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Reflection 4
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Reflection on *A Game-Like Activity for Learning Cantor's Theorem* .

Explaining Canto's Theorem to my Roomate.

I've just recently finished reading the article 'A Game-Like Activity for Learning Cantor's Theorem' by Shay Gueron and I'd like to explain what I got from it and bounce some ideas off you.

Admittedly, I do not quite fully understand the theorem or it's implications myself quite yet. I read through the paper twice and found myself just being more confused than when I started the first time. I tried watching a couple videos to supplement some knowledge or context that perhaps I was missing going into reading the paper and I do think it helped somewhat. This is all to say, my explanation and attempt at showing a proof may not be entirely accurate, but my hope is that in attempting to do so, I'll better understand what gaps exist in my understanding. That being said, let me give you a bit of context first.

I previously made an attempt to describe The Empty set to you, recall that I explained first that a set is a collection of elements. Cantor's Theorem is related to set theory and also relies on the idea of Power sets. A power set is a set that contains all possible sets of the original set. Lets say for example that the set A = { 1, 2, 3 }, its power set (denoted as $\mathcal{P}(A)$) would be equal to { {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 2, 3}, ∅} .

With that context, we can now state Cantor's Theorem which is that for every set A, its powerset contains more elements than the set itself.

When I first heard that, I thought it made perfect sense, but when asked to prove it, I find that I am really out of touch with what it is stating. You see, this theorem is true not only for finite sets (sets with a set number of elements), but it is also true for infinite sets as well, which is to suggest that the power set of an infinite set is larger than the original infinite set and that there are different sizes of infinity.

Now to actually prove this, if I can. If this is true, we should have a one-to-one relation with every element in A to its power set, but should not be able to have a one-to-one relation with every element in the power set with A. My understanding is that the best way to prove this is through contradiction. Perhaps this is the only way, though I am not sure. In any case, we can try to prove this is true by first assuming that we can use a function to successfully project A onto $\mathcal{P}(A)$. That is to say that for every element in $\mathcal{P}(A)$ has a relation with an element in A.

$f : A \to$ P(A)

Next, we define a set D as the set of all elements (x) that do not exist in the result of f(x). The problem is, that when attempting to project A onto its power set, you will find an element (x) in A that if plugged into the function will be equal to our set D. The problem here is that the definition of 'D' suggests that if x is in f(y), it should not be in D, but if it is equal to 'D', then it it has to. Because we have a contradiction to the assumption that A can be projected onto it's powerset, this suggests that the carnality (or size) of the power set is larger than A, even when both are infinite sets.

To be honest with you, I am still having a bit of trouble wrapping my head around this myself, as such, I'm note quite sure how to give an example of this. I know this explanation is not the most clear, but it goes to show that I still need to study it a bit more.

Explain why the same theorem or matching game cannot be used to try and map onto the power set minus the empty set.

My understanding of the proof is that it relies on our defined set D as being a subset of the powerset as this set is used to show a contraction to the idea that A can be projected onto the powerset. The problem is that by our definition, it may be possible for our set D to be equal to The Empty set. If we are trying to make the safe proof work for a projection onto the power set minus the empty set, this will not work since D will no longer be a subset of the power set.