P 1. *Compute* $\mathbf{u}+\mathbf{v}+\mathbf{w}$ *and* $2\mathbf{u}+2\mathbf{v}+\mathbf{w}$, *where* $\mathbf{u}=\begin{bmatrix}1\\2\\3\end{bmatrix}$, $\mathbf{v}=\begin{bmatrix}-3\\1\\-2\end{bmatrix}$, *and* $\mathbf{w}=\begin{bmatrix}2\\-3\\-1\end{bmatrix}$.

How do you know $\mathbf{u}$ *and* $\mathbf{v}$ *and* $\mathbf{w}$ *lie in a plane.*

$$u + v + w = \begin{bmatrix} u_1 + v_1 + w_1 \\ u_2 + v_2 + w_2 \\ u_3 + v_3 + w_3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 + (-3) + 2 \\ 2 + 1 + (-3) \\ 3 + (-2) + (-1) \end{bmatrix} \rightarrow \boxed{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

WE KNOW U+V+W LIE ON A PLANE DUE TO THE FACT THAT THEY EQUAL A CONSTAND WHEN ADDED (0 IN THIS CASE)

P 2. *Given* $\mathbf{v}=(1,-2,1)$ *and* $\mathbf{w}=(0,1,-1)$. *Find* c *and* d, *so that* $c\mathbf{v}+d\mathbf{w}=(3,3,-6)$. *Why is* $(3,3,6)$ *impossible?*

$cv + dw = (3,3,-6) \rightarrow c(1,-2,1) + d(0,1,-1) = (3,3,-6)$

$\rightarrow [(c + 0) + (-2c + d) + (c + (-d))] = (3,3,-6) \rightarrow \boxed{c = 3}$

$\rightarrow -2(3) + d = 3 \rightarrow -6 + d = 3 \rightarrow \boxed{d = 9}$

$$\begin{aligned} c &= 3 \\ -2c + d &= 3 \\ c - d &= -6 \end{aligned}$$

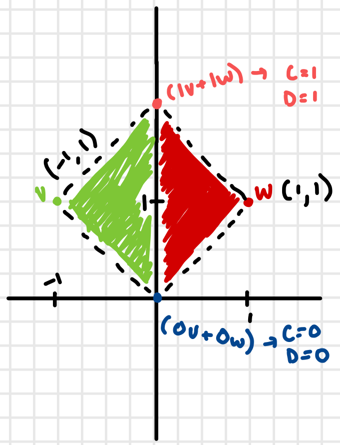
WORK CHECKING:

1. $c - d = -6 \rightarrow 3 - 9 = -6$ ✓
2. $3v + 9w = (3,3,-6) \rightarrow 3(1,-2,1) + 9(0,1,-1) = (3,3,-6)$

$$\rightarrow \begin{bmatrix} 3(1) + 9(0) \\ 3(-2) + 9(1) \\ 3(1) + 9(-1) \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix}$$ ✓

IMPOSSIBLE BECAUSE WE HAVE CONFIRMED THE VALUE OF C & D AND C - D WILL NEVER = -6

P 3. *Restricted by* $0 \le c \le 1$, *and* $0 \le d \le 1$, *shade in all combinations* $c\mathbf{v}+d\mathbf{w}$ *with* $\mathbf{v} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ *and* $\mathbf{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.



P 4. *What combination* $c \begin{bmatrix} 1 \\ 2 \end{bmatrix} + d \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ *produces* $\begin{bmatrix} 14 \\ 8 \end{bmatrix}$? *Express this question as two equations for the coefficients* c *and* d.

$$c(1,2) + d(3,1) = (14,8) \rightarrow \begin{cases} c+3d = 14 \rightarrow c = 14-3d \rightarrow c = 14-3(8-2c) \rightarrow c = 14-24+6c \\ \rightarrow 5c = 10 \rightarrow c = 2 \\ 2c + d = 8 \quad d = 8-2c \rightarrow d = 8-2(2) \rightarrow d = 4 \end{cases}$$

NOW THAT WE KNOW C=2

CHECK: $2(1,2) + 4(3,1) = (14,8) \rightarrow 2(1)+4(3) = 12$ ✓

$2(2)+4(1) = 8$ ✓

P 5. *Denote the vectors* $\mathbf{u} = \begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$, *and* $\mathbf{w} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. *Calculate the dot product* $\mathbf{u} \cdot \mathbf{v}$ *and* $\mathbf{u} \cdot \mathbf{w}$ *and* $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})$ *and* $\mathbf{w} \cdot \mathbf{v}$. *Compute the lengths and* $\|\mathbf{u}\|$, $\|\mathbf{v}\|$ *and* $\|\mathbf{w}\|$ *of those vectors. Check the Schwartz inequality* $|\mathbf{u} \cdot \mathbf{v}| \leq \|\mathbf{u}\|\|\mathbf{v}\|$ *and* $|\mathbf{v} \cdot \mathbf{w}| \leq \|\mathbf{v}\|\|\mathbf{w}\|$.

$u \cdot v \rightarrow u_1v_1 + u_2v_2 \rightarrow (-0.6, 0.8) \cdot (4, 3) \rightarrow (-.6 \times 4) + (.8 \times 3) \rightarrow -2.4 + 2.4 = 0$

$u \cdot w \rightarrow (-.6, 0.8) \cdot (1, 2) \rightarrow (-.6 \times 1) + (.8 \times 2) \rightarrow -.6 + 1.6 = 1$

$u \cdot (v + w) \rightarrow (-.6, .8) \cdot [(4,3) + (1,2)] \rightarrow (-.6, .8) \cdot (5, 5) \rightarrow 3 + 4 = 7$

$\|u\| = \sqrt{u \cdot u} \rightarrow \sqrt{(-.6, .8) \cdot (-.6, .8)} \rightarrow \sqrt{.36 + .64} \rightarrow \sqrt{1} = 1$

$\|v\| = \sqrt{(4,3) \cdot (4,.3)} \rightarrow \sqrt{16 + 9} \rightarrow \sqrt{25} = 5$

$\|w\| = \sqrt{(1,2) \cdot (1.2)} \rightarrow \sqrt{1 + 4} \rightarrow \sqrt{5} = \sqrt{5}$

$|u \cdot v| \leq \|u\| \|v\| \rightarrow |0| \leq 1 \cdot 5 \rightarrow 0 < 5$ ✓

$|v \cdot w| \leq \|v\| \|w\| \rightarrow 1 \cdot 10 \leq 5 \cdot \sqrt{5}$ ✓

P 6. *If* $\|\mathbf{v}\| = 5$ *and* $\|\mathbf{w}\| = 3$, *what are the smallest and largest possible values of* $\|\mathbf{v} - \mathbf{w}\|$? *What are the smallest and largest possible values of* $\mathbf{v} \cdot \mathbf{w}$?

SMALLEST VALUE OF $\|v - w\|$ IS WHEN THEY ARE PARALLEL & HAVE THE SAME DIRECTION:

$5 - 3 = 2$

LARGEST VALUE OF $\|v - w\|$ IS WHEN THEY ARE PARALLEL & HAVE AN OPPOSITE DIRECTION:

$5 + 3 = 8$

SMALLEST & LARGEST VALUES OF $v \cdot w$:

REMINDER: $v \cdot w = \|v\| \cdot \|w\| \cdot \cos\theta$ (FORMULA FOR ANGLE BETWEEN TWO VECTORS)

$$\left.\begin{aligned} &-1 \leq \cos(\theta) \leq 1 \\ &\rightarrow 5 \cdot 3 \cdot (-1) = -15 \\ &\rightarrow 5 \cdot 3 \cdot (1) = 15 \end{aligned}\right\} \quad -15 \leq v \cdot w \leq 15$$