HOMEWORK 7

1a

$$\int_0^{\infty} \frac{1}{x+1}\,dx \to \lim_{t\to\infty} \int_0^t \frac{1}{x+1}dx \to \lim_{t\to\infty} \ln|x+1| \Big|_0^t \to \lim_{t\to\infty} \ln|t+1| + \ln|1| \to \infty + 0$$

THIS INTEGRAL DIVERGES.

1b

$$\int_{-\infty}^{2} e^{2x}dx \to \lim_{t\to-\infty} \int_t^2 e^{2x}dx \to \lim_{t\to-\infty} \frac{1}{2}e^{2x}\Big|_t^2 \to \frac{1}{2}\left(\lim_{t\to-\infty}\left(e^4 - e^{2t}\right)\right) = \frac{1}{2}\left[\lim_{t\to-\infty} e^4 - \lim_{t\to-\infty} e^{2t}\right]$$

$$\lim_{t\to-\infty} e^4 \to e^4$$

$$\lim_{t\to-\infty} e^{2t} = 0$$

$$\frac{1}{2}\left[e^4 - 0\right] = \boxed{\frac{1}{2}e^4}$$

BECAUSE THERE IS A LIMIT, THE INTEGRAL DIVERGES.

1c

$$\int_0^2 \frac{1}{x^3}dx \to \lim_{a\to 0} \int_0^a \frac{1}{x^3}dx \to \lim_{a\to 0} \ln|x^3| \Big|_0^a \to \infty + 0 = \text{DIVERGES}$$

1d

$$\int_{-1}^{7} \frac{1}{\sqrt[3]{x+1}} \to \lim_{t\to-1} \int_t^7 (x+1)^{-\frac{1}{3}}dx \to \lim_{t\to-1} \int_t^7 u^{-\frac{1}{3}}dx \to \lim_{t\to-1} \frac{u^{2/3}}{\frac{2}{3}} \to$$

$u = (x+1)$

$du = 1dx$

$dx = du$

$$\to \lim_{t\to-1} \frac{3u^{2/3}}{2}\Big|_t^7 \to \lim_{t\to-1} \frac{3(x+1)^{2/3}}{2}\Big|_t^7 \to \frac{3(8)^{2/3}}{2} - \lim_{t\to-1} \frac{3(t+1)^{2/3}}{2}$$

$$\frac{3(4)}{2} - 0 = \boxed{6}$$

CONVERGES

1e

$$\int_{-\infty}^{2} e^{2x}\,dx \to \lim_{t\to-\infty} \int_t^2 e^{2x}dx \to \lim_{t\to\infty} \int_t^2 \frac{e^u}{2}du \to \lim_{t=-\infty} \frac{1}{2}\int_t^2 e^u du \to \lim_{t=-\infty} \frac{e^{2x}}{2}\Big|_t^2$$

$u = 2x$

$du = 2dx$

$dx = \frac{du}{2}$

$$\frac{e^4}{2} - \left(\text{HOW DOES THIS EQUAL ZERO?}\right)$$

1f

$$\int_{-\infty}^{\infty} \frac{4}{x^2+4} \to \int_{-\infty}^{\infty}$$

WE WANT TO CHOOSE AN INTERVAL SHORTER THAN -∞-∞

$$\int_{-\infty}^{k} \frac{4}{x^2+4}dk + \int_{k}^{\infty} \frac{4}{x^2+4}dk \to \lim_{a\to\infty} \int_a^k \frac{1}{x^2+4}dk + \lim_{b\to\infty} \int_k^b \frac{1}{x^2+4} \longrightarrow \int \frac{1}{x^2+4} \to \int \frac{2\sec^2\theta\, d\theta}{4\tan^2\theta+4} \to \frac{1}{2}\int \frac{\sec^2\theta}{\tan^2\theta+1}$$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$$\to \frac{1}{2}\int \frac{\sec^2\theta}{\sec^2\theta} \to \frac{1}{2}\int d\theta = \frac{1}{2}\theta = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

$$\lim_{a\to\infty} 4\left(\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\Big]_a^k\right) + \lim_{b\to\infty} 4\left(\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\Big]_k^b\right)$$

$k = 0$

$$\lim_{a\to-\infty} -2\tan\left(\frac{a}{2}\right) + \lim_{b\to\infty} 2\tan^{-1}\left(\frac{b}{2}\right)$$

$$-2\left(\frac{-\pi}{2}\right) + 2\left(\frac{\pi}{2}\right) = \boxed{2\pi}$$

1g

$$\int_{-\infty}^{\infty} \frac{2x}{(x^2+1)^2}dx$$

1h

$$\int_{-32}^{32} \frac{1}{(\sqrt[5]{x})^2}dx$$

HOMEWORK 7 (2/2)

1i

$\int_{-\infty}^{0} xe^{x}\,dx$

1j

$\int_0^1 2x\ln x\,dx \to \lim_{t\to0} 2\int_t^1 x\ln(x) \to 2\lim_{t\to0} \frac{x^2\ln(x)}{2} - \frac{1}{2}\int_t^1 x\,dx = 2\lim_{t\to0} \frac{x^2\ln(x)}{2} - \frac{1}{2}\left(\frac{x^2}{2}\right)\Big]_t^1$

$u = \ln(x)$ $\quad dv = x$

$du = \frac{1}{x}dx$ $\quad v = \frac{x^2}{2}$

$\to 2\lim_{t\to0} \frac{x^2\ln(x)}{2} - \frac{x^2}{4}\Big)_t^1 \to 2\lim_{t\to0} -\frac{1}{4} - \frac{t^2\ln(t)}{2} + \frac{t^2}{4}$

$\lim_{t\to0} t^2\ln(t) \to \frac{\ln(t)}{t^{-2}} = \frac{\infty}{\infty}$

LOPITALS THEOREM

1k

$\int_0^{\frac{\pi}{2}} \tan x\,dx$

1l

$\int_2^{\infty} \frac{1}{x\ln x}dx \to \lim_{t\to\infty} \int_2^t \frac{1}{x\ln(x)} \to \int_2^t \frac{1}{u}du \to \lim_{t\to\infty} \ln|u|\Big|_2^t \to \lim_{t\to\infty} \ln|\ln(x)|\Big|_2^t$

$u = \ln(x)$

$du = \frac{1}{x}dx$

$\to \ln|\ln(t)| - \ln|\ln(2)|$

$\infty - \ln|\ln(2)| = \boxed{\text{DIVERGES}}$

INTEGRATION BY PARTS

TRIG SUBSTITUTION

LOPITALS THEOREM

$\int u\,dv = uv - \int v\,du$

$x \geq 0$ $\quad$ all $n \in \mathbb{Z}$

$\int x^n \ln(x)\,dx \stackrel{?}{=} \frac{1}{n+1}x^{n+1}\ln(x) - \frac{1}{(x+1)^2}x^{n+1} + C$

$F(x) =$