HOMEWORK 5

1a)

$$\frac{5x-13}{x^2-5x+6} \rightarrow \frac{5x-13}{(x-3)(x-2)} \rightarrow \frac{A}{(x-3)}+\frac{B}{(x-2)} \rightarrow 5x-13=A(x-2)+B(x-3)$$

$$\boxed{\frac{2}{(x-3)}+\frac{3}{(x-2)}}$$

$x=3 \rightarrow 15-13=1A+0$

$2=1A \rightarrow \boxed{A=2}$

$x=2 \rightarrow 10-13=0-1B \rightarrow -3=-1B \rightarrow B=3$

1b)

$$\frac{x+4}{x^2+2x+1} \rightarrow \frac{(x+4)}{(x+1)^2} \rightarrow \frac{A}{(x+1)}+\frac{B}{(x+1)^2} \rightarrow x+4=A(x+1)+B \rightarrow \boxed{\frac{1}{(x+1)}+\frac{3}{(x+1)^2}}$$

LET $x=-1 \rightarrow 3=B$

LET $x=0 \rightarrow 4=A+3 \rightarrow A=1$

1c)

$$\frac{1}{x^3-x} \rightarrow \frac{1}{x(x^2-1)} \rightarrow \frac{1}{x(x+1)(x-1)} \rightarrow \frac{A}{x}+\frac{B}{(x+1)}+\frac{C}{(x-1)} \rightarrow 1=A(x+1)(x-1)+Bx(x-1)+Cx(x+1) \rightarrow \boxed{-\frac{1}{x}+\frac{1}{2(x+1)}+\frac{1}{2(x-1)}}$$

LET $x=1 \rightarrow 1=0+0+2C \rightarrow C=\frac{1}{2}$

LET $x=-1 \rightarrow 1=0+2B+0 \rightarrow B=\frac{1}{2}$

LET $x=0$ $1=-A \rightarrow A=-1$

1d)

$$\frac{3x^3-2x^2+x-1}{x^4+x^2} \rightarrow \frac{3x^3-2x^2+x-1}{x^2(x^2+1)} \rightarrow \frac{A}{(x)}+\frac{B}{(x^2)}+\frac{Cx+D}{(x^2+1)} \rightarrow 3x^3-2x^2+x-1=Ax(x^2+1)+B(x^2+1)+Cx^3+Dx^2$$

$$\boxed{\frac{1}{x}-\frac{1}{x^2}+\frac{2(x)}{x^2+1}-\frac{1}{x^2+1}}$$

$3x^3-2x^2+x-1=Ax^3+Ax-x^2-1+Cx^3+Dx^2$

$3x^3-2x^2+x-1=(A+C)x^3+(D-1)x^2+Ax-1 \rightarrow A+C=3,\ D-1=-2,\ A=1$

LET $x=0 \rightarrow -1=B \rightarrow B=-1$ $\quad D=-1 \quad C=2$

LET $x=$

1e)

$$\frac{x^2-2x-1}{x^2-1} \rightarrow \frac{x^2-2x-1}{(x+1)(x-1)} \rightarrow \frac{A}{x+1}+\frac{B}{(x-1)} \rightarrow x^2-2x-1=A(x-1)+B(x+1)$$

$$x^2-1 \overline{\smash{)}\,\cancel{x^2}-2x-\cancel{1}} \quad 1 \qquad -(\cancel{x^2}\ \cancel{+1})$$

$$\rightarrow 1-\frac{2x}{x^2-1} \rightarrow 1-\frac{A}{(x+1)}+\frac{B}{(x-1)} \rightarrow -2x=A(x-1)+B(x+1) \rightarrow 1-\frac{1}{x+1}-\frac{1}{x-1}$$

LET $x=1 \rightarrow 2=2B \rightarrow \boxed{B=1}$

LET $x=-1$ $\quad -2=-2A \rightarrow \boxed{A=1}$

1f)

$$\frac{x^3+x-6}{x^2+3x} \rightarrow x^2+3x \overline{\smash{)}\,x^3+x-6} \quad x-3$$

$-(x^3+3x^2)$

$0-3x^2$

$-(-3x^2+9x)$

$-9x-6$

$$x-3-\frac{9x+6}{x^2+3x} \rightarrow \frac{9x+6}{x(x+3)} \rightarrow \frac{A}{x}+\frac{B}{x+3} \rightarrow 9x+6=A(x+3)+Bx$$

$x=0 \rightarrow 6=3A \rightarrow A=2$

$x=3 \rightarrow 33=3B \rightarrow B=11$

$$\frac{1}{x^3-x} \rightarrow \frac{1}{x(x^2-1)} \quad \frac{1}{x(x-1)(x+1)} \rightarrow \frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+1}$$

$$1 = A(x-1)(x+1)+B(x+1)(x)+C(x-1)(x)$$

$$\rightarrow A \rightarrow \quad 1=-A \rightarrow A=-1$$

$$-\frac{1}{x}+\frac{1}{2(x-1)}+\frac{1}{2(x+1)} \rightarrow -\int_2^3 \frac{1}{x}dx+\frac{1}{2}\int_2^3\frac{1}{x-1}dx+\frac{1}{2}\int_2^3\frac{1}{x+1}dx$$

$$\rightarrow -\ln|x|+\frac{1}{2}\ln|x-1|+\frac{1}{2}\ln|x+1| \rightarrow \overbrace{-\ln|3|}^{1}+\overbrace{\frac{1}{2}\ln|3-1|}^{2}+\overbrace{\frac{1}{2}|3+1|}^{3}+\overbrace{\ln(2)}^{2}-\cancelto{0}{\frac{1}{2}\ln(1)}\overbrace{-\frac{1}{2}\ln(3)}^{1}$$

$$\overbrace{-\frac{3}{2}\ln(3)}^{1}+\overbrace{\frac{3}{2}\ln(2)}^{2}+\overbrace{\ln(2)}^{3}$$

$$\rightarrow \boxed{-\frac{3}{2}\ln(3)+\frac{5}{2}\ln(2)}$$

$$\frac{x^3+x-6}{x^2+3x} \rightarrow \frac{x^3+x-6}{x(x+3)} \rightarrow$$

$$\begin{array}{r} x-3 \\ x^2+3x\,\overline{\big)\,x^3+x-6} \\ -(x^3+3x^2) \\ \hline 0-3x^2+x-6 \\ -(-3x-9x) \\ \hline 0 \quad 10x-6 \end{array}$$

$$\rightarrow x-3+\frac{10x-6}{x^2+3x} \rightarrow x-3+\frac{10x-6}{x(x+3)} \rightarrow \frac{A}{x}+\frac{B}{x+3}$$

$$10x-6=A(x+3)+B(x)$$

$$-6=3A - A=-2$$

$$-36=-3B \rightarrow B=12$$

$$x-3-\frac{2}{x}+\frac{12}{x-3}$$

$$\int x\,dx-3\int dx-2\int\frac{1}{x}dx+12\int\frac{1}{x+3}dx$$

$$\boxed{\frac{x^2}{2}-3x-2\ln\left(\frac{1}{x}\right)+12\ln(x+3)+C}$$

$$\int\frac{88t^2}{t^2+1} \rightarrow \begin{array}{r} 88 \\ t^2+1\,\overline{\big)\,88t^2} \\ -\ 88t^2-88 \\ \hline -88 \end{array} \rightarrow 88-\frac{88}{t^2}dt \rightarrow 88\int dt-88\int\frac{1}{t^2+1}dt$$

$$\int\frac{1}{t^2+1}dt \rightarrow \int\frac{\sec^2(\theta)d\theta}{\tan^2\theta+1} \rightarrow \int\frac{\sec^2\theta\, d\theta}{\sec^2\theta\, d\theta} \rightarrow \theta=\tan^{-1}(x)$$

Let $x=\tan(\theta)$
$dx=\sec^2(\theta)d\theta$

$$88t-88\tan^{-1}(t)\Big|_0^5$$

$$88(5)-88\tan^{-1}(5)-0+88\tan^{-1}(0)$$

$$\boxed{440-88\tan^{-1}(5)}$$

3a $\frac{x}{x^2-9} \to \frac{x}{(x-3)(x+3)} \to$ $A(x+3)+B(x-3)$ $\to$

3b $\to \frac{2x-4}{x^2-4x-5} = \frac{2x-4}{(x-5)(x+1)} \to \frac{A}{x-5} + \frac{B}{x+1}$

$2x-4 = A(x+1) + B(x-5)$

$x=-1 \to -6 = -6B \to B=1$
$x=5 \to 6 = 6A \to A=1$

HW 5: 1D

$\frac{3x^3-2x^2+x-1}{x^4+x^2} \to \frac{3x^3-2x^2+x-1}{x^2(x^2+1)} \to \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$

$3x^3-2x^2+x-1 = A(x)(x^2+1) + B(x^2+1) + (Cx+D)(x^2)$
$C0+D(0+1)$

HOW TO TELL IF QUADRATIC FACTORS:

ax^2+bx+c

$Ax^3+Ax-x^2-1+Cx^3+Dx^2 \to (A+C)x^3+(D-1)x^2+Ax-1 = 3x^3-2x^2+x-1$

$x=0 \to \boxed{-1=B}$

$A=1$ $D=1$ $C=2$

$\int \frac{1}{x} - \frac{1}{x^2} + \frac{2x+1}{x^2+1}$

$D-1=2 \to D=1$

$1+C=3$

$\ln|x| + \frac{1}{x} + \int \frac{2x+1}{x^2+1}$

$\int_2^3 \frac{1}{x^3-x} dx \to \int_2^3 \frac{1}{x(x^2-1)} = \frac{1}{x(x-1)(x+1)} \to \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1} \to$ $1 = A(x-1)(x+1) + B(x)(x+1) + C(x-1)x$

$\int -\frac{1}{x} + \frac{1}{2(x-1)} + \frac{1}{2(x+1)}$

$x=0 \to$ $1=-A \to A=-1$

$x=1 \to$ $1=0+2B \to B=\frac{1}{2}$

$\to -\ln|x| \frac{1}{2}\ln|x-1| + \frac{1}{2}\ln|x+1|$ $x=-1 \to 1=0+0+2C \to C=\frac{1}{2}$