DIRECT COMPARISON THEOREM
TRAPIZOIDAL RULE
TRIG IDENTITIES
TRIG ANTIDERIVATIVES
TRIG DERIVATIVES
ERROR ______ $\rightarrow E_T$

IMPROPER INTEGRALS CONT.

Ex. DOES THE INT $\int_3^\infty \frac{1}{x-1}dx$ CONVERGE OR DIVERGE?

NOT CONTINUOUS IN $\mathbb{R}$ BUT IS ON $[3,\infty)$

SOLUTION:

$f(x) = \frac{1}{x-1}$, YOU CAN CHOOSE

$g(x) \geq f(x) \geq 0$ SO THAT $\int_3^\infty g(x)dx$ CONVERGES

OR

$g(x)$ SO THAT $g(x) \leq f(x)$ and $\int_3^b g(x)dx$ DIVERGES.

CANDIDATES OF g

WE KNOW THAT $x \geq 3 \rightarrow$

$x-1 \rightarrow$ $x-1 \leq x$ WHENEVER $x \geq 3 \rightarrow \underbrace{\frac{1}{x}}_{g(x)} \leq \frac{1}{x-1}$ $x \geq 3$

$\infty = \int_3^\infty \frac{1}{x}dx \leq \int_3^\infty \frac{1}{x-1}dx \rightarrow$ CONVERGES BY DIRECT COMPARISON THEOREM

Ex: $\int_0^3 \frac{1}{x-1}dx$

$\int_0^3 \frac{1}{x-1}dx = \underbrace{\lim_{a\to1^-} \int_0^a \frac{1}{x-1}dx}_{\text{TRYING TO APPROACH ASYMPTOTE FROM THE LEFT}} + \underbrace{\lim_{a\to1^+} \int_a^3 \frac{1}{x-1}dx}_{\text{TRYING TO APPROACH THE ASYMPTOTE FROM THE RIGHT.}}$

$\rightarrow \lim_{a\to1^-} \ln|x-1| \Big|_0^a + \lim_{a\to1^+} \ln|x-1| \Big|_a^3$

$\rightarrow \lim_{a\to1^-} \ln|a-1| + \lim_{b\to1^-} (\ln|2| - \ln|b-1|)$

$= -\infty + \infty$ DIVERGES.

DEFINING INTEGRALS ON AN INTERVAL THAT CONTAINS A DISCONTINUITY

1) IF f IS CONT. ON $(a,b]$ AND DISCONT. AT a, then $\int_a^b f(x)dx = \lim_{t\to a^+} \int_t^b f(x)dx$

2) IF f IS CONT. ON $[a,b)$ AND DISCONT AT b, THEN $\int_a^b f(x)dx = \lim_{t\to b^-} \int_a^t f(x)dx$

3) IF f IS DISCONT at c in (a,b) and $\int_a^c f(x)dx$ and $\int_c^b f(x)dx$ ARE CONVERGENT THEN
$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

EXAMPLE: $\int_0^1 \frac{1}{\sqrt{x}}dx = \lim_{a\to0^+} \int_0^1 x^{-\frac{1}{2}}dx$

$\lim_{a\to0^+} 2x^{\frac{1}{2}} \Big|_0^1 \rightarrow \lim_{a\to0^+} (2-2\sqrt{a}) = \boxed{2}$

END OF EXAM 1 CONTENT

WED, JAN 29, 2020

WHAT VALUE OF n SHOULD BE USED TO GUARANTEE THAT AN ESTIMATE OF $\int_0^{?} e^{x^2} dx$ IS ACCURATE TO WITHIN 0.01 IF WE USE TRAPIZOIDAL RULE.

$$|E_T| = \left|\int_0^1 e^{x^2} dx - \text{TRAPIZOIDAL RULE}\right| \leq \frac{M(b-a)^2}{12n^2}$$

$b = 1$ $\qquad |f''(x)| \leq M$

$a = 0$ $\qquad f'(x) = 2xe^{x^2}$

$n = ?$ $\qquad f''(x) = 2e^{x^2} + 4x^2e^{x^2}$

$\qquad |f''(x)| = 2e^{x^2}(1+2x)$