JANUARY 31, 2020

SEQUENCES:

DEFINITION: AN ORDERED LIST OF NUMBERS

IN A RIGOROUS WAY, WE CAN DEFINE A SEQUENCE a FUNCTION $f(n)\left\{ f: \mathbb{N} \to \mathbb{R} \right.$

$\{f(1), f(2), f(3), \ldots, f(n), f(n+1)\}$ THIS IS ORDERED

Ex: THE SEQUENCE $a = \{3,4,5,6,7,8,9\} \longrightarrow a_1 = 3,\ a_2 = 4, \ldots, a_7 = 9$

Ex: THE SEQUENCE $b = \{4, 7, 11\}$ $b_1 = 4,\ b_2 = 7,\ b_3 = 11$

$c = \{11, 7, 4\} \to$ SAME SET, DIFFERENT SEQUENCE.

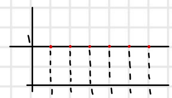
$c \mathrel{!=} b$

FROM OUR EXAMPLES, A, B, & C ARE FINITE SEQUENCES.

CONCEPT

SEQUENCES CAN HAVE AN INFINITE NUMBER OF TERMS. A SEQUENCE CAN BE DESCRIBED BY A RULE OF THE n^{th} TERM.

Ex: CONSIDER SEQUENCE $\{1,1,1,1,\ldots,1,1,\ldots\} \to$ CONSTANT SEQUENCE. $\to f(n) = 1 \to a_n = 1 \to f(n) = a_1$



Ex: $\{1,2,3,4,5,6,7,8,9,10,\ldots,n+1, n+2,\ldots\} \to f(n) = n$

$\{n\}_{n \geq 1}$

Ex: $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$ $a_n = \frac{1}{n}$

DEFINITION: A SEQUENCE IS INCREASING $a_n \leq a_{n+1}$ & DECREASING $a_n \geq a_{n+1}$ FOR EVERY n

$\{(-1)^n\}_{n \geq 1} \to \{-1, 1, -1, 1, -1, \ldots\} \to$ OSCILLATORY SEQUENCE

LIMIT OF A SEQUENCE: WE SAY A SEQUENCE $\{a_n\}_{n \geq 1}$ IS CONVERGENT IF $\forall \epsilon > 0, \exists N \in \mathbb{N}$ SUCH THAT $\forall n \geq N$ THEN $|a_n - l| < \epsilon$

$l =$ limit

I.E. $\to$ IF THE TERMS OF A SEQUENCE $\{a_n\}$ GET ARBITRARILY CLOSE TO A SINGLE NUMBER L as n GETS LARGE, WE WRITE

$$\lim_{n \to \infty} a_n = L \text{ or } a_n \to L \text{ as } n \to \infty$$

AND SAY $\{a_n\}$ CONVERGES TO L. IF NO L EXISTS, WE SAY $\{a_n\}$ DIVERGES.

Ex: $a_n = 1/n \to \lim_{n \to \infty} \frac{1}{n} = 0$, IT CONVERGES, $L = 0$

Ex: $a_n = \frac{(-1)^n}{n} \to \lim_{n \to \infty} \frac{(-1)^n}{n} = 0$, CONVERGES, $L = 0$

RECURSIVELY DEFINED SEQUENCE → SEQUENCE WHOSE TERMS ARE DEFINED BY OTHER TERMS OF THE SEQUENCE.

EXAMPLE: $a_1 = 1,\ a_2 = 1,\ a_n = a_{n-1} + a_{n-2}$ FOR $n \geq 3$

$a_3 = a_2 - a_1\ ,\ a_4 = a_3 - a_2$

EXAMPLE: WRITE DOWN THE FIRST n TERMS OF THE SEQUENCE:

$a_n = a_{n-1} \cdot a_{n-2} \quad n \geq 3 \quad a_1 = -2,\ a_2 = -3$

$a_3 = a_2 \cdot a_1 = 6$ IF

$a_4 = -18$

$a_5 =$

*PROPERTIES OF LIMITS OF SEQUENCES:

ASSUME $a_n \to L,\ b_n \to M$, THEN

1) $a_n + b_n \to L+M$

2) $C \cdot a_n \to C(L)$

3) $a_n b_n \to (a)(b)$

4) $\frac{a_n}{b_n} \to \frac{L}{M}$. FOR $M \neq 0$

5) IF $L=M$ AND $\{c_n\}$ SATISFIES $a_n \leq c_n \leq b_n$ FOR EVERY n, $c_n \to L$