JANUARY 27, 2020 (MONDAY)

## IMPROPER INTEGRALS

$$\boxed{\int_{-\infty}^{\infty} f(x)dx,\ \int_{a}^{\infty} f(x)dx,\ \int_{-\infty}^{a} f(x)dx}$$

$\int_a^b f(x)dx \rightarrow$ BUT $f(x)$ IS NOT DEFINED AT SOME POINT C OF $[a,b]$

### 1) DEFINING PRIMARY (INFINITY) FORMS

IF $\int_a^t f(x)dx$ EXISTS FOR EVERY $t$, $t \geq a$ THEN:

$$\int_a^{\infty} f(x)dx = \lim_{t\to\infty} \int_a^t f(x)dx$$

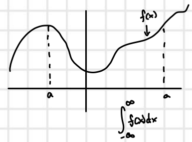
IF $\int_a^b f(x)dx$ FOR EVERY NUMBER T, $t \leq b$ THEN

$$\int_{-\infty}^{b} f(x)dx = \lim_{t\to-\infty} \int_t^b f(x)dx$$

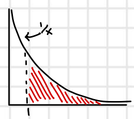
IF LIMIT EXISTS, WE SAY THAT THE INTERVAL CONVERGES. OTHERWISE, THE INTEGRAL DIVERGES OR IS DIVERGENTS.

IF BOTH $\int_{-\infty}^{a} f(x)dx$ & $\int_{a}^{\infty} f(x)dx$ ARE CONVERGENT, WHERE $A \in \mathbb{R}$ THEN:

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{a} f(x)dx + \int_{a}^{\infty} f(x)dx$$




Ex: EVALUATE $\int_1^{\infty} \frac{1}{x}dx \rightarrow \lim_{t\to\infty} \int_1^t \frac{1}{x}dx \rightarrow \lim_{t\to\infty} \ln x \Big|_1^t \rightarrow \infty$. MEANING THE INTEGRAL IS DIVERGENT.




Ex: EVALUATE $\int_1^{\infty} \frac{1}{x^2}dx \rightarrow \lim_{t\to\infty} \int_1^t \frac{1}{x^2}dx \rightarrow \lim_{t\to\infty} -\frac{1}{x}\Big|_1^t \rightarrow \lim_{t\to\infty}\left(-\frac{1}{t}+1\right) = 1 \rightarrow$ CONVERGENT

Ex: COMPUTE: $\int_0^{\infty} e^{-x}dx \rightarrow \lim_{t\to\infty} \int_0^t e^{-x}dx \rightarrow \lim_{t\to\infty}\left(-e^{-x}\Big|_0^t\right) \rightarrow 1 \rightarrow$ CONVERGENT

Ex: SUPPOSE $p$ IS A REAL NUMBER, FOR WHAT VALUE OF $p$ IS $\int_1^{\infty} \frac{1}{x^p}dx$ IS CONVERGENT.

PROOF $\int_1^{\infty} \frac{1}{x^p}dx = \lim_{t\to\infty} \int_1^t x^{-p}dx = \lim_{t\to\infty} \frac{x^{1-p}}{1-p}\Big|_1^t \rightarrow \lim_{t\to\infty}\left(\frac{t^{1-p}}{1-p} - \frac{1}{1-p}\right) \rightarrow \frac{1}{1-p}\lim_{t\to\infty}\left(t^{1-p}-1\right)$

$\left.\begin{array}{l}\text{CONVERGES WHEN } P>1 \\ \text{DIVERGES WHEN } P<1\end{array}\right\} \rightarrow \int_1^{\infty} \frac{1}{x^p}dx$

$\lim_{t\to\infty} t^{1-p}$

IF $P>1$, THEN $0>1-P$

$\lim_{t\to\infty} \frac{1}{t^{1-p}} = 0$

IF $P<1$, THEN $0<1-P$

$\lim_{t\to\infty} t^{1-p} = \infty$

DIRECT COMPARISON THEOREM:

SUPPOSE THAT $f$ & $g$ ARE CONTINUOUS FUNCTIONS WITH $f(x) \geq g(x) \geq 0$ FOR ALL $x \geq a$

1) IF $\int_a^{\infty} f(x)dx$ IS CONVERGENT, THEN $\int_a^{\infty} g(x)dx$ IS CONVERGENT.

2) IF $\int_a^{\infty} f(x)dx$ IS DIVERGENT, THEN $\int_a^{\infty} g(x)dx$ IS DIVERGENT.

Ex: DOES THE $\int_1^{\infty} \frac{1}{x^2+x+1}dx$ C OR D?

SOLUTION:

$g x = \frac{1}{x^2+x+1} \leq \frac{1}{x^2} \rightarrow f(x)$

BECAUSE $[1,\infty]$

$$\int_1^{\infty} \frac{1}{x^2+x+1}dx \leq \int_1^{\infty} \frac{1}{x^2}dx < \infty$$

THIS INTEGRAL CONVERGES.