EXAM 1: NEXT WEEK, THURSDAY 30, FRIDAY 31, SATURDAY 1ST.

MATERIAL COVERS UP TO NUMERIC INTEGRATION

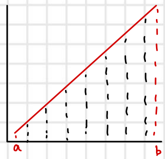
TIME LIMIT: 90 MINUTES

HOW MANY QUESTIONS:

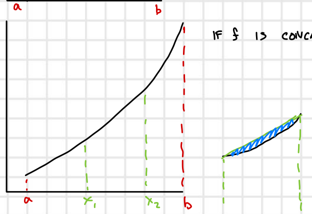
NUMERICAL INTEGRATION (CONT.) (TRAPIZOIDAL RULE)

$$\int_a^b f(x)dx \approx \frac{1}{2}\Delta x\left[f(x_0)+2f(x_1), \cdots, +2f(x_{n-1})+f(x_n)\right]$$

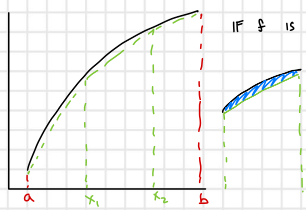
EXAMPLE: DRAW SOME DIFFERENT CURVES & TRY TO FIGURE OUT WHEN USING THE TRAPIZOIDAL RULE WILL RESULT IN OVER OR UNDER APPROXIMATION.



IF f IS LINEAR, TRAPIZOIDAL RULE WILL GIVE US AN EXACT VALUE OF $\int_a^b f(x)dx$



IF f IS CONCAVE UP, TRAPIZOIDAL RULE WILL OVER-APPROXIMATE



IF f IS CONCAVE DOWN, TRAPIZOIDAL RULE WILL UNDER-APPROXIMATE

IF A FUNCTION HAS PERIODS WHERE IT IS CONCAVE UP & CONCAVE DOWN, WE DONT KNOW IF WE ARE OVER OR UNDER-APPROXIMATING. IT DEPENDS ON f

THEOREM: IF $f''(x)$ IS CONTINUOUS & $M>0$ IS A NUMBER S.T $|f''(x)|\le M$ FOR ALL $x\in[a,b]$ THEN $|E_T|\le \frac{M(b-a)^3}{12n^2}$ WHERE E_T IS THE ERROR OF THE TRAPIZOIDAL APPROX. OF $\int_a^b f(x)dx$

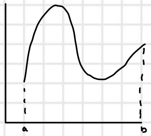
- $|E_T| = |\text{TRAPIZOIDAL RULE} - \int_a^b f(x)dx|$

  APPROX VALUE   EXACT VALUE.

- M = UPPER BOUND FOR $f''(x)$
  (MAXIMUM FOR $f''(x)$)

BIGGER THE n, THE SMALLER THE ERROR.

IF f IS CONTINUOUS ON $[a,b]$



<u>EX:</u> FOR $\int_0^2 (x^2+x)\,dx$, FIND THE MINIMUM n SO THAT THE MAGNITUDE OF THE ERROR FOR THE n-th TRAPZOIDAL APPROXIMATION IS $< 10^{-4}$

$a=0,\ b=2,\ f(x)=(x^3+x)$

$M=?\ ,\quad n=?$

$f'(x)=3x^2+6,\ f''(x)=6x \rightarrow |6x| \le \underset{\substack{\uparrow \\ M}}{12}$ FOR ANY x IN $[0,2]$

USE THEOREM $\rightarrow |E_T| = \frac{\cancel{12}(2-0)^3}{\cancel{12}n^2} \rightarrow \frac{8}{n^2} < 10^{-4} \rightarrow \frac{8}{10^{-4}} < n^2 \rightarrow 8\times10^4 < n^2 \rightarrow \sqrt{80{,}000} < n \rightarrow \sqrt{80{,}000} \approx 282.8 < n \rightarrow \boxed{N=283}$

LEARN CONVENTION FOR INTEGERS.

EXAMPLE: