3 x + C$

   (g) $\int_0^{\pi} \sin^2 x \, dx$

   Solution: $\int_0^{\pi} \sin^2 x \, dx \ = \ \frac{\pi}{2}$

   (h) $\int_0^{\pi/8} \sin^2 x \ \cos^2 x \, dx$

   Solution: $\int_0^{\pi/8} \sin^2 x \ \cos^2 x \, dx \ = \ \frac{\pi - 2}{64}$

   (i) $\int_{\pi/36}^{\pi/12} \cos^2(3x) \, dx$

   Solution: $\int_{\pi/36}^{\pi/12} \cos^2(3x) \, dx \ = \ \frac{2\pi + 3}{72}$

2. Find the area of the region bounded by the graphs of the equations $y = \cos^2 x$, $y = \sin^2 x$, $x = -\frac{\pi}{4}$, and $x = \frac{\pi}{4}$

   Solution: Area $= 1$

$$\int \sin^3(x)\cos(x)\,dx \to \int u^3 \cancel{\cos(x)} \frac{du}{\cancel{\cos(x)}} \to \int u^3\,du \to \tfrac{1}{4}u^4 \to \boxed{\tfrac{1}{4}\sin^4(x) + C}$$

$u = \sin(x)$
$du = \cos(x)\,dx$
$dx = \frac{du}{\cos(x)}$

$$\int \tan^5(2x)\sec^2(2x)\,dx \to \frac{1}{2}\int u^5 \cancel{\sec^2(2x)} \frac{du}{\cancel{\sec^2(2x)}} \to \frac{1}{2}\left(\frac{1}{6}u^6\right) \to \frac{u^6}{12} \to \boxed{\frac{\tan^6(x)}{12} + C}$$

$u = \tan(2x)$
$du = 2\sec^2(2x)\,dx$
$dx = \frac{du}{2\sec^2(x)}$

$$\int \sin^3(x)\,dx$$

$$\int \sin^2(x)\sin(x) \to \int (1-\cos^2(x))\sin(x) \to -\int (1-u^2)\cancel{\sin(x)}\frac{du}{\cancel{\sin(x)}} \to -\left(u - \frac{1}{3}u^3\right) \to \boxed{-\cos(x) + \frac{1}{3}\cos^3(x) + C}$$

$u = \cos$
$du = -\sin(x)\,dx$
$dx = -\frac{du}{\sin(x}$

$$\int \cos^5 x \to \int \cos^4 x \cos(x) \to \int (1-\sin^2(x))^2\cos(x) \to \int (1-u^2)^2 \cancel{\cos(x)}\frac{du}{\cancel{\cos(x)}} \to \int u^4 - u^2 - u^2 + 1 \to \int \sin(x)^4 - 2\int \sin^2(x) + \int du$$

$$\frac{1}{5}\sin(x)^5 - \frac{2}{3}\sin^3(x) + \sin(x) + C$$

$u = \sin(x)$
$du = \cos(x)$
$dx = \frac{du}{\cos(x)}$

$$\int \sin^3(x)\cos^3(x)$$

$$\int \cos(x)(\sin^3(x))(1-\sin^2(x)) \to \int \cancel{\cos(x)}[u^3(1-u^2)]\frac{du}{\cancel{\cos(x)}} \to \int u^3(1-u^2)\,du \to \int u^3 - u^5\,du \to \frac{1}{4}u^4 - \frac{1}{6}u^6 \to \boxed{\frac{\sin^4(x)}{4} - \frac{\sin^6(x)}{6} + C}$$

$u = \sin(x)$
$du = \cos(x)$
$dx = \frac{du}{\cos(x)}$

$$\int \tan(x)\sec^3(x)\,dx \to \int \tan(x)\sec(x)\sec^2(x) \to \int \cancel{\tan(x)\sec(x)}(u^2)\frac{du}{\cancel{\sec(x)\tan(x)}} \to \int u^2\,du \to \frac{1}{3}u^3 + C \to \boxed{\frac{1}{3}\sec^3(x) + C}$$

$u = \sec(x)$
$du = \sec(x)\tan(x)$
$dx = \frac{du}{\sec(x)\tan(x)}$

$$\int_0^\pi \sin^2(x) \to \frac{1}{2}\int_0^\pi (1-\cos(2x))\,dx \to \frac{1}{2}\left(\int 1\,dx - \int \cos(2x)\right)dx$$

$$\to \frac{1}{2}\left(x - \frac{\sin(2x)}{2}\right)$$

$$\frac{x}{2} - \frac{\sin(2x)}{4} \to \frac{2x - \sin(2x)}{4}$$

$$F(\pi) - F(0) = \frac{2\pi - \sin(2\pi)}{4} - \frac{2(0) - \sin(2(0))}{4} \to \frac{2\pi}{4} \to \boxed{\frac{\pi}{2}}$$

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$$\int_0^{\pi/8} \sin^2(x)\cos^2(x)\,dx \rightarrow \int_0^{\pi/8} (1-\cos^2(x))\cos^2(x)\,dx \rightarrow \int_0^{\pi/8} \cos^2(x) - \cos^4(x)\,dx \rightarrow \int_0^{\pi/8} \cos^2(x)\,dx - \int \cos^4(x)\,dx$$

$$\int \cos^2(x) = \frac{1}{2}\int 1+\cos(2x) \rightarrow \frac{1}{2}\int 1 + \frac{1}{2}\int \cos(2x)$$

$$\rightarrow \frac{1}{2}x + \frac{\sin(x)}{4}$$

$$\int \cos^4(x)\,dx \rightarrow \frac{1}{2}\int (1+\cos(2x))^2\,dx \rightarrow \frac{1}{4}\int (1+\cos(2x))(1+\cos(2x)) \rightarrow \frac{1}{4}\int 1+2\cos(2x)+\cos(2x)^2$$

$$\frac{x}{4} + \frac{\sin(x)}{8} + \int 1-$$