Handout 2

Chapters 6-12

Momentum: $\vec{p} = m\vec{v}$ Kinetic energy: $K = \frac{1}{2}mv^2$ Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$

Potential energies: relation between F(x) and U(x). In 1D: $F = {}^{-dU}/_{dx}$ Spring: $U = \frac{1}{2}kx^2$

Gravity near Earth: $U = mgy$ $G = 6.67 \times 10^{-11} Nm^2/kg^2$ Gravity in general: $U = -\frac{GMm}{r}$

Newton's Law of Gravitation: $\vec{F} = -\frac{GMm}{r^2}\hat{r}$ Circular orbits: $\frac{mv^2}{r} = \frac{GMm}{r^2}$

Work: $W = \int \vec{F} \cdot d\vec{r} = \Delta K = K - K_0$ Potential Energy (conservative forces): $\Delta U_{AB} = -\int_A^B \vec{F} \cdot d\vec{r}$

Mechanical Energy: $\Delta E_{Mech} = \Delta K + \Delta U = W_{nc}$ Conservation: $E = K + U = K_0 + U_0 = constant$

Center of mass: $x_{CM} = \frac{1}{M}\sum_i m_i x_i$ $y_{CM} = \frac{1}{M}\sum_i m_i y_i$

Collisions: conservation of momentum and kinetic energy $\vec{P}_{final} = \vec{P}_{initial}$ and $K_{final} = K_{initial}$

Impulse-Momentum Theorem: $Impulse \equiv \vec{J} = \int_{t_i}^{t_f} \vec{F}(t)dt = \bar{\vec{F}}\Delta t = \Delta\vec{p} \equiv Momentum\ change$

Dot product: $\vec{A} \cdot \vec{B} = AB\cos\theta$ or $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

Cross product: $|\vec{A} \times \vec{B}| = AB\sin\theta$ plus right-hand rule for direction or

$\vec{A} \times \vec{B} = (A_x B_y - A_y B_x)\hat{k} + (A_y B_z - A_z B_y)\hat{\imath} + (A_z B_x - A_x B_z)\hat{\jmath}$

Rotational Kinematics: $\omega = \omega_0 + \alpha\,\Delta t$ $\theta = \theta_0 + \omega_0\,\Delta t + \frac{1}{2}\,\alpha\,(\Delta t)^2$ $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$a_r = \frac{v^2}{r} = \omega^2 r$ $v = r\omega$ $\alpha = \frac{dw}{dt}$ $a_t = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha$

Angular momentum: $\vec{L} = \sum_i \vec{r_i} \times \vec{p_i} = I\,\vec{\omega}$ Moment of inertia: $I = \sum_i m_i r_i^2$

Newton's 2nd law for rotation: $\frac{d\vec{L}}{dt} = \sum_i \vec{r_i} \times \vec{F_i} = \sum_i \vec{\tau_i}$ or $I\alpha = \tau_{net}$ for a rigid body

Kinetic energy of a rotating rigid body: $K_{rot} = \frac{1}{2}I\omega^2$ Rolling no-slip: $v = R\omega$

Angular momentum conservation: $\tau_{net} = 0$ when $L_{final} = L_{initial}$ or $I_{final}\omega_{final} = I_{initial}\omega_{initial}$

Moments of inertia: $I_{spherical\ shell} = \frac{2}{3}MR^2$ $I_{disk} = \frac{1}{2}MR^2$ $I_{hoop} = MR^2$ $I_{sphere} = \frac{2}{5}MR^2$
$I_{rod,center} = \frac{1}{12}ML^2$ $I_{rod,end} = \frac{1}{3}ML^2$ Density: $\rho = \frac{m}{V}$ Volume of a sphere: $V = \frac{4}{3}\pi R^3$

Static equilibrium: $\vec{\tau} = \vec{r} \times \vec{F}$ $|\vec{\tau}| = |\vec{r}||\vec{F}|\sin\theta$ plus right-hand rule for direction

Conditions for static equilibrium: $\sum_i \vec{\tau_i} = 0$ and $\sum_i \vec{F_i} = 0$

Conditions for stable equilibrium in 1D: $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} \geq 0$