

INTRODUCTION

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-FROM COLUMBIA

3 EXAMS (INCLUDING FINAL)

2 MIDTERMS (50%)

1 FINAL (25%)

QUIZES (25%)

QUIZES ARE ON THURSDAYS

NO QUIZ THE FIRST WEEK

NO GRADING CURVES

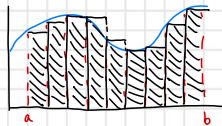
2 LOWEST QUIZES ARE DROPPED

REVIEW:

CONTINUOUS = NO HOLES

DEFINITION OF DEFINITE INTEGRAL

Let f be a continuous function on a closed interval $[a, b]$. Divide $[a, b]$ into n equal parts of width $\Delta x = \frac{b-a}{n}$. Let x_0, x_1, \dots, x_n be the endpoints of this subdivision.



THE DEFINITE INTEGRAL OF f FROM $x=a$ TO $x=b$ IS DENOTED BY $\int_a^b f(x) dx$ AND CAN BE DEFINED AS:

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$$

MORE RECTANGLES = CLOSER APPROX

WE CANNOT FIND INFINITE BUT WE CAN APPROXIMATE IT.

THE EVALUATION THEORY:

Let f be a cont. function on $[a, b]$ and let F be any antiderivative of f then:

$$\int_a^b f(x) dx = F(b) - F(a)$$

EXAMPLES:

- 1) USE THE RIGHT ENDPOINT DEF OF DEFINITE INTEGRAL TO WRITE THE INTEGRAL $\int_{-2}^2 (2x+2) dx$ AS THE LIMIT OF A SUM

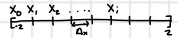
$$\Delta x = \frac{b-a}{n} \rightarrow \Delta x = \frac{2-(-2)}{n} \rightarrow \Delta x = \frac{4}{n}$$

$$f(x) = 2x+2 \rightarrow f(x_i) = 2x_i+2$$

$$x_i = -2 + i \Delta x \rightarrow -2 + i \left(\frac{4}{n}\right) \rightarrow -2 + \frac{4i}{n}$$

$$\int_{-2}^2 (2x+2) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n (2(x_i)+2) \Delta x \rightarrow \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(2 \left(-2 + \frac{4i}{n} \right) + 2 \right) \frac{4}{n}$$

$$\rightarrow \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(-2 + \frac{8i}{n} \right) \frac{4}{n} \rightarrow \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{32i}{n^2} - \frac{8}{n} \right)$$



$$x_0 = -2 = a$$

$$x_i = -2 + i \Delta x$$

$$x_n = -2 + 2 \Delta x$$

$$x_i = -2 + i \Delta x$$

- 2) USE EVALUATION THEORY OF PREVIOUS EXAMPLE:

$$\int_{-2}^2 (2x+2) dx = F(2) - F(-2)$$

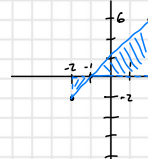
$$F(x) = x^2 + 2x$$

$$F(2) = 2^2 + 2(2) \rightarrow 8$$

$$F(-2) = (-2)^2 + 2(-2) \rightarrow 0$$

$$8 - 0 = 8$$

- 3) GEOMETRIC WAY:



FIND AREA OF TRIANGLES

$$\frac{6 \times 3}{2} = 9$$

$$\frac{1 \times 2}{2} = 1$$

$$9 - 1 = 8$$